Series 1 - Solutions

TAs: *Cemre Celikbudak, Soroush Rafiei, Sokratis Anagnostopoulos, Ramin Mohammadi, Ellen Jamil Dagher, Veronika Pak, El Ghali Jaidi, Coline Jeanne Leteurtre*

1.46

1.46 The density of oxygen contained in a tank is 2.0 kg/m³ when the temperature is 25 °C. Determine the gage pressure of the gas if the atmospheric pressure is 97 kPa.

$$p = \rho R T = \left(2.0 \frac{kg}{m^3}\right)\left(259.8 \frac{J}{kg \cdot K}\right)\left[(25°C + 273)K\right]$$

$$= 155\ kPa\ (abs)$$

$$p\,(gage) = p_{abs} - p_{atm} = 155\ kPa - 97\ kPa = \underline{\underline{58\ kPa}}$$

1.63

1.63 For air at standard atmospheric pressure the values of the constants that appear in the Sutherland equation (Eq. 1.10) are $C = 1.458 \times 10^{-6}\ kg/(m \cdot s \cdot K^{1/2})$ and $S = 110.4$ K. Use these values to predict the viscosity of air at 10 °C and 90 °C and compare with values given in Table B.4 in Appendix B.

$$\mu = \frac{C T^{\frac{3}{2}}}{T + S} = \frac{\left(1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}}\right) T^{\frac{3}{2}}}{T + 110.4\ K}$$

For $T = 10°C = 10°C + 273.15 = 283.15\ K$,

$$\mu = \frac{(1.458 \times 10^{-6})(283.15\ K)^{3/2}}{283.15\ K + 110.4} = \underline{\underline{1.765 \times 10^{-5}\ \frac{N \cdot s}{m^2}}}$$

From Table B.4, $\mu = 1.76 \times 10^{-5}\ \frac{N \cdot s}{m^2}$

For $T = 90°C = 90°C + 273.15 = 363.15\ K$,

$$\mu = \frac{(1.458 \times 10^{-6})(363.15\ K)^{3/2}}{363.15\ K + 110.4} = \underline{\underline{2.13 \times 10^{-5}\ \frac{N \cdot s}{m^2}}}$$

From Table B.4, $\mu = 2.14 \times 10^{-5}\ \frac{N \cdot s}{m^2}$

1.69:

1.69 Two flat plates are oriented parallel above a fixed lower plate as shown in Fig. P1.69. The top plate, located a distance b above the fixed plate, is pulled along with speed V. The other thin plate is located a distance cb, where $0 < c < 1$, above the fixed plate. This plate moves with speed V_1, which is determined by the viscous shear forces imposed on it by the fluids on its top and bottom. The fluid on the top is twice as viscous as that on the bottom. Plot the ratio V_1/V as a function of c for $0 < c < 1$.

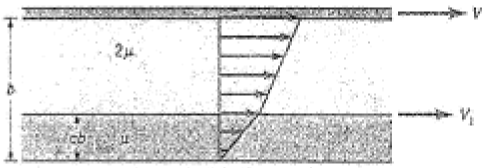


FIGURE P1.69

For constant speed, V_1, of the middle plate, the net force on the plate is 0. Hence, $F_{top} = F_{bottom}$, where $F = \tau A$. Thus, the shear stress on the top and bottom of the plate must be equal.

$$\tau_{top} = \tau_{bottom} \quad \text{where} \quad \tau = \mu \frac{du}{dy} \qquad (1)$$

For the bottom fluid $\frac{du}{dy} = \frac{V_1}{cb}$, while for the top fluid $\frac{du}{dy} = \frac{(V-V_1)}{b-cb}$

