Series 3 - Solutions

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3.12

3.12 Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in Fig. P3.12. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

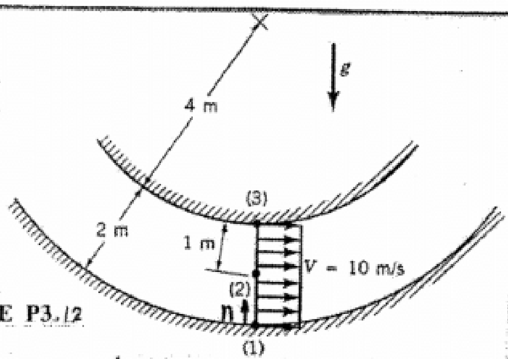


FIGURE P3.12

$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}}$ with $\frac{dz}{dn} = 1$ and $V = 10\,m/s$

Thus, with $\mathcal{R} = 6 - n$

$$\frac{dp}{dn} = -\gamma - \frac{\rho V^2}{6-n} \quad \text{or}$$

$$\int_{n=0}^{n} \frac{dp}{dn}\,dn = -\int_{n=0}^{n} \gamma\,dn - \int_{n=0}^{n} \frac{\rho V^2\,dn}{6-n}$$

so that since γ and V are constants

$$p - p_1 = -\gamma n - \rho V^2 \int_{n=0}^{n} \frac{dn}{6-n}$$

Thus,

$$p = p_1 - \gamma n - \rho V^2 \ln\left(\frac{6}{6-n}\right)$$

With $p_1 = 40\,kPa$ and $n_2 = 1\,m$: $p_2 = 40\,kPa - 9.8\times10^3\,\frac{N}{m^3}\,(1\,m) - 999\,\frac{kg}{m^3}\left(10\,\frac{m}{s}\right)^2 \ln\left(\frac{6}{5}\right)$

or

$$p_2 = \underline{\underline{12.0\,kPa}}$$

and

with $p_1 = 40\,kPa$ and $n_3 = 2\,m$: $p_3 = 40\,kPa - 9.80\times10^3\,\frac{N}{m^3}\,(2\,m) - 999\,\frac{kg}{m^3}\left(10\,\frac{m}{s}\right)^2 \ln\left(\frac{6}{4}\right)$

or

$$p_3 = \underline{\underline{-20.1\,kPa}}$$

3.65 Water flows steadily from a large, closed tank as shown in Fig. P3.65. The deflection in the mercury manometer is 2.5 cm and viscous effects are negligible. (a) Determine the volume flowrate. (b) Determine the air pressure in the space above the surface of the water in the tank.

(a) From the Bernoulli eqn.,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $V_2 = 0$, $z_1 = z_2$

Also, for the manometer,

$$p_2 + \gamma_{H_2O} h = p_1 + \gamma_{H_2O}(h - 2.5\text{ cm}) + \gamma_{Hg}(2.5\text{ cm})$$

or

$$p_2 - p_1 = (\gamma_{Hg} - \gamma_{H_2O})(0.025\text{ m}) = \gamma_{H_2O}(SG_{Hg} - 1)(0.025\text{ m})$$

$$= \left(9.796\ \frac{kN}{m^3}\right)(13.56 - 1)(0.025\text{ m})$$

$$= 3.08\ \frac{kN}{m^2} = 3.1\ \frac{kN}{m^2}$$

Thus, from Eq. (1)

$$\frac{V_1^2}{2g} = \frac{p_2 - p_1}{\gamma} = \frac{3.1\ \frac{kN}{m^2}}{9.796\ \frac{kN}{m^3}} = 0.32\text{ m}$$

so that

$$V_1 = \sqrt{2(9.81\text{ m/s}^2)(0.32\text{ m})} = 2.48\text{ m/s} = 2.5\text{ m/s}$$

Hence, $Q = A_1 V_1 = \frac{\pi}{4}(0.3\text{ m})^2 (2.5\ \frac{m}{s}) = \underline{\underline{0.18\ \frac{m^3}{s}}}$

