Series 12 - Solutions

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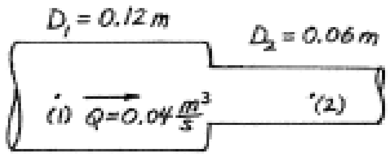
8.58

8.58 Water flows at a rate of $0.040\ m^3/s$ in a 0.12-m-diameter pipe that contains a sudden contraction to a 0.06-m-diameter pipe. Determine the pressure drop across the contraction section. How much of this pressure difference is due to losses and how much is due to kinetic energy changes?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + K_L\frac{V_2^2}{2g}, \text{ where } z_1 = z_2 \qquad (1)$$

and

$$V_1 = \frac{Q}{A_1} = \frac{0.04\frac{m^3}{s}}{\frac{\pi}{4}(0.12m)^2} = 3.54\frac{m}{s}, \quad V_2 = \frac{Q}{A_2} = \frac{0.04\frac{m^3}{s}}{\frac{\pi}{4}(0.06m)^2} = 14.1\frac{m}{s}$$

Thus, with $\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{0.06m}{0.12m}\right)^2 = 0.25$ we obtain from Fig. 8.30

$K_L = 0.40$

Hence, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\rho\left[K_L V_2^2 + V_2^2 - V_1^2\right] = \frac{1}{2}\left(999\frac{kg}{m^3}\right)\left[0.40\left(14.1\frac{m}{s}\right)^2 + \left(14.1\frac{m}{s}\right)^2 - \left(3.54\frac{m}{s}\right)^2\right]$$

or

$$p_1 - p_2 = 39.7\times10^3\frac{N}{m^2} + 93.0\times10^3\frac{N}{m^2} = \underline{\underline{133\ kPa}}$$

This represents a <u>39.7 kPa</u> drop from losses and a <u>93.0 kPa</u> drop due to an increase in kinetic energy.

8-61

8.79

8.79 Water at 10 °C is pumped from a lake as shown in Fig. P8.79. If the flowrate is 0.011 m^3/s, what is the maximum length inlet pipe, ℓ, that can be used without cavitation occurring?

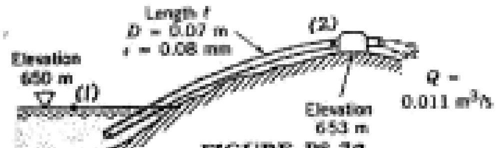


FIGURE P8.79

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}, \text{ where } p_1=101\text{ kPa},\ z_1=650\text{m} \quad (1)$$

$V_1=0,\ V_2=V,\ z_2=653\,m$, and from Table B.2 $p_2=p_v=1.228$ kPa

Also, $V=\frac{Q}{A}=\frac{0.011\frac{m^3}{s}}{\frac{\pi}{4}(0.07m)^2}=2.86\frac{m}{s}$ so that

$Re=\frac{VD}{\nu}=\frac{(2.86\frac{m}{s})(0.07m)}{1.307\times10^{-6}\frac{m^2}{s}}=1.53\times10^5$. With this Re and from Table 8.1 with

$\frac{\varepsilon}{D}=\frac{0.08\,mm}{70\,mm}=0.00114$ we obtain $f=0.0216$ (see Fig. 8.20)

Hence, with $\Sigma K_L=0.8$ for the entrance, Eq. (1) becomes

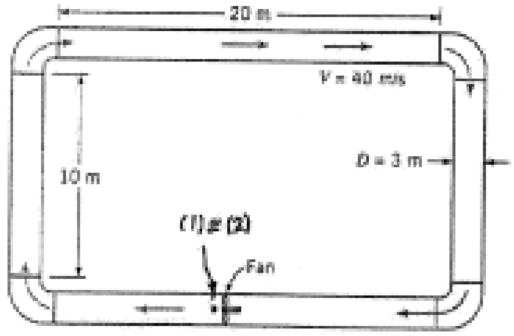
$$\frac{(101-1.228)\times10^3\frac{N}{m^2}}{9.80\times10^3\frac{N}{m^3}}+650\,m=653\,m+\left(1+(0.0216)\left(\frac{\ell}{0.07m}\right)+0.8\right)\frac{(2.86\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}$$

or $\ell=\underline{50.0\ m}$

8-88

8.88

8.88 A fan is to produce a constant air speed of 40 m/s throughout the pipe loop shown in Fig. P8.88. The 3-m-diameter pipes are smooth, and each of the four 90-degree elbows has a loss coefficient of 0.30. Determine the power that the fan adds to the air.



