Series 11 - Solutions

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***8.15**

*8.15 Some fluids behave as a non-Newtonian power-law fluid characterized by $\tau = -C(du/dr)^n$, where $n = 1, 3, 5$, and so on, and C is a constant. (If $n = 1$, the fluid is the customary Newtonian fluid.) (a) For flow in a round pipe of a diameter D, integrate the force balance equation (Eq. 8.3) to obtain the velocity profile

$$u(r) = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{1/n}\left[r^{(n+1)/n} - \left(\frac{D}{2}\right)^{(n+1)/n}\right]$$

(b) Plot the dimensionless velocity profile u/V_c, where V_c is the centerline velocity (at $r = 0$), as a function of the dimensionless radial coordinate $r/(D/2)$, where D is the pipe diameter. Consider values of $n = 1, 3, 5$, and 7.

(a) For any fluid $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ so that with $\tau = -C\left(\frac{du}{dr}\right)^n$ we obtain

$$\frac{\Delta p}{\ell} = -\frac{2C}{r}\left(\frac{du}{dr}\right)^n \quad \text{or} \quad \frac{du}{dr} = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}} r^{\frac{1}{n}} \quad *$$

or

$$-\int du = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}} \int r^{\frac{1}{n}} dr$$ which integrates to give

$$u = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}} \frac{n}{(n+1)} r^{\left(\frac{n+1}{n}\right)} + C_1 \quad \text{, where } C_1 \text{ is a constant.} \qquad (1)$$

The fluid sticks to the pipe so that $u = 0$ at $r = \frac{D}{2}$.

Hence, from Eq. (1)

$$C_1 = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}} \frac{n}{(n+1)} \left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}$$

so that

$$u = \frac{n}{(n+1)}\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\left[-r^{\left(\frac{n+1}{n}\right)} + \left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}\right]$$

*Note: Since we are considering only odd integer values for n we can use the fact that if

$$\left(\frac{du}{dr}\right)^n = -K \text{, where } K > 0 \text{, then } \frac{du}{dr} = -K^{\frac{1}{n}}$$

so that $\frac{du}{dr} < 0$. $K = \frac{\Delta P\, r}{2\, C\, l} > 0$, assuming C>0

(b) From part (a):

$$u(r) = \frac{n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left[-r^{\left(\frac{n+1}{n}\right)} + \left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}\right] \qquad (2)$$

Let $V_c = u(r=0)$, or $V_c = \frac{n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}$ (3)

Note: For $\tau = -C\left(\frac{du}{dr}\right)^n$ with $\frac{du}{dr} < 0$ and n an odd integer, to have $\tau > 0$, we must have $C > 0$. Thus, from Eq. (2), $V_c > 0$ as it must.

By dividing Eq. (2) by Eq. (3) we obtain

(con't)

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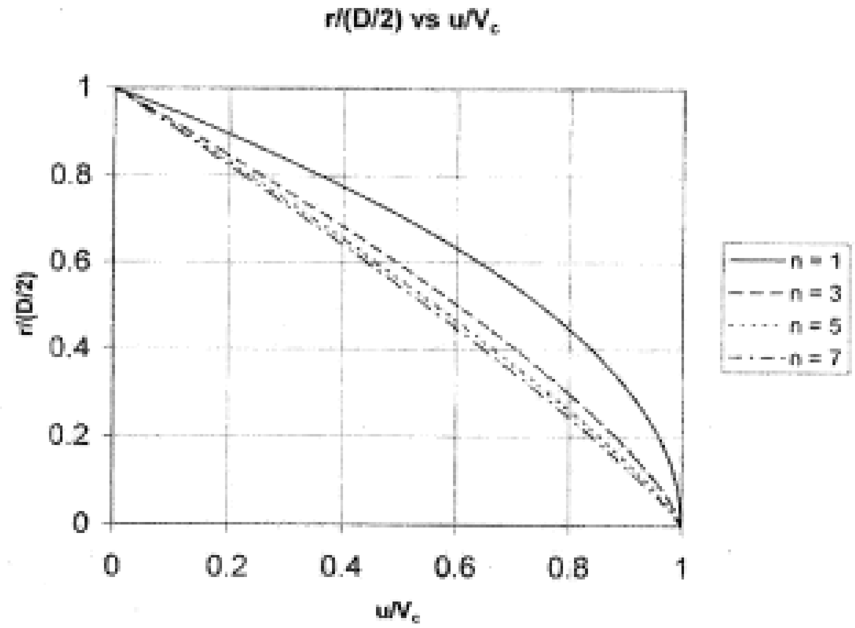
*8.15 (con't)

$$\frac{u}{V_c} = 1 - \left[\frac{r}{\left(\frac{D}{2}\right)}\right]^{\left(\frac{n+1}{n}\right)}$$

This result is plotted below for $n = 1, 3, 5,$ and 7, with $0 \le \frac{r}{\left(\frac{D}{2}\right)} \le 1$.