3-69

(b) From the Bernoulli equation,

$$\frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3, \qquad (2)$$

where $V_4 = 0$, $p_3 = 0$, and $V_3 = \frac{Q}{A_3}$

Thus,

$$V_3 = \frac{0.18 \frac{m^3}{s}}{\frac{\pi}{4}(0.076 \ m)^2} = 39 \frac{m}{s}$$

Hence, from Eq. (2),

$$\frac{p_4}{\gamma} + z_4 = \frac{V_3^2}{2g} + z_3$$

or

$$p_4 = \frac{1}{2} \rho V_3^2 + \gamma (z_3 - z_4)$$

Hence,

$$p_4 = \frac{1}{2} \left(1001.21 \frac{kg}{m^3}\right) (39 \ m)^2 + 9.796 \frac{kN}{m^3} (-2.4 \ m)$$

$$= \underline{\underline{738 \frac{kN}{m^2}}}$$

3.69

3.69 Determine the flowrate through the pipe in Fig. P3.69.

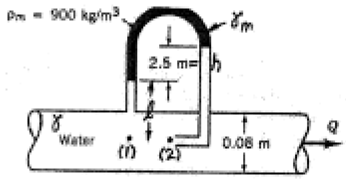


FIGURE P3.69

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } z_1=z_2 \text{ and } V_2=0$$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma} \quad \text{or} \quad V_1=\sqrt{2g\frac{(p_2-p_1)}{\gamma}}$$

but,

$$p_1-\gamma\ell-\gamma_m h+\gamma(\ell+h)=p_2 \quad \text{or} \quad p_2-p_1=(\gamma-\gamma_m)h$$

so that

$$V_1=\sqrt{2g\left(1-\frac{\gamma_m}{\gamma}\right)h}=\left[2\left(9.81\,\frac{m}{s^2}\right)\left(1-\frac{900\,\frac{kg}{m^3}}{999\,\frac{kg}{m^3}}\right)(2.5\,m)\right]^{1/2}$$

$$=2.20\,\frac{m}{s}$$

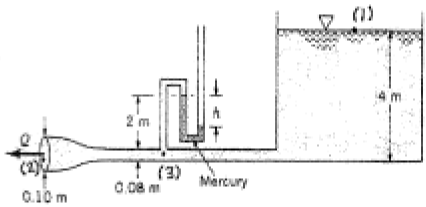
Thus,

$$Q=A_1V_1=\frac{\pi}{4}(0.08\,m)^2\left(2.20\,\frac{m}{s}\right)=\underline{\underline{0.0111\,\frac{m^3}{s}}}$$

3-74

3.76

3.76 Water flows steadily from the large open tank shown in Fig. P3.76. If viscous effects are negligible, determine (a) the flowrate, Q, and (b) the manometer reading, h.



■ FIGURE P3.76

(a) From the Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2, \text{ where } p_1 = p_2 = 0,\ V_1 = 0,\ z_1 = 4m, \text{ and } z_2 = 0.$$

Thus,

$$\gamma z_1 = \frac{1}{2}\rho V_2^2, \text{ or } \rho g z_1 = \frac{1}{2}\rho V_2^2 \text{ so that } V_2 = \sqrt{2 g z_1}$$

or

$$V_2 = \sqrt{2(9.81 m/s^2)(4m)} = 8.86\ m/s$$

Hence,

$$Q = A_2 V_2 = \frac{\pi}{4}(0.10m)^2(8.86 m/s) = \underline{\underline{0.0696\ m^3/s}}$$

(b) From the Bernoulli equation,

$$p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_3, \text{ where } z_2 = z_3 \text{ and } p_2 = 0$$

so that

$$p_3 = \frac{1}{2}\rho(V_2^2 - V_3^2)$$

Also, $A_2 V_2 = A_3 V_3$ so that $V_3 = \frac{A_2}{A_3} V_2 = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{0.1m}{0.08m}\right)^2 8.86 m/s = 13.84 m/s$

Hence,

$$p_3 = \frac{1}{2}(999\ kg/m^3)\left[(8.86 m/s)^2 - (13.84 m/s)^2\right] = -56{,}500\ N/m^2 \qquad (1)$$

Also, from the manometer,

$$p_3 = -\gamma_{Hg} h + \gamma_{H_2O}(2m + (0.08/2)m)$$
$$= -(133\times10^3 N/m^3)h + (9.80\times10^3 N/m^3)(2.04m)$$
$$= -133\times10^3 h + 1.99\times10^4\ N/m^2, \text{ where } h \sim m \qquad (2)$$

Thus, from Eqs. (1) and (2):

$$-5.65\times10^4\ N/m^2 = -133\times10^3 h + 1.99\times10^4\ N/m^2$$

or

$$h = \underline{\underline{0.574\ m}}$$

3-82

3.94

3.94 An ancient device for measuring time is shown in Fig. P3.94 The axisymmetric vessel is shaped so that the water level falls at a constant rate. Determine the shape of the vessel, $R = R(z)$, if the water level is to decrease at a rate of 0.10 m/hr and the drain hole is 5.0 mm in diameter. The device is to operate for 12 hr without needing refilling. Make a scale drawing of the shape of the vessel.

