Series 2 - Solutions

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2.4 Blood pressure is usually given as a ratio of the maximum pressure (systolic pressure) to the minimum pressure (diastolic pressure). As shown in Video V2.3, such pressures are commonly measured with a mercury manometer. A typical value for this ratio for a human would be 120/70, where the pressures are in mm Hg. (a) What would these pressures be in pascals? (b) If your car tire was inflated to 120 mm Hg, would it be sufficient for normal driving?

$$p = \gamma h$$

(a) For 120 mm Hg,

$$p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.120\ m)$$

$$p = \underline{16.0\ kPa}$$

For 70 mm Hg,

$$p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.070)$$

$$= \underline{9.31\ kPa}$$

(b) For 120 mm Hg, the value of p obtained was 16.0 kPa

Since a typical tire pressure is 200–250 kPa, 120 mm Hg is <u>not sufficient</u> for normal driving.

2.39

2.39 A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in Fig. P2.39. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure gage reading at A is 60 kPa, determine: (a) the pressure in pipe B, and (b) the pressure head, in millimeters of mercury, at the top of the dome (point C).

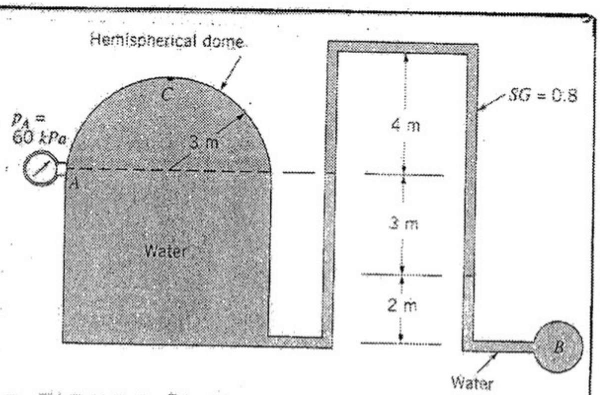


FIGURE P2.39

(a)
$$p_A + (SG)(\gamma_{H_2O})(3\ m) + \gamma_{H_2O}(2\ m) = p_B$$

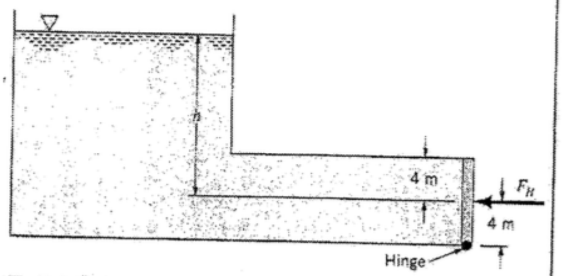
$$p_B = 60\ kPa + (0.8)\left(9.81 \times 10^3 \frac{N}{m^3}\right)(3\ m) + \left(9.80 \times 10^3 \frac{N}{m^3}\right)(2\ m)$$

$$= \underline{103\ kPa}$$

2-39 (con't)

(b)
$$p_C = p_A - \gamma_{H_2O}\,(3\,m)$$
$$= 60\,kPa - \left(9.80\times10^{3}\,\frac{N}{m^3}\right)(3\,m)$$
$$= 30.6\times10^{3}\,\frac{N}{m^2}$$
$$h = \frac{p_c}{\gamma_{Hg}} = \frac{30.6\times10^{3}\,\frac{N}{m^2}}{133\times10^{3}\,\frac{N}{m^3}} = 0.230\,m$$
$$= 0.230\,m\left(\frac{10^3\,mm}{m}\right) = \underline{\underline{230\,mm}}$$

2.93 A 3-m-wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Fig. P2.93. The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum value for F_H is 3500 kN. (a) Determine the maximum water depth, h, above the center of the gate that can exist without the gate opening. (b) Is the answer the same if the gate is hinged at the top? Explain your answer.



■ FIGURE P2.93

For gate hinged at <u>bottom</u>

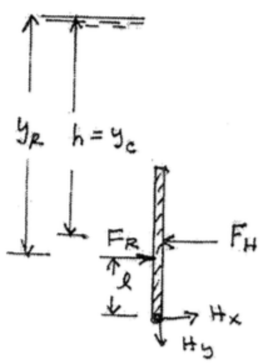
$$\sum M_H = 0$$

so that

$$(4\,m)\,F_H = \ell\,F_R \quad (\text{see figure}) \quad (1)$$

and

$$F_R = \gamma h_c A = \left(9.80\,\frac{kN}{m^3}\right)(h)(3\,m\times8\,m)$$
$$= (9.80\times24\,h)\,kN$$



$$\ell = h + 4 - y_R$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(3\,m)(8\,m)^3}{h\,(3\,m\times8\,m)} + h$$
$$= \frac{5.33}{h} + h$$

Thus,

$$\ell\,(m) = h + 4 - \left(\frac{5.33}{h} + h\right) = 4 - \frac{5.33}{h}$$

and from Eq. (1)

$$(4\,m)(3500\,kN) = \left(4 - \frac{5.33}{h}\right)(9.80\times24)(h)\ kN$$

so that

$$\underline{\underline{h = 16.2\,m}}$$

2-93 (con't)