Hence, from Eqn. (1),

$$(2\mu)\frac{(V-V_1)}{b(1-c)} = (\mu)\frac{V_1}{cb}$$, which can be written as:

$$2cV - 2cV_1 = V_1 - cV_1$$

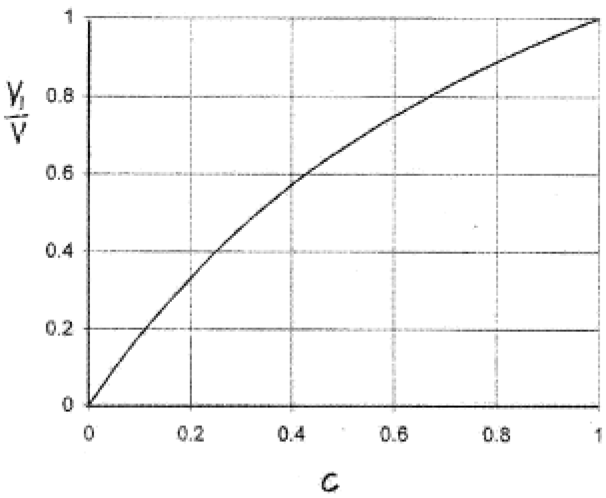
or

$$\frac{V_1}{V} = \frac{2c}{c+1}$$

Note: If $c = 0$, $\frac{V_1}{V} = 0$

If $c = \frac{1}{2}$, $\frac{V_1}{V} = \frac{2}{3}$

If $c = 1$, $\frac{V_1}{V} = 1$



1.72

1.72 A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Fig. P1.72 The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of 8.0×10^{-4} m²/s and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.

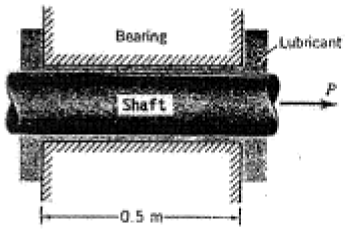
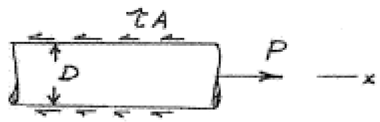


FIGURE P1.72



$$\sum F_x = 0$$

Thus, $P = \tau A$

where $A = \pi D \times (\text{shaft length in bearing}) = \pi D \ell$

and $\tau = \mu \dfrac{(\text{velocity of shaft})}{(\text{gap width})} = \mu \dfrac{V}{b}$

so that

$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell)$$

Since $\mu = \nu \rho = \nu (SG)(\rho_{H_2O @ 4°C})$,

$$P = \frac{(8.0 \times 10^{-4} \frac{m^2}{s})(0.91 \times 10^3 \frac{kg}{m^3})(3 \frac{m}{s})(\pi)(0.025m)(0.5m)}{(0.0003m)}$$

$$= \underline{\underline{286\ N}}$$

1.76 A thin layer of glycerin flows down an inclined, wide plate with the velocity distribution shown in Fig. P1.82. For $h = 0.75$ cm and $\alpha = 20°$, determine the surface velocity, U. Note that for equilibrium, the component of weight acting parallel to the plate surface must be balanced by the shearing force developed along the plate surface. In your analysis assume a unit plate width.

SOLUTION

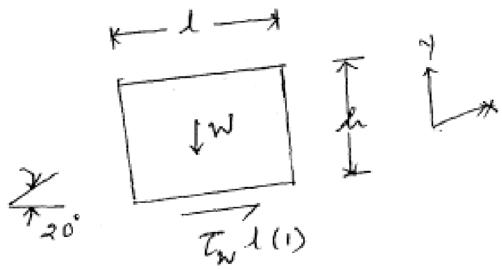
$$\Sigma F_x = 0$$

Thus,

$$W \sin 20° = \tau_w \, l \, (1)$$

and with $W = \gamma l h (1)$

$$\gamma l \, h(1) \sin 20° = \tau_w \, l(1)$$



or

$$\gamma h \sin 20° = \tau_w \longrightarrow (1)$$

At the plate

$$\tau_w = \mu \left(\frac{du}{dy}\right)_y = 0$$

$$\text{since} \quad \frac{du}{dy} = \frac{2V}{h} = \frac{2Vy}{h^2}$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2V}{h}$$

1.76 (cont'd)