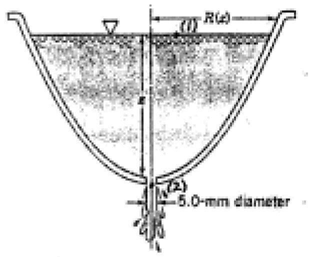


FIGURE P3.94

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ if the flow is assumed to be quasi-steady.

Also, $p_1 = 0$, $p_2 = 0$, $z_1 = z$, and $z_2 = 0$

Thus,

$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} + z$$ which, if $V_1 \ll V_2$ (i.e. $R \gg 5.0mm$), becomes

$$V_2 = \sqrt{2gz}$$

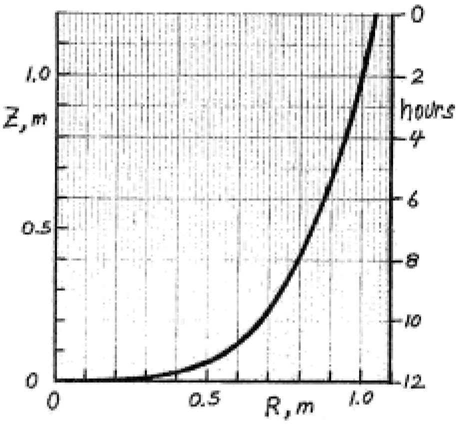
Since $A_1 V_1 = A_2 V_2$ and $V_1 = \left|\frac{dz}{dt}\right| = 0.1 \frac{m}{hr}\left(\frac{1 hr}{3,600 s}\right)$
we obtain $= 2.78 \times 10^{-5} \frac{m}{s}$

$$\pi R^2 (2.78 \times 10^{-5} \tfrac{m}{s}) = \frac{\pi}{4}(0.005m)^2 \sqrt{2(9.81 \tfrac{m}{s^2})\, z}\ ,$$
where R and z are ~m

Thus,

$$\underline{R = 0.998\, z^{1/4}}$$
or

z, m	R, m
0	0
0.1	0.561
0.2	0.667
0.4	0.794
0.6	0.878
0.8	0.944
1.0	0.998
1.2	1.045



3.107

3.107 Water flows from the pipe shown in Fig. P3.107 as a free jet and strikes a circular flat plate. The flow geometry shown is axisymmetrical. Determine the flowrate and the manometer reading, H.

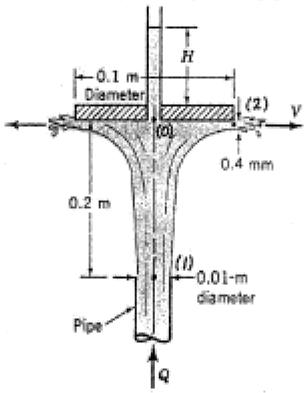


FIGURE P3.107

$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$, where $p_1=0$, $p_2=0$, $z_1=0$, and $z_2=0.2\,m$

Thus,

$\frac{V_1^2}{2g}=\frac{V_2^2}{2g}+z_2$ where $A_1V_1=A_2V_2=Q$ (1)

or $V_1=\frac{A_2}{A_1}V_2=\frac{\pi D_2 h}{\frac{\pi}{4}D_1^2}V_2=\frac{4D_2h}{D_1^2}V=\frac{4(0.1m)(4\times10^{-4}m)}{(0.01m)^2}V_2=1.6V_2$

Hence, Eq. (1) gives

$(1.60V_2)^2=V_2^2+2(9.81\frac{m}{s^2})(0.2m)$ or $V_2=1.59\frac{m}{s}$

so that

$Q=A_2V_2=\pi(0.1m)(4\times10^{-4}m)(1.59\frac{m}{s})=\underline{\underline{2.00\times10^{-4}\frac{m^3}{s}}}$

Also,

$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0$, where $V_0=0$, $z_0=0.2m$, $V_1=1.60\,V_2$ or $V_1=1.60(1.59\frac{m}{s})=2.54\frac{m}{s}$, and $p_1=0$

Thus,

$H=\frac{p_0}{\gamma}=\frac{V_1^2}{2g}-z_0=\frac{(2.54\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}-0.2m=\underline{\underline{0.129\ m}}$

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