For gate hinged at <u>top</u>

$\sum M_H = 0$

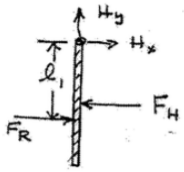
so that

$(4\,m)F_H = \ell_1 F_R$ (see figure) (1)

where

$\ell_1 = y_R - (h-4) = \left(\frac{5.33}{h} + h\right) - (h-4)$

$= \frac{5.33}{h} + 4$



$\ell_1 = y_R - (h-4)$

Thus, from Eq. (1)

$(4m)(3500\,kN) = \left(\frac{5.33}{h} + 4\right)(9.80 \times 24)(h)\ kN$

and

$$\underline{\underline{h = 13.5\,m}}$$

Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.

2.110 The concrete dam of Fig. P2.110 weighs 23.6 kN/m³ and rests on a solid foundation. Determine the minimum coefficient of friction between the dam and the foundation required to keep the dam from sliding at the water depth shown. Assume no fluid uplift pressure along the base. Base your analysis on a unit length of the dam.

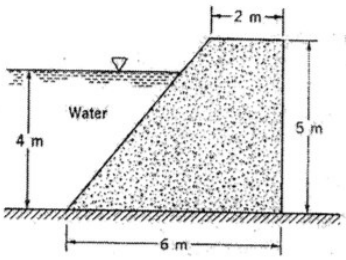
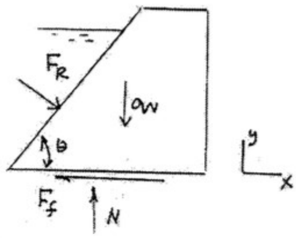


FIGURE P2.110

$F_R = \gamma h_c A$

where $A = \left(\frac{4\,m}{\sin 51.3°}\right)(1\ \ )$



$\tan\theta = \frac{5\,m}{4\,m}$

$\theta = 51.3°$

so that

$F_R = \left(9.80\,\frac{kN}{m^3}\right)\left(\frac{4\,m}{2}\right)\left(\frac{4\,m}{\sin 51.3°}\right)(1\,m)$

$= 100\,kN$

For equilibrium,

$\sum F_x = 0$

or

$F_R \sin 51.3° = F_f = \eta N$ where $\eta \sim$ coefficient of friction.

Also,

$\sum F_y = 0$

so that

$N = \mathcal{W} + F_R \cos 51.3°$ where

$\mathcal{W} = (\gamma_{concrete})(\text{volume of concrete})$

Thus,

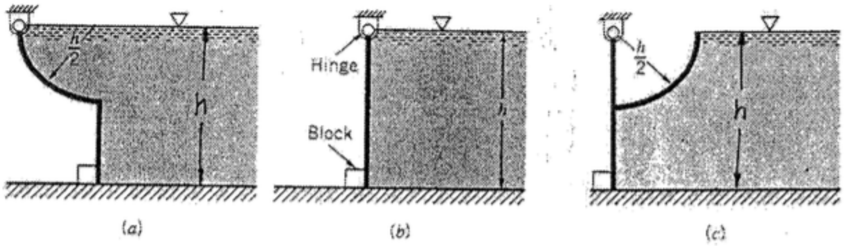
$N = \left(23.6\,\frac{kN}{m^3}\right)(20\,m^3) + (100\,kN)\cos 51.3° = 534\,kN$

and

$\eta = \frac{F_R \sin 51.3°}{N} = \frac{(100\,kN)\sin 51.3°}{534\,kN} = \underline{\underline{0.146}}$

2.131

2.131 Three gates of negligible weight are used to hold back water in a channel of width b as shown in Fig. P2.131. The force of the gate against the block for gate (b) is R. Determine (in terms of R) the force against the blocks for the other two gates.

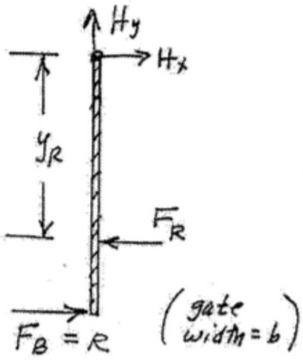


■ FIGURE P2.131

For case (b)

$$F_R = \gamma h_c A = \gamma \left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and $y_R = \frac{2}{3} h$



Thus,

$$\Sigma M_H = 0$$

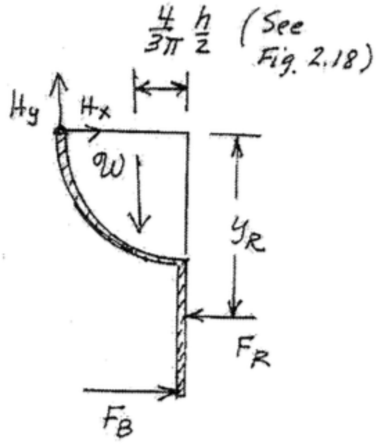
so that

$$h R = \left(\frac{2}{3} h\right) F_R$$

$$h R = \left(\frac{2}{3} h\right)\left(\frac{\gamma h^2 b}{2}\right)$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$

