# Series 5 - Solutions

TAs: *Cemre Celikbudak, Soroush Rafiei, Sokratis Anagnostopoulos, Ramin Mohammadi, Ellen Jamil Dagher, Veronika Pak, El Ghali Jaidi, Coline Jeanne Leteurtre*

5.10

**5.10** Air flows steadily between two cross sections in a long, straight section of 0.1-m inside diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.10. If the average air velocity at section (1) is 205 m/s, determine the average air velocity at section (2).

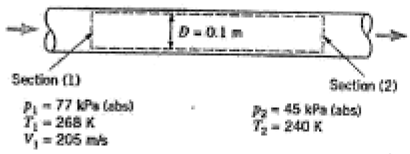



FIGURE P5.10

This analysis is similar to the one of Example 5.2.
For steady flow between sections (1) and (2)

$$\dot{m}_2 = \dot{m}_1$$

or

$$\rho_2 A_2 \bar{V}_2 = \rho_1 A_1 \bar{V}_1$$

Thus

$$\bar{V}_2 = \frac{\rho_1}{\rho_2}\frac{A_1}{A_2}\bar{V}_1 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.8) to get

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}\frac{T_2}{T_1} \qquad (2)$$

Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\bar{V}_2 = \frac{p_1}{p_2}\frac{T_2}{T_1}\bar{V}_1 = \frac{[77\ kPa(abs)](240K)}{[45\ kPa(abs)](268K)}\left(205\ \frac{m}{s}\right)$$

$$\bar{V}_2 = 314\ \frac{m}{s}$$

5-10

**5.23**

**5.23** As shown in Fig. P5.23 at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity $V$. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where $u$ is in ft/s and $y$ is in ft. Determine the value of $V$.

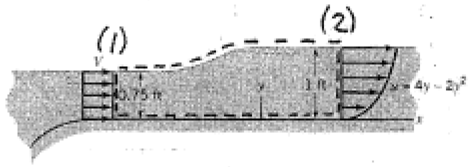



FIGURE P5.23

Use the control volume indicated by the broken lines in the sketch above.

From the conservation of mass principle

$$Q_1 = Q_2$$

$$V_1 A_1 = \int_{A_2} u\,dA \quad \int_0^{1\,ft} (4y - 2y^2)\, b\, dy$$

$$V(0.75\,ft)\,b = 3\left[\frac{4y^2}{2} - \frac{2y^3}{3}\right]_0^{1\,ft} b = \frac{4b}{3}\,\frac{ft^3}{s}$$

$$V = \frac{4}{3(0.75)} = \underline{\underline{1.78}}\ \frac{ft}{s}$$

5-24

5.36

5.36 A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in Video V5.6 and Fig. P5.36. Determine the minimum volume flowrate needed to tip the block.

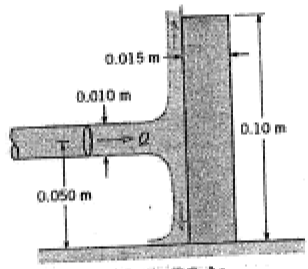
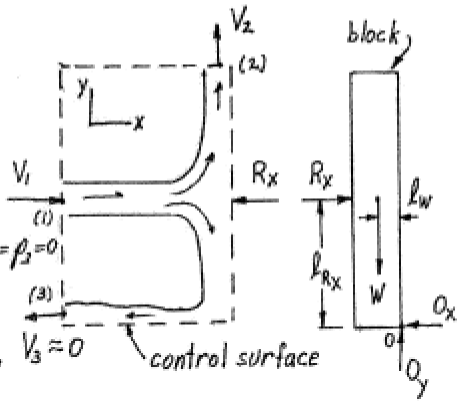



FIGURE P5.36




From the free body diagram of the block when it is ready to tip $\Sigma M_0 = 0$, or

$R_x \ell_{R_x} = W \ell_w$ where $R_x$ is the fore that the water puts on the block.

Thus,

$$R_x = \frac{W \ell_w}{\ell_{R_x}} = \frac{6N\left(\frac{0.015}{2}m\right)}{0.050m} = 0.90N$$

For the control volume shown the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \Sigma F_x$$

becomes

$$V_1 \rho (-V_1) A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

$$V_1 = \sqrt{\frac{0.9N}{\left(999 \frac{kg}{m^3}\right) \frac{\pi}{4} (0.01m)^2}} = 3.39 \frac{m}{s}$$

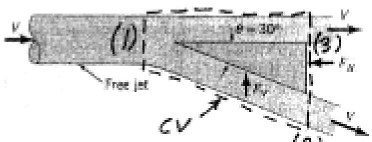
Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.01m)^2 \left(3.39 \frac{m}{s}\right) = \underline{\underline{2.66 \times 10^{-4} \frac{m^3}{s}}}$$

5-36

5.41

5.41 A free jet of fluid strikes a wedge as shown in Fig. P5.41. Of the total flow, a portion is deflected 30°; the remainder is not deflected. The horizontal and vertical components of force needed to hold the wedge stationary are $F_H$ and $F_V$, respectively. Gravity is negligible, and the fluid speed remains constant. Determine the force ratio, $F_H/F_V$.




