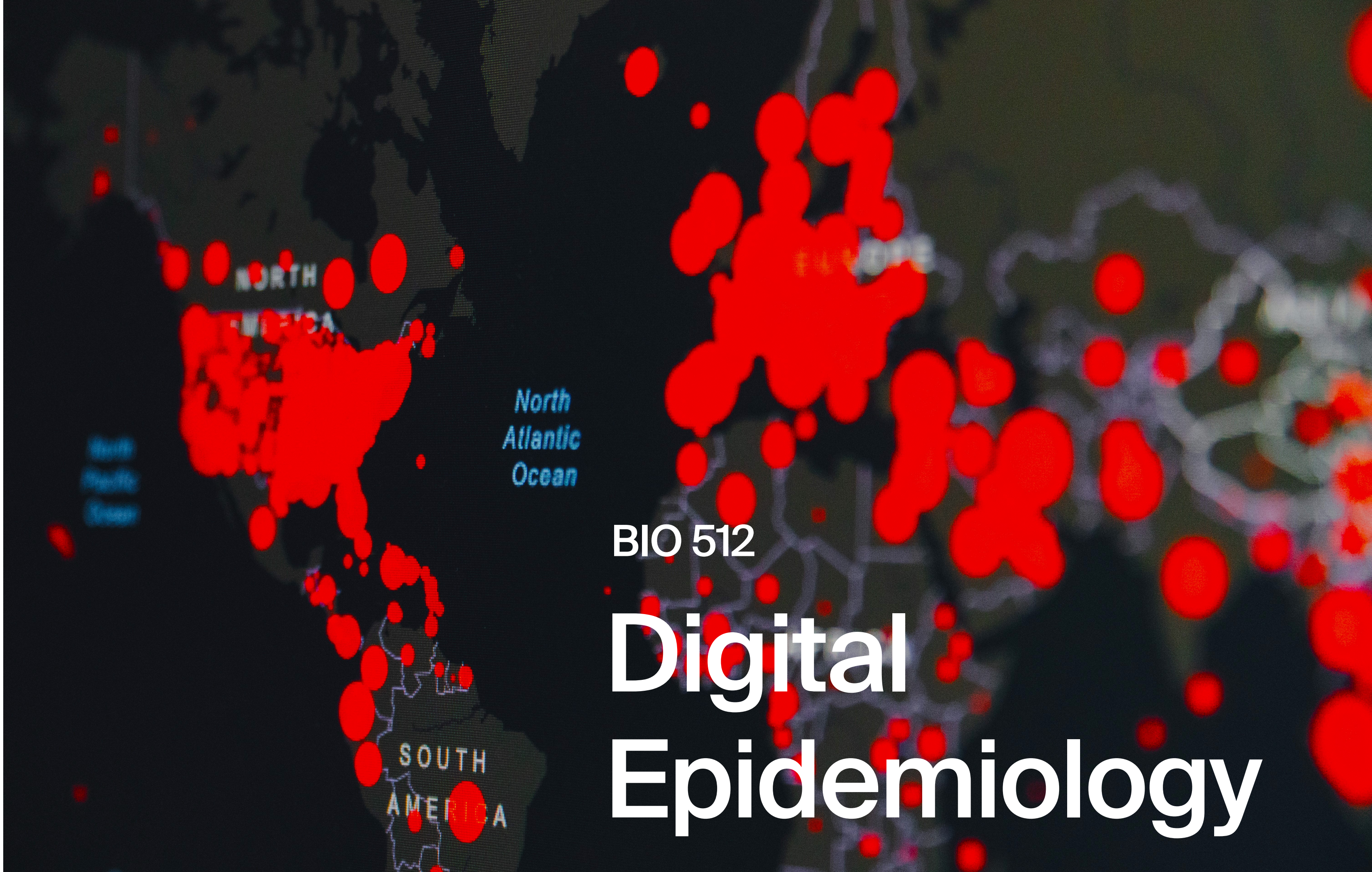
EPFL

Digital Epidemiology Lab
BIO-521
Spring semester 2025



BIO 512

Digital Epidemiology

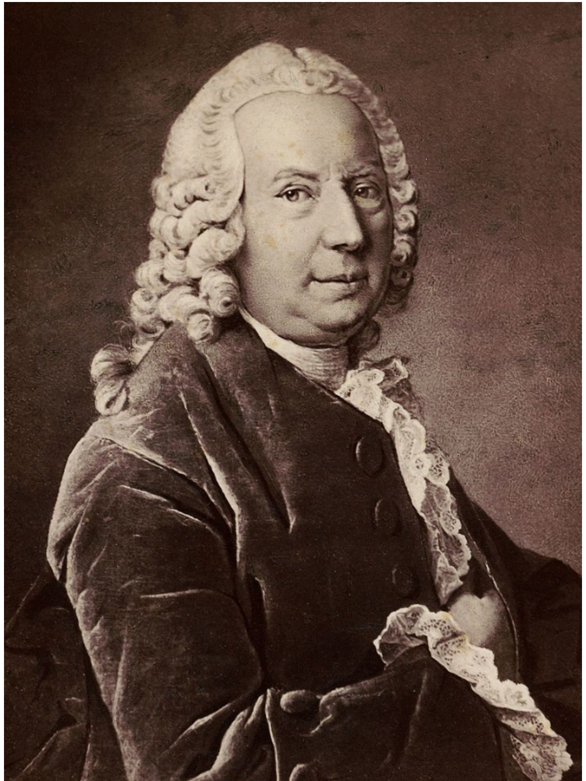
Modeling Infectious Diseases

Learning Objectives

- Develop basic infectious disease models and understand their dynamics
- S(E)IR(S) models
- closed vs. open models