An EXCEL program was used to do the calculations and plotting.

	n = 1	n = 3	n = 5	n = 7
r/(D/2)	u/V_c	u/V_c	u/V_c	u/V_c
0	1	1	1	1
0.05	0.998	0.982	0.973	0.967
0.1	0.990	0.954	0.937	0.928
0.15	0.978	0.920	0.897	0.886
0.2	0.960	0.883	0.855	0.841
0.25	0.938	0.843	0.811	0.795
0.3	0.910	0.799	0.764	0.747
0.35	0.878	0.753	0.716	0.699
0.4	0.840	0.705	0.667	0.649
0.45	0.798	0.655	0.616	0.599
0.5	0.750	0.603	0.565	0.547
0.55	0.698	0.549	0.512	0.495
0.6	0.640	0.494	0.458	0.442
0.65	0.578	0.437	0.404	0.389
0.7	0.510	0.378	0.348	0.335
0.75	0.438	0.319	0.292	0.280
0.8	0.360	0.257	0.235	0.225
0.85	0.278	0.195	0.177	0.170
0.9	0.190	0.131	0.119	0.113
0.95	0.097	0.066	0.060	0.057
1	0.000	0.000	0.000	0.000



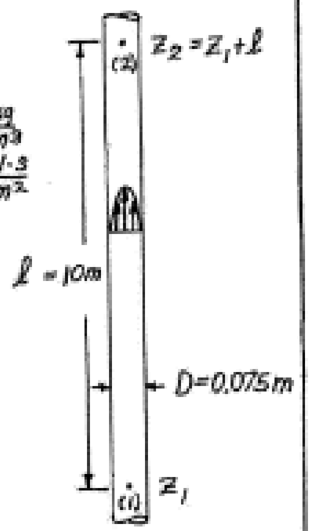
8-15

8.18

8.18 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

$\rho = 1260\ \frac{kg}{m^3}$

$\mu = 1.50\ \frac{N\cdot s}{m^2}$



For laminar flow in a pipe,

V = average velocity $= \frac{1}{2}V_{max} = \frac{1}{2}(1\frac{m}{s}) = 0.5\frac{m}{s}$

Thus,

$$Re = \frac{\rho V D}{\mu} = \frac{(1260\frac{kg}{m^3})(0.5\frac{m}{s})(0.075\,m)}{1.50\ \frac{N\cdot s}{m^2}} = 31.5 < 2100$$

The flow is laminar so that

$$V = \frac{(\Delta p - \gamma \ell \sin\theta)D^2}{32\mu\ell}, \text{ where } \theta = 90°$$

Thus,

$$\Delta p = \frac{32\mu\ell V}{D^2} + \gamma\ell = \frac{32(1.50\frac{N\cdot s}{m^2})(10m)(0.5\frac{m}{s})}{(0.075\,m)^2} + (9.81\tfrac{m}{s^2})(1260\tfrac{kg}{m^3})(10m)$$

$$= 1.66\times10^5\ \frac{N}{m^2}, \text{ or } \Delta p = \underline{\underline{166\ kPa}}$$

Also,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L, \text{ or with } V_1 = V_2,\ z_2 - z_1 = \ell, \text{ and}$$

$p_1 = p_2 + \Delta p$ this gives

$$h_L = \frac{\Delta p}{\gamma} - \ell = \frac{1.66\times10^5\ \frac{N}{m^2}}{(9.81\frac{m}{s^2})(1260\frac{kg}{m^3})} - 10m = \underline{\underline{3.43\,m}}$$

8-18

8.20 At time $t = 0$ the level of water in tank A shown in Fig. P8.20 is 0.6 m above that in tank B. Plot the elevation of the water in tank A as a function of time until the free surfaces in both tanks are at the same elevation. Assume quasisteady conditions—that is, the steady pipe flow equations are assumed valid at any time, even though the flowrate does change (slowly) in time. Neglect minor losses. *Note:* Verify and use the fact that the flow is laminar.