■ FIGURE P5.41

The horizontal and vertical components of the linear momentum equation are applied to the contents of the control volume shown to get

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 + V_3 \cos 30° \rho V_3 A_3 = -F_H \quad (1)$$

$$-V_3 \sin 30° \rho V_3 A_3 = F_V \quad (2)$$

However $V_1 = V_2 = V_3 = V$ so eqs. (1) and (2) become

$$V^2 \rho \left(A_2 + A_3 \cos 30° - A_1\right) = -F_H$$

$$V^2 \rho A_3 \sin 30° = -F_V$$

and

$$\frac{F_H}{F_V} = \frac{A_2 + A_3 \cos 30° - A_1}{A_3 \sin 30°} \quad (3)$$

From conservation of mass we get

$$Q_1 = Q_2 + Q_3$$

or

$$A_1 V = A_2 V + A_3 V$$

and

$$A_1 = A_2 + A_3 \quad (4)$$

Combining Eqs. (3) and (4) we get

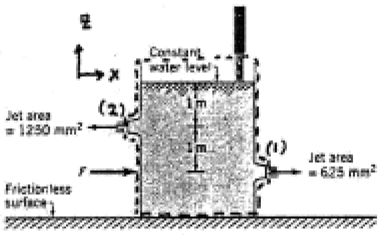
$$\frac{F_H}{F_V} = \frac{A_2 + A_3 \cos 30° - A_2 - A_3}{A_3 \sin 30°} = \frac{A_3 (\cos 30° - 1)}{A_3 \sin 30°} = \underline{\underline{-0.27}}$$

The negative sign indicates that $F_V$ is down rather than up as shown in the sketch.

5-42

5.48

5.48 Water is added to the tank shown in Fig. P5.48 through a vertical pipe to maintain a constant (water) level. The tank is placed on a horizontal plane which has a frictionless surface. Determine the horizontal force, $F$, required to hold the tank stationary. Neglect all losses.




■ FIGURE P5.48

Applying the x-direction component of the linear momentum equation to the contents of the control volume sketched above we get

$$V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F \qquad (1)$$

Using Bernoulli's equation to describe the frictionless flow from the constant water surface level to the flow leaving at stations (1) and (2) we obtain

$$V_2 = \sqrt{2 g h_2} \qquad (2)$$

and

$$V_1 = \sqrt{2 g h_1} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$F = 2 g h_1 \rho A_1 - 2 g h_2 \rho A_2$$

or

$$F = 2\left(9.81 \frac{m}{s^2}\right)\left(999 \frac{kg}{m^3}\right)\left[\frac{(2\ m)(625\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2} - \frac{(1\ m)(1250\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2}\right]$$

and

$$F = \underline{\underline{0}}\ N$$

5-49

5.53

**5.53** A vertical jet of water leaves a nozzle at a speed of 10 m/s and a diameter of 20 mm. It suspends a plate having a mass of 1.5 kg as indicated in Fig. P5.53. What is the vertical distance $h$?

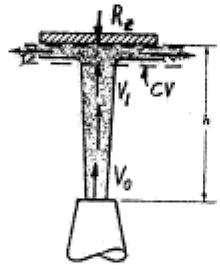



**FIGURE P5.53**

To determine the vertical distance $h$ we apply the vertical direction component of the linear momentum equation (Eq. 5.22) to the water in the control volume shown in the sketch above. Thus,

$$-R_z - \rho g \forall_{water} = -V_1 \rho A_1 V_1 = -\rho V_1^2 \frac{\pi}{4} D_1^2 \qquad (1)$$

The vertical reaction force of the plate on the water is equal in magnitude to the weight of the plate, or

$$R_z = g\, m_{plate} = (9.81 \tfrac{m}{s^2})(1.5\,kg) = 14.7\ N$$

Also, the weight of the water within the control volume, $\rho g \forall_{water}$, is negligible, and the mass flowrate is

$$\dot{m} = \rho A_1 V_1 = \rho A_0 V_0 = (999 \tfrac{kg}{m^3}) \tfrac{\pi}{4} (0.02\,m)^2 (10 \tfrac{m}{s}) = 3.13 \tfrac{kg}{s}$$

Thus, Eq 1 becomes

$$-14.7\ N = -V_1 \dot{m} \quad \text{or} \quad V_1 = \frac{14.7\ N}{3.13\ kg/s} = 4.70 \tfrac{m}{s}$$

From the Bernoulli Equation (Eq. 3.7) we have

$$p_0 + \tfrac{1}{2}\rho V_0^2 + \gamma z_0 = p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 \text{ , where } p_0 = p_1 = 0$$
$$z_0 = 0 \text{ , } z_1 = h$$

Thus,

$$\tfrac{1}{2}\rho V_0^2 = \tfrac{1}{2}\rho V_1^2 \quad h$$

or since $\gamma = \rho g$

$$h = \frac{1}{2g}(V_0^2 - V_1^2) = \frac{1}{2(9.81 \frac{m}{s^2})}(10^2 - 4.70^2)\frac{m^2}{s^2} = \underline{\underline{3.97\ m}}$$

5-55