Series 10 - Solutions

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7.13

7.13 The drag, $\mathscr{D}$, on a washer shaped plate placed normal to a stream of fluid can be expressed as

$$\mathscr{D} = f(d_1, d_2, V, \mu, \rho)$$

where d_1 is the outer diameter, d_2 the inner diameter, V the fluid velocity, μ the fluid viscosity, and ρ the fluid density. Some experiments are to be performed in a wind tunnel to determine the drag. What dimensionless parameters would you use to organize these data?

$\mathscr{D} \doteq F \qquad d_1 \doteq L \qquad d_2 \doteq L \qquad V \doteq LT^{-1} \qquad \mu \doteq FL^{-2}T \qquad \rho \doteq FL^{-4}T^2$

From the pi theorem, $6-3=3$ pi terms required. Use d_1, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \mathscr{D}\, d_1^a V^b \rho^c$$

and

$$(F)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad (\text{for } F)$$
$$a + b - 4c = 0 \qquad (\text{for } L)$$
$$-b + 2c = 0 \qquad (\text{for } T)$$

It follows that $a = -2$, $b = -2$, $c = -1$, and therefore

$$\Pi_1 = \frac{\mathscr{D}}{d_1^2 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{\mathscr{D}}{d_1^2 V^2 \rho} \doteq \frac{MLT^{-2}}{(L)^2(LT^{-1})^2(ML^{-3})} \doteq M^0L^0T^0 \qquad \therefore \text{ok}$$

For Π_2:

$$\Pi_2 = d_2\, d_1^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad (\text{for } F)$$
$$1 + a + b - 4c = 0 \qquad (\text{for } L)$$
$$-b + 2c = 0 \qquad (\text{for } T)$$

(con't)

7.13 (con't)

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{d_2}{d_1}$$

which is obviously dimensionless.

For Π_3:

$$\Pi_3 = \mu\, d_1^{a} V^{b} \rho^{c}$$

$$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-2 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$1 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=-1$, $c=-1$, and therefore

$$\Pi_3 = \frac{\mu}{d_1 V \rho}$$

Check dimensions using MLT system:

$$\frac{\mu}{d_1 V\rho} \doteq \frac{ML^{-1}T^{-1}}{(L)(LT^{-1})(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

Thus,

$$\frac{\mathcal{D}}{d_1^2 V^2 \rho} = \phi\left(\frac{d_2}{d_1}, \frac{\mu}{d_1 V \rho}\right) \qquad (1)$$

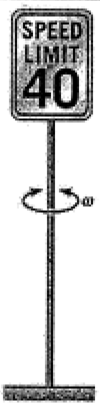
Since $\frac{\rho V d_1}{\mu}$ is a standard dimensionless parameter (Reynolds number), Eq. (1) would more commonly be expressed as

$$\underline{\underline{\frac{\mathcal{D}}{d_1^2 V^2 \rho} = \phi\left(\frac{d_2}{d_1}, \frac{\rho V d_1}{\mu}\right)}} \qquad (2)$$

As far as dimensional analysis is concerned, Eqs. (1) and (2) are equivalent.

7.19

7.19 Under certain conditions, wind blowing past a rectangular speed limit sign can cause the sign to oscillate with a frequency ω. (See Fig. P7.19 and Video V9.9.) Assume that ω is a function of the sign width, b, sign height, h, wind velocity, V, air density, ρ, and an elastic constant, k, for the supporting pole. The constant, k, has dimensions of FL. Develop a suitable set of pi terms for this problem.



■ FIGURE P7.19

$$\omega = f(b, h, V, \rho, k)$$

$\omega \doteq T^{-1}$ $b \doteq L$ $h \doteq L$ $V \doteq LT^{-1}$ $\rho \doteq FL^{-4}T^{2}$ $k \doteq FL$

From the pi theorem $6-3=3$ pi terms required. Use b, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \omega b^a V^b \rho^c$$

and

$$(T^{-1})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$c = 0$ (for F)

$a + b - 4c = 0$ (for L)

$-1 - b + 2c = 0$ (for T)

It follows that $a = 1$, $b = -1$, $c = 0$, and therefore

$$\Pi_1 = \frac{\omega b}{V}$$

Check dimensions:

$$\frac{\omega b}{V} \doteq \frac{(T^{-1})(L)}{(LT^{-1})} \doteq L^0T^0 \quad \therefore OK$$

For Π_2:

$$\Pi_2 = h b^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$c = 0$ (for F)