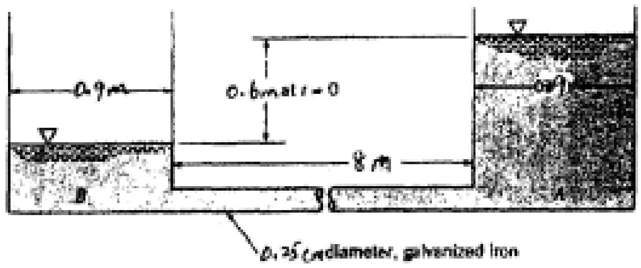
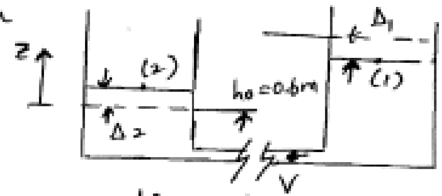


FIGURE P8.20

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{l}{D}\frac{V^2}{2g}, \quad \text{where } p_1 = p_2 = 0 \text{ and } V_1 = V_2 \approx 0 \qquad (1)$$

At $t=0$, $z_2 = 0$ and $z_1 = h_0 = 0.6$ m



Because the tanks are the same diameter

$\Delta_1 = \Delta_2$ and with $z_2 = \Delta_2$, $z_1 = h_0 - \Delta_2$

we obtain $z_1 = h_0 - z_2$.

Thus Eq. (1) becomes

$$z_1 = z_2 + f\frac{l}{D}\frac{V^2}{2g} \quad \text{or} \quad 2z_1 - h_0 = f\frac{l}{D}\frac{V^2}{2g} \qquad (2)$$

Also, $A_1\left(-\frac{dz_1}{dt}\right) = Q = \frac{\pi}{4}(D^2)V$, where $A_1 = \frac{\pi}{4}D_T^2$ with $D_T = 0.9$ m = tank diameter

Thus,

$$V = -\left(\frac{D_T}{D}\right)^2 \frac{dz_1}{dt} \qquad (3)$$

The maximum $Re = \frac{\rho V D}{\mu}$ occurs when the head, $z_1 - z_2$, is greatest.

From Eq. (2) (with $z_1 = h_0$), $h_0 = f\frac{l}{D}\frac{V_{max}^2}{2g}$

8.20 con't

Assume laminar flow so that $f = \frac{64}{Re}$ or $f = \frac{64\mu}{\rho V D}$ (4)

Thus, from Eq. (4)

$$h_o = \frac{64\mu}{\rho V_{max} D} \cdot \frac{l}{D} \cdot \frac{V_{max}^2}{2g} = \frac{32\mu l V_{max}}{\gamma D^2},$$

or

$$V_{max} = \frac{\gamma D^2 h_o}{32\mu l} = \frac{\left(9.796\ \frac{kN}{m^3}\right)(0.0025\ m)^2 (0.6\ m)}{32\left(1.0\times10^{-3}\ \frac{N\cdot s}{m^2}\right)(8\ m)}$$

$$= 0.143\ \frac{m}{s}$$

or

$$Re_{max} = \frac{\left(1001.21\ \frac{kg}{m^3}\right)\left(0.143\ \frac{m}{s}\right)(0.0025\ m)}{1.0\times10^{-3}\ \frac{N\cdot s}{m^2}}$$

$= 358 < 2100$. The flow remains laminar.

Thus, Eqs. (2) and (4) give

$$2z_1 - h_o = \frac{64\mu}{\rho V D}\frac{l}{D}\frac{V^2}{2g} = \frac{32\mu l V}{\gamma D^2},$$ or

by using Eq. (3)

$$2z_1 - h_o = -\left(\frac{D_T}{D}\right)^2 \frac{32\mu l}{\gamma D^2}\frac{dz_1}{dt} \qquad (5)$$

Let $F = z_1 - \frac{h_o}{2}$ so that $\frac{dF}{dt} = \frac{dz_1}{dt}$ and

Eq. (5) becomes

$$2F = -\left(\frac{D_T}{D}\right)^2 \frac{32\mu l}{\gamma D^2}\frac{dF}{dt}$$

or

$$\alpha\frac{dF}{dt} + F = 0, \text{ where } \alpha = \frac{16\mu l}{\gamma D^2}\left(\frac{D_T}{D}\right)^2$$

Thus,

$$\alpha\int\frac{dF}{F} = -\int dt \quad \text{or} \quad \alpha \ln F = -t + \tilde{C}, \text{ where } \tilde{C} = \text{constant}$$

[8.20 con't]

Hence,

$$F = Ce^{(-t/\alpha)}.$$

That is $z_1 - \frac{h_0}{2} = Ce^{-(t/\alpha)}$ with the initial conditions

$z_1 = h_0$ when $t=0$, or $C = \frac{h_0}{2}$

Thus,

$$z_1 - \frac{h_0}{2} = \frac{h_0}{2} e^{-(t/\alpha)}$$

or

$$z_1 = \frac{h_0}{2}\left[1 + e^{-(t/\alpha)}\right]$$

Note:

As $t \to \infty$, $z_1 \to \frac{h_0}{2}$

For the conditions given,

$h_0 = 0.6$ m and

$$\alpha = \frac{16\left(1.0\times10^{-3}\ \frac{N\cdot s}{m^2}\right)(8m)}{\left(9.796\ \frac{kN}{m^3}\right)(0.0025\,m)^2}\left(\frac{0.9m}{0.0025m}\right)^2$$

$$= 2.7\times10^5\ s$$

Hence,

$$\underline{\underline{z_1 = 1 + e^{-\left(\frac{t}{2.7\times10^5\ s}\right)}}},$$ where $z_1 \sim ft$ and $t \sim s$

This result is plotted (Note: $\lim_{t\to\infty} z_1 = 0.3$ m)

8.25

8.25 Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4}\ m^2/s$ flows through the vertical pipe shown in Fig. P8.25 at a rate of $4 \times 10^{-4}\ m^3/s$. Determine the manometer reading, h.

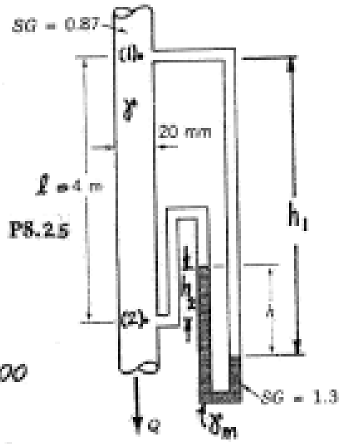


FIGURE P8.25

$V = \frac{Q}{A} = \frac{4\times10^{-4}\frac{m^3}{s}}{\frac{\pi}{4}(0.02m)^2} = 1.27\frac{m}{s}$ so that

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(1.27\frac{m}{s})(0.02m)}{2.2\times10^{-4}\frac{m^2}{s}} = 115 < 2100$

The flow is laminar with

$Q = \frac{\pi(\Delta p + \gamma \ell)D^4}{128\mu\ell}$, or $\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4} - \gamma\ell$ (1)

Hence, with $\gamma = SG\,\gamma_{H_2O} = 0.87(9.81\frac{kN}{m^3}) = 8.53\frac{kN}{m^3}$ and

$\mu = \nu\rho = \nu\,SG\,\rho_{H_2O} = (2.2\times10^{-4}\frac{m^2}{s})(0.87)(1000\frac{kg}{m^3}) = 0.191\ \frac{N\cdot s}{m^2}$

Eq. (1) gives

$\Delta p = \frac{128(0.191\frac{N\cdot s}{m^2})(4m)(4\times10^{-4}\frac{m^3}{s})}{\pi(0.020m)^4} - (8.53\frac{kN}{m^3})(4m)(10^3\frac{N}{kN})$

or $\Delta p = 4.37\times10^4\frac{N}{m^2} = 43.7\frac{kN}{m^2}$ (2)

From manometer considerations

$p_1 + \gamma h_1 - \gamma_m h + \gamma h_2 = p_2$, where $\gamma_m = SG_m\gamma_{H_2O} = 1.3(9.81\frac{kN}{m^3}) = 12.74\frac{kN}{m^3}$

and $h_1 = h - h_2 + \ell$, or $h_2 + h_1 = h + \ell$

Thus,

$p_1 - p_2 = \Delta p = -\gamma(h_2 + h_1) + \gamma_m h = (\gamma_m - \gamma)h - \gamma\ell$ (3)

Combine Eqs. (2) and (3) to give

$43.7\frac{kN}{m^2} = (12.74 - 8.53)\frac{kN}{m^3}h - (8.53\frac{kN}{m^3})(4m)$

or $h = \underline{\underline{18.5\ m}}$

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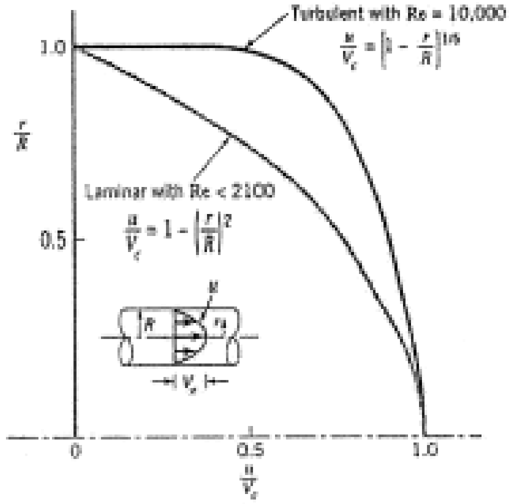
8.30

8.30 As shown in Video V8.10 and Fig P8.30 the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at Re = 10,000 the velocity profile can be approximated by the power-law profile shown in the figure. (a) For laminar flow, determine at what radial loaction you would place a Pitot tube if it is to measure the average velocity in the pipe. (b) Repeat part (a) for turbulent flow with Re = 10,000.