■ FIGURE P8.88

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_s = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

If we locate (1) and (2) at the same place it follows that

$p_1 = p_2$, $V_1 = V_2$, and $z_1 = z_2$.

Thus,

$$h_s = h_L = \left(f\frac{\ell}{D} + \sum_i K_{L_i}\right)\frac{V^2}{2g} \quad \text{where } \sum_i K_{L_i} = 4(0.30) = 1.2$$

Also, $Re = \frac{\rho V D}{\mu} = \frac{1.23\,\frac{kg}{m^3}(40\,\frac{m}{s})(3\,m)}{1.79\times10^{-5}\,\frac{N\cdot s}{m^2}} = 8.25\times10^6$

and $\frac{\varepsilon}{D} = 0$ so that from Fig. 8.20, $f = 0.0083$

Hence,

$$h_s = \left(0.0083\,\frac{(20+20+10+10)\,m}{3\,m} + 1.2\right)\frac{(40\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})} = 111\,m$$

so that

$$\dot{W}_s = \gamma Q h_s = \rho g Q h_s = (1.23\,\tfrac{kg}{m^3})(9.81\,\tfrac{m}{s^2})\left[\tfrac{\pi}{4}(3\,m)^2(40\,\tfrac{m}{s})\right]111\,m$$

$$= 3.79\times10^5\,\frac{N\cdot m}{s} = \underline{\underline{379\,kW}}$$

8-97

8.90

8.90 Water flows from the nozzle attached to the spray tank shown in Fig. P8.90. Determine the flowrate if the loss coefficient for the nozzle (based on upstream conditions) is 0.75 and the friction factor for the rough hose is 0.11.

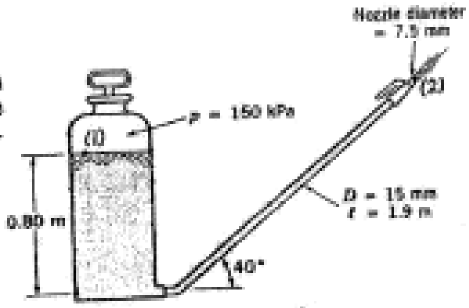


FIGURE P8.90

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + K_L\right)\frac{V^2}{2g}, \text{ where } p_1 = 150\,kPa,\ p_2 = 0, \quad (1)$$

$z_1 = 0.8\,m$, $z_2 = \ell \sin 40° = (1.9\,m)\sin 40° = 1.22\,m$, $V_1 = 0$,

$V = \frac{Q}{A}$, and $V_2 = \frac{Q}{A_2} = \left(\frac{A}{A_2}\right)V = \left(\frac{D}{D_2}\right)^2 V = \left(\frac{15\,mm}{7.5\,mm}\right)^2 V = 4V$

Thus, with $f = 0.11$ and $K_L = 0.75$ Eq. (1) gives

$$\frac{150 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}} + 0.8\,m = 1.22\,m + \left(4^2 + 0.11\left(\frac{1.9\,m}{0.015\,m}\right) + 0.75\right)\frac{V^2}{2(9.81\frac{m}{s^2})}$$

or

$V = 3.09\,\frac{m}{s}$

Thus, $Q = AV = \frac{\pi}{4}(0.015\,m)^2(3.09\,\frac{m}{s}) = \underline{\underline{5.46 \times 10^{-4}\,\frac{m^3}{s}}}$

8-99

8.94

8.94 The pump shown in Fig. P8.94 adds 25 kW to the water and causes a flowrate of 0.04 m³/s. Determine the flowrate expected if the pump is removed from the system. Assume $f = 0.016$ for either case and neglect minor losses.

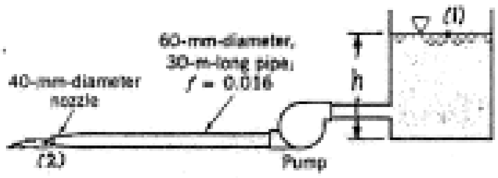


FIGURE P8.94

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}\text{, where } p_1=p_2=0,\ z_1=h,\ z_2=0$$

$$V_1=0,\quad V_2=\frac{Q}{A_2}=\frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.04\,m)^2}=31.8\,\frac{m}{s},\quad V=\frac{Q}{A}=\frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.06\,m)^2}=14.15\,\frac{m}{s}$$