- seasonality
- stochastic vs. deterministic models

Modeling Infectious Diseases

History



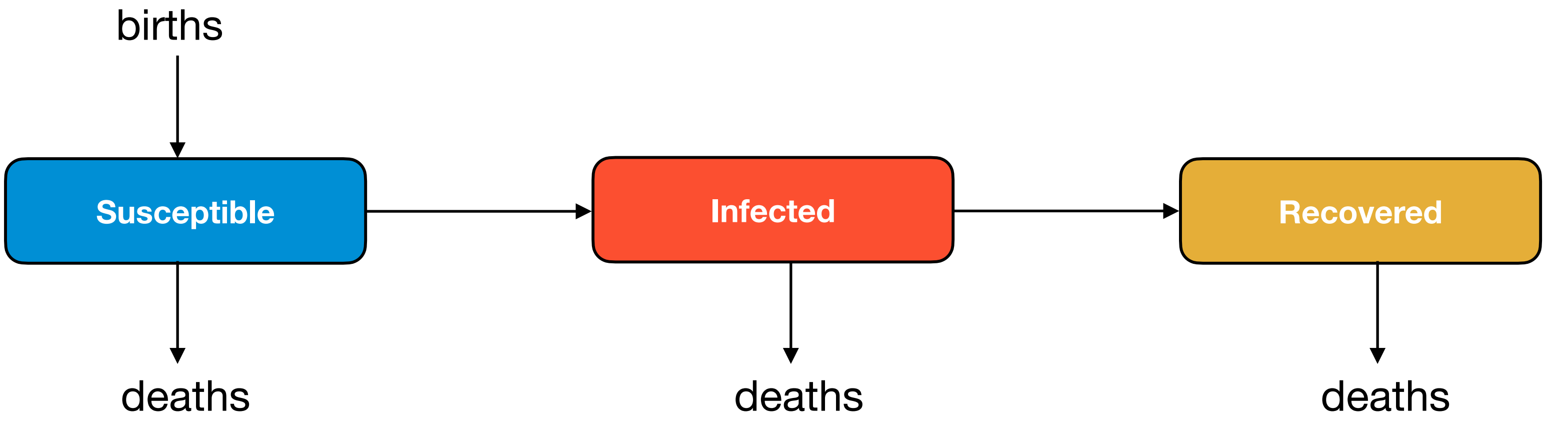
- Daniel Bernoulli, 1766: first mathematical model (specific to smallpox variolation)
- 1917/18: Ross & Hudson, first general epidemiological models
- 1927: Kermack & McKendrick, first SIR model