■ FIGURE P8.30

For laminar or turbulent flow,

$$Q = AV = \pi R^2 V = \int u\,dA = \int u(2\pi r\,dr) = 2\pi \int_{r=0}^{R} u\,r\,dr$$

a) Laminar flow:

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1-\left(\frac{r}{R}\right)^2\right]dr = 2\pi V_c\left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi \frac{R^2}{2} V_c$$

Thus, $V = \frac{1}{2}V_c$ For $u = V = \frac{V_c}{2}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{1}{2} = 1 - \left(\frac{r}{R}\right)^2, \text{ or } \left(\frac{r}{R}\right)^2 = \frac{1}{2} \quad \text{Thus, } r = \frac{1}{\sqrt{2}}R = \underline{\underline{0.707R}}$$

b) Turbulent flow

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1-\frac{r}{R}\right]^{1/5} dr = 2\pi R^2 V_c \int_0^1 \left(\frac{r}{R}\right)\left[1-\left(\frac{r}{R}\right)\right]^{1/5} d\left(\frac{r}{R}\right)$$

Let $y = 1 - \left(\frac{r}{R}\right)$ so that $\left(\frac{r}{R}\right) = 1 - y$ and $d\left(\frac{r}{R}\right) = -dy$

Thus,

$$\pi R^2 V = 2\pi R^2 V_c \int_{y=1}^{y=0} (1-y)\,y^{1/5}(-dy) = 2\pi R^2 V_c \int_0^1 \left(y^{1/5} - y^{6/5}\right)dy$$

$$= 2\pi R^2 V_c\left[\frac{5}{6} - \frac{5}{11}\right] = 2\pi R^2 V_c\left(\frac{25}{66}\right)$$

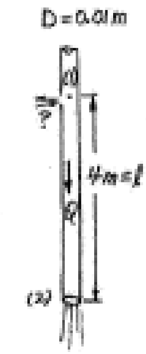
or $V = \frac{50}{66}V_c$ For $u = V = \frac{50}{66}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{50}{66} = \left[1 - \frac{r}{R}\right]^{1/5} \text{ or } \frac{r}{R} = 0.750 \text{ so that } r = \underline{\underline{0.750\,R}}$$

8-33

8.43

8.43 Water flows downward through a vertical 10-mm-diameter galvanized iron pipe with an average velocity of 5.0 m/s and exits as a free jet. There is a small hole in the pipe 4 m above the outlet. Will water leak out of the pipe through this hole, or will air enter into the pipe through the hole? Repeat the problem if the average velocity is 0.5 m/s.



$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}$, where $p_2=0$, $z_2=0$,

$z_1=4m$, $V_1=V_2=V$. Thus,

$\frac{p_1}{\gamma}=f\frac{\ell}{D}\frac{V^2}{2g}-z_1$, or $p_1=f\frac{\ell}{D}\frac{1}{2}\rho V^2-\gamma\ell$ With ε from Table 8.1, (1)

$\frac{\varepsilon}{D}=\frac{0.15\,mm}{10\,mm}=0.015$ so that with $Re=\frac{VD}{\nu}=\frac{(5\frac{m}{s})(0.01m)}{1.12\times10^{-6}\frac{m^2}{s}}=4.46\times10^4$

we obtain $f=0.045$ (see Fig. 8.20).

Thus, from Eq.(1)

$p_1=0.045\left(\frac{4m}{0.01m}\right)\frac{1}{2}(999\frac{kg}{m^3})(5\frac{m}{s})^2-9800\frac{N}{m^3}(4m)=1.86\times10^5\frac{N}{m^2}$

Since $p_1>0$, water will leak out of the pipe when $V=5\frac{m}{s}$

If $V=0.5\frac{m}{s}$, then $Re=4.46\times10^3$ and $f=0.052$

Thus, from Eq.(1)

$p_1=0.052\left(\frac{4m}{0.01m}\right)\frac{1}{2}(999\frac{kg}{m^3})(0.5\frac{m}{s})^2-9800\frac{N}{m^3}(4m)=-3.66\times10^4\frac{N}{m^2}$

Since $p_1<0$, air will enter the pipe when $V=0.5\frac{m}{s}$

Note: The above conclusion is valid regardless of the length, ℓ.

8-45