Thus,

$$h+h_p=\frac{(31.8\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})}+0.016\left(\frac{30\,m}{0.06\,m}\right)\frac{(14.15\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})}=133.2\,m$$

but,

$$h_p=\frac{P}{\gamma Q}=\frac{2.5\times10^3\,\frac{N\cdot m}{s}}{(9.80\times10^3\,\frac{N}{m^3})(0.04\,\frac{m^3}{s})}=63.8\,m$$

Hence,

$$h=133.2\,m-63.8\,m=69.5\,m$$

Without the pump $h_p=0$ and $z_1=\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g}$ where $h=69.5\,m=z_1$

and

$$V_2=\frac{AV}{A_2}=\left(\frac{D}{D_2}\right)^2V\quad\text{or}\quad V_2=\left(\frac{60\,mm}{40\,mm}\right)^2V=2.25\,V$$

Thus,

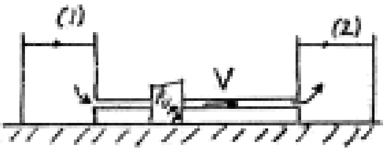
$$69.5\,m=\frac{(2.25V)^2+0.016\left(\frac{30\,m}{0.06\,m}\right)V^2}{2(9.81\,\frac{m}{s^2})}\quad\text{or } V=10.22\,\frac{m}{s}$$

so that

$$Q=AV=\frac{\pi}{4}(0.06\,m)^2(10.22\,\frac{m}{s})=\underline{\underline{0.0289\,\frac{m^3}{s}}}$$

8.101

8.101 Water is pumped between two large open reservoirs through 1.5 km of smooth pipe. The water surfaces in the two reservoirs are at the same elevation. When the pump adds 20 kW to the water the flowrate is 1 m³/s. If minor losses are negligible, determine the pipe diameter.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 = z_2$$

Thus,

(1) $h_s = h_L$ where $h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{20 \times 10^3\,N\cdot m/s}{(9.80 \times 10^3 \frac{N}{m^3})(1 \frac{m^3}{s})} = 2.04\,m$

and

$h_L = f \frac{\ell}{D}\frac{V^2}{2g}$ with $V = \frac{Q}{A} = \frac{1\,m^3/s}{\frac{\pi}{4}D^2} = \frac{1.273}{D^2}$ m/s with D~m

Hence,

(2) $h_L = f\,\frac{1.5 \times 10^3 m}{D}\;\frac{(1.23/D^2)^2\,m^2/s^2}{2\,(9.81\,m/s^2)} = 123.9\,f/D^5\ m$

From Eqs (1) and (2), $2.04 = 123.9\,f/D^5$ or $f = 0.0165\,D^5$

(3) $D = 2.27\,f^{1/5}$

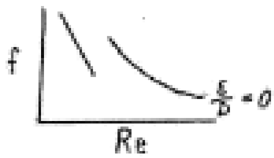
Also,

$Re = \frac{\rho V D}{\mu} = \frac{999 \frac{kg}{m^3}\,(1.273/D^2)m\;D\,m}{1.12 \times 10^{-3}\,\frac{N\cdot s}{m^2}}$ or

(4) $Re = 1.14 \times 10^6/D$

Finally, with $\varepsilon/D = 0$ the Moody chart (Fig. 8.20) is the final equation.

(5)



Trial and error solution of Eqs. (3), (4), and (5) for f, Re, and D:
Assume f = 0.02 so Eq (3) gives $D = 2.27\,(0.02)^{1/5} = 1.04\,m$ and Eq(4) gives $Re = 1.14\times10^6/1.04 = 1.10\times10^6$. Thus, from Eq (5), f = 0.0115 which is not equal to the assumed f = 0.02. Try again with f = 0.0115 which gives D = 0.931 m, $Re = 1.22\times10^4$, and $f = 0.0113 \neq 0.0115$. One final try with f = 0.0113 gives D = 0.927 m, $Re = 1.23\times10^4$, and f = 0.0113 as assumed. Thus, $\underline{\underline{D = 0.927\,m}}$.

An alternate method is to use the Colebrook formula (Eq (8.35)) rather than the Moody chart (Eq(5)). Thus, with $\varepsilon/D = 0$,

(con't)

8-112

8.101 (con't)

Eq(8.35) is

$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{Re\sqrt{f}}\right)$ which, when combined with Eqs. (3) and (4), gives

(6) $$\frac{1}{(0.0165\,D^5)^{1/2}} = -2.0 \log\left[\frac{2.51\,D}{1.14\times10^6(0.0165\,D^5)^{1/2}}\right]$$

Using a computer root-finding program to solve Eq.(6) gives $D = 0.926$, which is consistent with the trial and error solution given above.