Modeling Infectious Diseases
A Basic SIR Model

Modeling Infectious Diseases
A Basic SIR Model

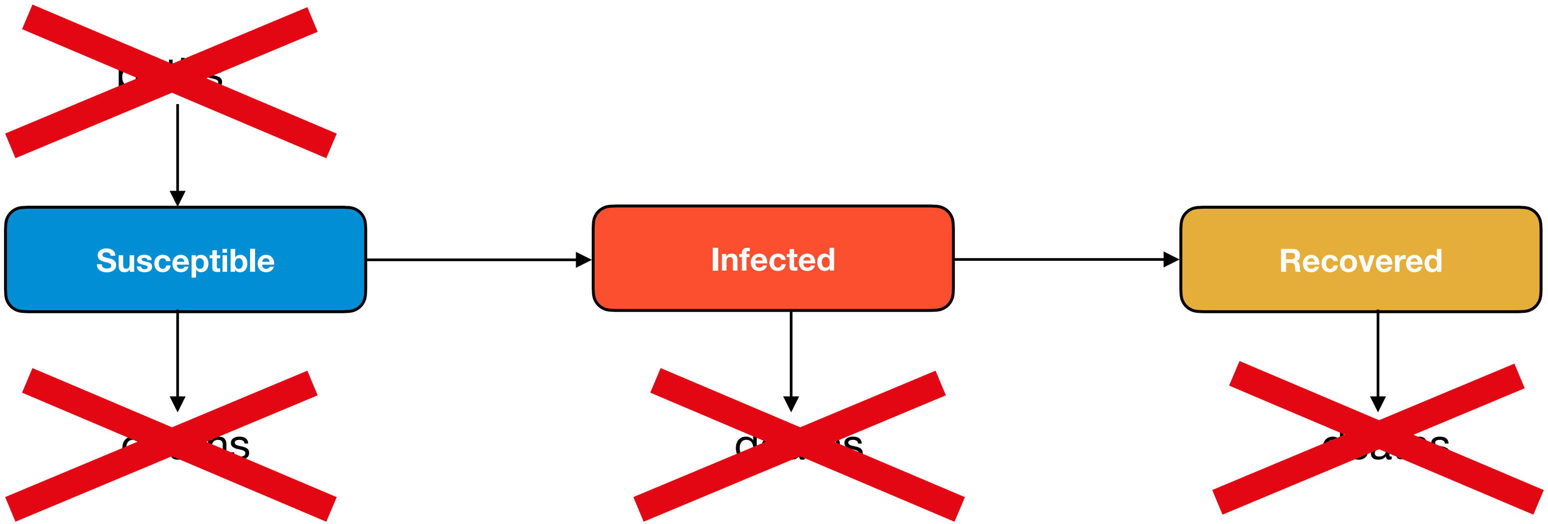
EPFL

Modeling Infectious Diseases
A Basic SIR Model



Modeling Infectious Diseases

A Basic SIR Model



Modeling Infectious Diseases
A Basic SIR Model

β: per capita contact and disease transmission rate per unit time
γ: per capita recovery rate per unit time

Modeling Infectious Diseases
A Basic SIR Model

$$\frac{dS}{dt} = -\beta I \frac{S}{N}$$

$$\frac{dI}{dt} = \beta I \frac{S}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