$1 + a + b - 4c = 0$ (for L)

$-b + 2c = 0$ (for T)

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{h}{b}$$

which is obviously dimensionless. (con't)

7-24

7.19 (con't)

For Π_3:

$$\Pi_3 = k\, b^a V^b \rho^c$$

$$(FL)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -3$, $b = -2$, $c = -1$, and therefore

$$\Pi_3 = \frac{k}{b^3 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{k}{b^3V^2\rho} \doteq \frac{ML^2T^{-2}}{(L^3)(LT^{-1})^2(ML^{-3})} \doteq M^0L^0T^0 \therefore OK$$

Thus,

$$\underline{\underline{\frac{\omega b}{V} = \phi\left(\frac{h}{b}, \frac{k}{b^3V^2\rho}\right)}}$$

7.22

7.22 The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, \ell, V, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \ell \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p\, D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p\, D}{\mu V} \doteq \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing ℓ):

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p\, D}{\mu V} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

From the statement of the problem, $\Delta p \propto \ell$ so that Eq. (1) must be of the form

$$\frac{\Delta p\, D}{\mu V} = K\frac{\ell}{D}$$

where K is some constant. It thus follows that

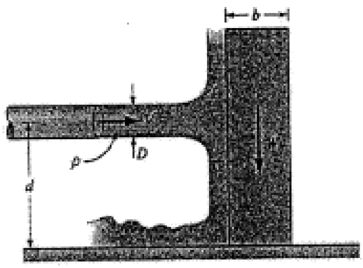
$$\underline{\underline{\Delta p \propto \frac{1}{D^2}}}$$

for a given velocity.

7-28

7.28

7.28 As shown in Fig. P7.28 and Video V5.6, a jet of liquid directed against a block can tip over the block. Assume that the velocity, V, needed to tip over the block is a function of the fluid density, ρ, the diameter of the jet, D, the weight of the block, $\mathcal{W}$, the width of the block, b, and the distance, d, between the jet and the bottom of the block. **(a)** Determine a set of dimensionless parameters for this problem. Form the dimensionless parameters by inspection. **(b)** Use the momentum equation to determine an equation for V in terms of the other variables. **(c)** Compare the results of parts **(a)** and **(b)**.



■ FIGURE P7.28

(a) $V = f(\rho, D, \mathcal{W}, b, d)$

$V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad D \doteq L$ $\quad \mathcal{W} \doteq F$ $\quad b \doteq L$ $\quad d \doteq L$

From the pi theorem, $6-3 = 3$ pi terms required.

By inspection for Π_1 (containing V)

$$\Pi_1 = VD\sqrt{\frac{\rho}{\mathcal{W}}} \doteq (LT^{-1})(L)\left(\sqrt{\frac{FL^{-4}T^{2}}{F}}\right) \doteq F^0L^0T^0$$

Check using MLT:

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = (LT^{-1})(L)\left(\sqrt{\frac{ML^{-3}}{MLT^{-2}}}\right) \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2 let

$$\Pi_2 = \frac{b}{d}$$

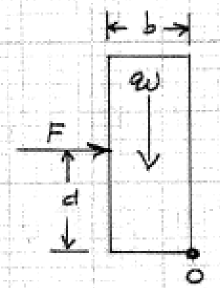
and for Π_3

$$\Pi_3 = \frac{d}{D}$$

and both Π_2 and Π_3 are obviously dimensionless.

Thus,

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$



(b) For impending tipping around o

$$\Sigma M_o = 0$$

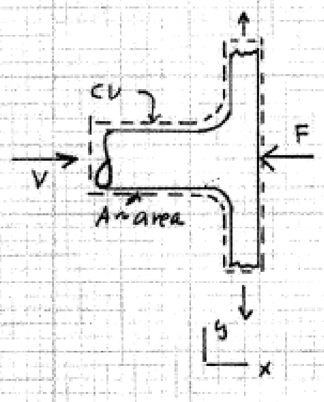
so that

$$Fd = \mathcal{W}\left(\frac{b}{2}\right) \qquad (1)$$

(con't)

7.2B (con't)

From momentum considerations using the CV shown



$(\overset{+}{\rightarrow})$ $\int \rho u \vec{V} \cdot \hat{n} \, dA = \sum F_x$

$$\rho V^2 A = F$$

Thus, from Eq. (1)

$$(\rho V^2 A)(d) = \mathcal{W}\left(\frac{b}{2}\right)$$

so that

$$V = \sqrt{\frac{\mathcal{W}(b)}{2\rho A d}}$$

and with $A = \pi/4 \, D^2$

$$\underline{\underline{V = \sqrt{\frac{2\mathcal{W} b}{\pi \rho d D^2}}}} \qquad (2)$$