Thus, from Eq. (1)

$$\gamma h \sin 20^\circ = \mu \frac{2V}{h}$$

and

$$V = \frac{\gamma h^2 \sin 20^\circ}{2\mu}$$! where γ=ρg (specific weight)

$$= \frac{\left(1259.18 \frac{kg}{m^3}\right)\left(\frac{0.75}{100}\right)^2 m^2 \sin 20^\circ *g}{2\,(0.62)\cdot\frac{N\,s}{m^2}}$$

$$= \frac{\left(12440 \frac{N}{m^3}\right)(0.0075)^2\, 0.34}{1.24}$$

$$V = 0.19 \text{ m/s}$$

For the above equation, the values for the physical properties of glycerin (γ=12440 N/m3 and μ=0.62 Pa*s) were taken from a handbook different than the Fluid Mechanics 7th ed book that we are following.
Measurements of viscosity can be made using different types of viscometers, which causes slight disagreements in the reported values.
Please note:
1. Density ρ=1259.18 kg/m3 still needs to be multiplied with g, in order to yield specific weight γ!
2. Following the Fluid Mechanics book, you will find
(Table 1.3) γ=12400 N/m3 and μ=1.5 Pa*s, which gives the result V=0.08 m/s .

1.81

1.81 The viscosity of liquids can be measured through the use of a *rotating cylinder viscometer* of the type illustrated in Fig. P1.81. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The torque $\mathcal{T}$ required to develop ω is measured and the viscosity is calculated from these two measurements. (a) Develop an equation relating μ, ω, $\mathcal{T}$, ℓ, R_o, and R_i. Neglect end effects and assume the velocity distribution in the gap is linear. (b) The following torque-angular velocity data were obtained with a rotating cylinder viscometer of the type discussed in part (a).

Torque (N-m)	17.8	35.3	53.6	71.5	89.9	106.5
Angular velocity (rad/s)	1.0	2.0	3.0	4.0	5.0	6.0

For this viscometer $R_o = 6.35$ cm, $R_i = 6.22$ cm and $\ell = 12.7$ cm. Make use of these data and a standard curve-fitting program to determine the viscosity of the liquid contained in the viscometer.

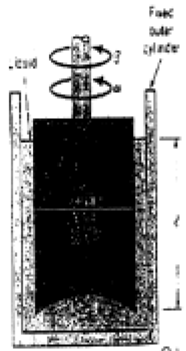
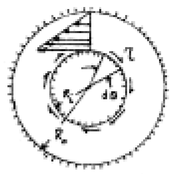


FIGURE P1.81



top view
(ℓ ~ cylinder length)

(a) Torque, $d\mathcal{T}$, due to shearing stress on inner cylinder is equal to

$$d\mathcal{T} = R_i \tau \, dA$$

where $dA = (R_i \, d\theta)\ell$. Thus,

$$d\mathcal{T} = R_i^2 \ell \tau \, d\theta$$

and torque required to rotate inner cylinder is

$$\mathcal{T} = R_i^2 \ell \tau \int_0^{2\pi} d\theta = 2\pi R_i^2 \ell \tau$$

(ℓ ~ cylinder length)

For a linear velocity distribution in the gap

$$\tau = \mu \frac{R_i \omega}{R_o - R_i}$$ so that

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu \omega}{R_o - R_i} \qquad (1)$$

(b) Thus, for a fixed geometry and a given viscosity, Eq (1) is of the form

$$y = bx \qquad (y \sim \mathcal{T} \text{ and } x \sim \omega)$$

where b is a constant equal to

$$b = \frac{2\pi R_i^3 \ell \mu}{R_o - R_i} \qquad (2)$$

To obtain b, fit the data to a linear equation of the form $y = bx$ using a standard curve-fitting program such as found in EXCEL.

1.81 cont

Thus, from Eq. (2)

$$\mu = \frac{(b)\ (R_o - R_i)}{2\pi R_i^3\ l}$$

and with the data given, $b = 17.69\ N{\cdot}m{\cdot}s$, so that

$$\mu = \frac{(17.69\ N{\cdot}m{\cdot}s)\ (0.0635\ m - 0.0622\ m)}{2\pi\ (0.0622\ m)^3\ (0.127\ m)}$$

and

$$\mu = 119.8\ \frac{N{\cdot}s}{m^2}$$