For case (a) on free-body-diagram shown



$F_R = \frac{\gamma h^2 b}{2}$ (from above) and

$y_R = \frac{2}{3} h$

and

$$\mathcal{W} = \gamma \times \text{Vol}$$

$$= \gamma \left[\frac{\pi \left(\frac{h}{2}\right)^2}{4}(b)\right]$$

$$= \frac{\pi \gamma h^2 b}{16}$$

Thus, $\Sigma M_H = 0$

so that

$$\mathcal{W}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3} h\right) = F_B\, h$$

and

$$\frac{\pi \gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3} h\right) = F_B\, h$$

2.131 (con't)

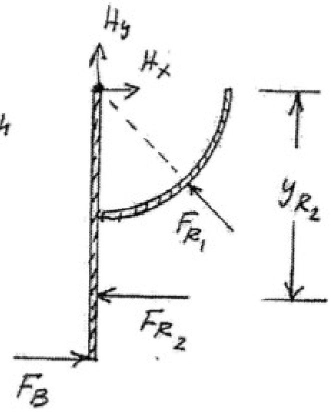
It follows that

$$F_B = \gamma h^2 b\,(0.390)$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \underline{\underline{1.17R}}$$

For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate



$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right) = \frac{3}{8}\gamma h^2 b$$

and

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus, $\Sigma M_H = 0$

so that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B\, h$$

or

$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

2.144

2.144 The thin-walled, 1-m-diameter tank of Fig. P2.144 is closed at one end and has a mass of 90 kg. The open end of the tank is lowered into the water and held in the position shown by a steel block having a density of 7840 kg/m³. Assume that the air that is trapped in the tank is compressed at a constant temperature. Determine: **(a)** the reading on the pressure gage at the top of the tank, and **(b)** the volume of the steel block.

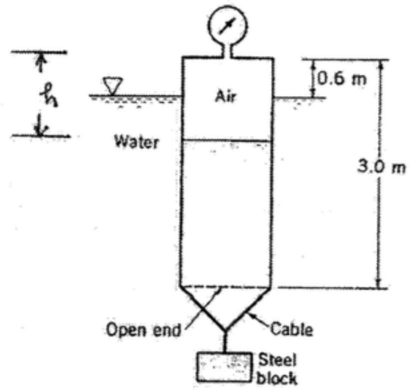


FIGURE P2.144

(a) For constant temperature compression,

$$p_i \forall_i = p_f \forall_f \quad \text{where } i \sim \text{initial state and } f \sim \text{final state.}$$

Let $\forall_f = A_t h$ (see figure) where A_t is the cross sectional area of tank, and

$$p_f = \gamma(h - 0.6) + p_{atm} \quad \text{(where all lengths are in m).} \qquad (1)$$

Thus,

$$\forall_f = A_t h = \frac{p_i \forall_i}{p_f}$$

Since $p_i = p_{atm}$ and $\forall_i = A_t(3)$

$$h = \frac{p_{atm}}{p_f}\frac{A_t(3)}{A_t} = \frac{3\,p_{atm}}{\gamma(h-0.6) + p_{atm}}$$

so that

$$h^2 + \left(\frac{p_{atm}}{\gamma} - 0.6\right)h - 3\frac{p_{atm}}{\gamma} = 0$$

For $\gamma = 9.80 \frac{kN}{m^3}$ and $p_{atm} = 101\,kPa$,

$$h^2 + \left(\frac{101\,kPa}{9.80\frac{kN}{m^3}} - 0.6\,m\right)h - \frac{3\,(101\,kPa)}{9.80\frac{kN}{m^3}} = 0$$

or

$$h^2 + 9.71\,h - 30.9 = 0$$

so that

$$h = \frac{-9.71 \pm \sqrt{(9.71)^2 + 4\,(30.9)}}{2} = 2.53\,m$$

Thus, from Eq. (1)

$$p_f\,(gage) = \left(9.80\frac{kN}{m^3}\right)(2.53\,m - 0.6\,m) = \underline{\underline{18.9\,kPa}}$$

(con't)

2.144 (con't)

(b) For equilibrium of tank (see free-body-diagram),

$$T = p_f A_t - \mathcal{W}_t$$

where $\mathcal{W}_t \sim$ tank weight, and for steel block

$$T = \mathcal{W}_s - F_{B_s} = \forall_s (\gamma_s - \gamma)$$

Thus,

$$\forall_s = \frac{T}{\gamma_s - \gamma} = \frac{p_f A_t - W_t}{\gamma_s - \gamma}$$

$$= \frac{\left(18.9 \times 10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(1\,m)^2 - (90\,kg)\left(9.81 \frac{m}{s^2}\right)}{\left(7.840 \times 10^3 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) - 9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.208\ m^3}}$$