(c) From part (a)

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}} \, \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$

Eq. (2) can be written as

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}} \left(\sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}\right) \qquad (3)$$

It follows by comparing Eqs. (2) and (3) that

$$\phi\left(\frac{b}{d}, \frac{d}{D}\right) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}$$

so that $\phi\left(\frac{b}{d}, \frac{d}{D}\right)$ is actually independent of $\frac{d}{D}$.

*7.34 The pressure drop across a short hollowed plug placed in a circular tube through which a liquid is flowing (see Fig. P7.34) can be expressed as

$$\Delta p = f(\rho, V, D, d)$$

where ρ is the fluid density, and V is the mean velocity in the tube. Some experimental data obtained with $D = 0.06$ m, $\rho = 1030$ kg/m³, and $V = 0.6$ m/s are given in the following table:

d (m)	0.01	0.02	0.03	0.04
Δp (N/m²)	24	7.5	3	0.6

Plot the results of these tests, using suitable dimensionless parameters, on log-log graph paper. Use a standard curve-fitting technique to determine a general equation for Δp. What are the limits of applicability of the equation?

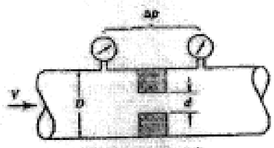


FIGURE P7.34

$\Delta P \doteq FL^{-2}$ $\quad \rho \doteq FL^{-4}T^{-2}$ $\quad V \doteq LT^{-1}$ $\quad D \doteq L$ $\quad d \doteq L$

From the pi theorem, $5-3=2$ pi terms determined.

By inspection for Π_1 (containing ΔP):

$$\Pi_1 = \frac{\Delta P}{\rho V^2} = \frac{FL^{-2}}{(FL^{-4}T^{2})(LT^{-1})^2} = F^0L^0T^0$$

check using MLT:

$$\frac{\Delta P}{\rho V^2} = \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} = M^0L^0T^0 \qquad \therefore \text{OK}$$

For Π_2 (containing D and d):

$$\Pi_2 = \frac{D}{d}$$

(which is obviously dimensionless). Thus,

$$\frac{\Delta P}{\rho V^2} = \phi\left(\frac{D}{d}\right)$$

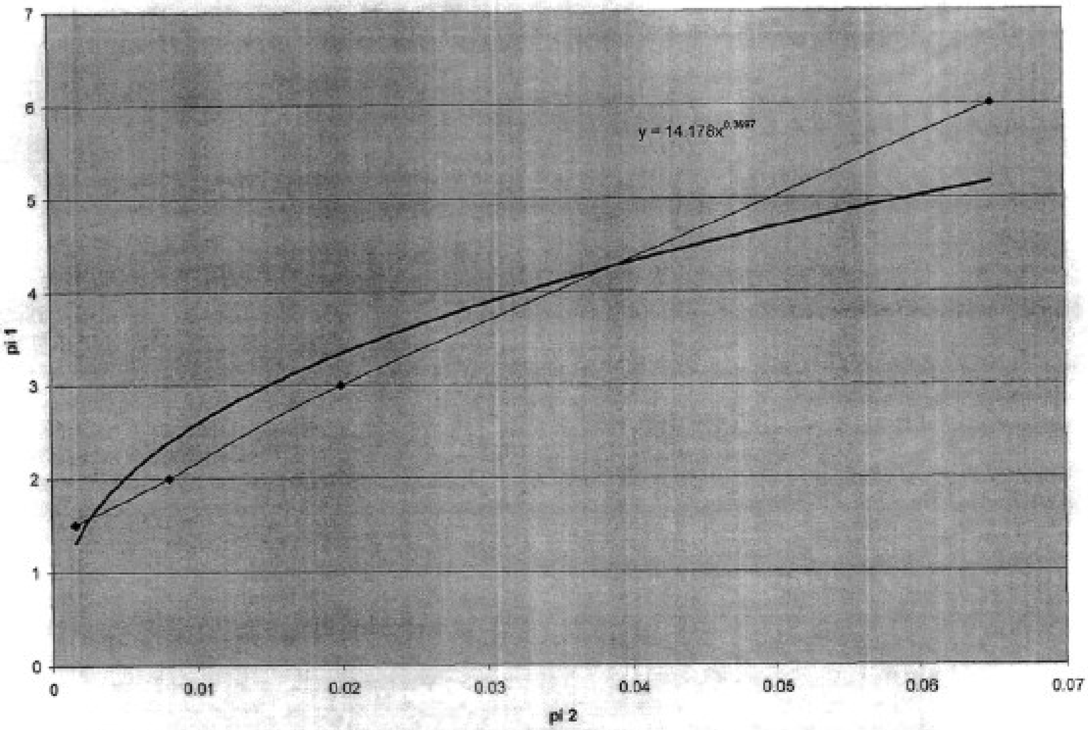
For the data given:

D/d	6	3	2	1.5	(y-axis)
$\Delta P/\rho V^2$	0.065	0.020	0.0081	0.0016	(x-axis)

A log-log plot of these data is shown on the next page.

Also see attached excel file for a more detailed explanation on curve fitting for this exercise.

Problem 7.34



*7.34 con't

since the data plot ≈ straight line on a log-log plot, the equation for the data is of the form

$$\Pi_1 = a \Pi_2^{\,b}$$

where $\Pi_1 = \frac{\Delta P}{\rho V^2}$ and $\Pi_2 = \frac{D}{d}$.

A power law fit of the data gives

$a = 14.178$ and $b = 0.3697$

Thus,

$$\frac{\Delta P}{\rho V^2} = 14.178 \left(\frac{D}{d}\right)^{0.3697}$$

This equation is applicable over the range of data $1.5 \le \frac{D}{d} \le 6$.

7.69

7.69 A thin layer of spherical particles rests on the bottom of a horizontal tube as shown in Fig. P7.69. When an incompressible fluid flows through the tube, it is observed that at some critical velocity the particles will rise and be transported along the tube. A model is to be used to determine this critical velocity. Assume the critical velocity, V_c to be a function of the pipe diameter, D, particle diameter, d, the fluid density, ρ, and viscosity, μ, the density of the particles, ρ_p, and the acceleration of gravity, g. **(a)** Determine the similarity requirements for the model, and the relationship between the critical velocity for model and prototype (the prediction equation). **(b)** For a length scale of $\frac{1}{2}$ and a fluid density scale of 1.0, what will be the critical velocity scale (assuming all similarity requirements are satisfied)?

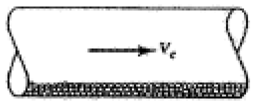


FIGURE P7.69

(a)
$$V_c = f(D, d, \rho, \mu, \rho_p, g)$$

$V_c \doteq LT^{-1}$ $\quad D \doteq L$ $\quad d \doteq L$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$ $\quad \rho_p \doteq FL^{-4}T^2$ $\quad g \doteq LT^{-2}$

From the pi theorem, 7-3 = 4 pi terms required, and a dimensional analysis yields

$$\frac{\rho V_c D}{\mu} = \phi\left(\frac{d}{D}, \frac{\rho}{\rho_p}, \frac{g d^3 \rho^2}{\mu^2}\right)$$

Thus, the similarity requirements are

$$\underline{\underline{\frac{d_m}{D_m} = \frac{d}{D}}} \qquad \underline{\underline{\frac{\rho_m}{\rho_{pm}} = \frac{\rho}{\rho_p}}} \qquad \underline{\underline{\frac{g_m d_m^3 \rho_m^2}{\mu_m^2} = \frac{g d^3 \rho^2}{\mu^2}}}$$

The prediction equation is

$$\underline{\underline{\frac{\rho V_c D}{\mu} = \frac{\rho_m V_{cm} D_m}{\mu_m}}}$$

(b) If all similarity requirements are satisfied, the prediction equation indicates that

$$\frac{V_{cm}}{V_c} = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{D}{D_m} = (1.0)\left(\frac{\mu_m}{\mu}\right)(2) = 2\frac{\mu_m}{\mu} \qquad (1)$$

From the third similarity requirement (with $g = g_m$),

$$\frac{\mu_m}{\mu} = \sqrt{\left(\frac{d_m}{d}\right)^3\left(\frac{\rho_m}{\rho}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^3(1.0)^2} = \sqrt{\frac{1}{8}}$$

Thus, from Eq. (1)

$$\frac{V_{cm}}{V_c} = 2\sqrt{\frac{1}{8}} = \underline{\underline{0.707}}$$