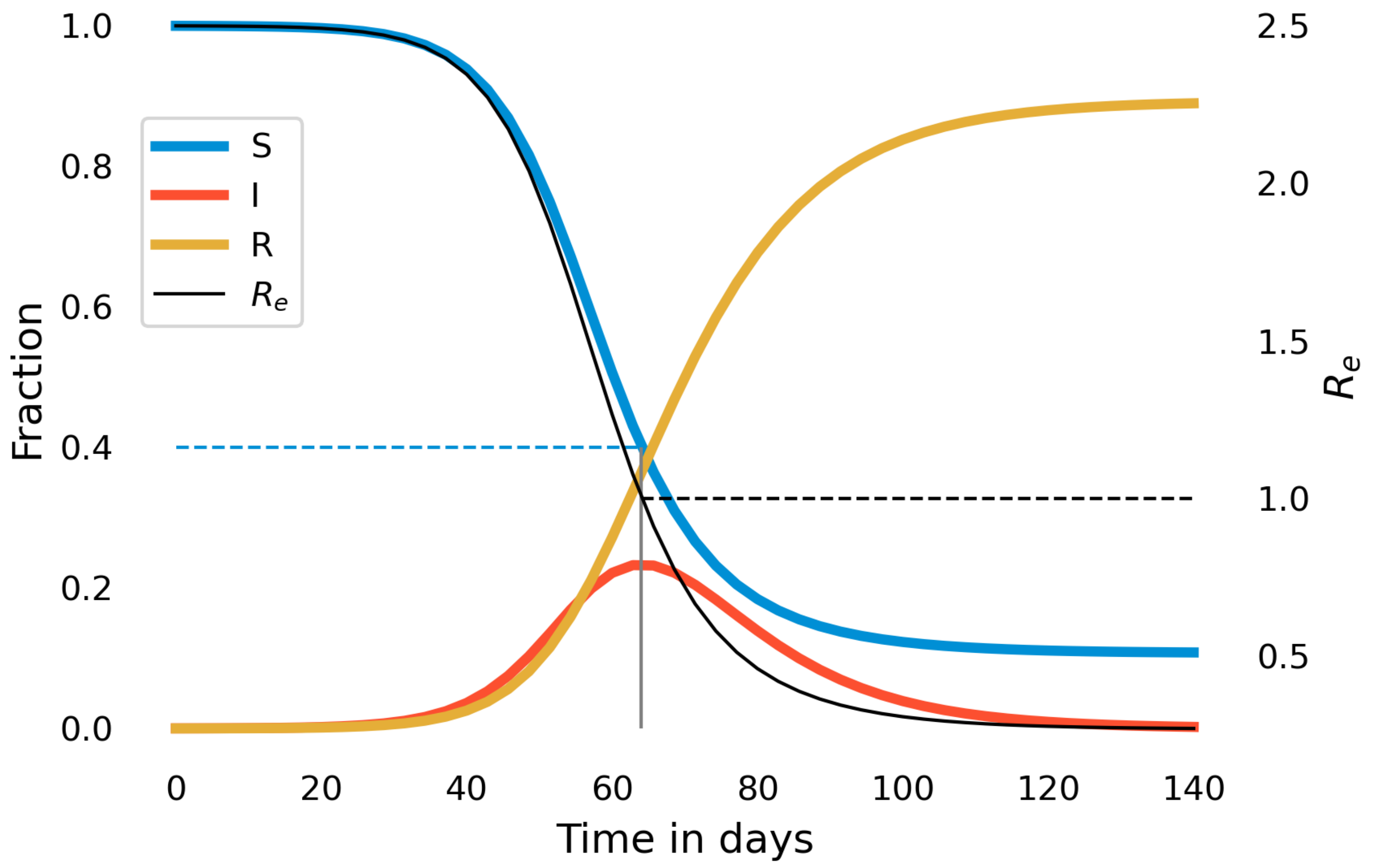
Modeling Infectious Diseases

A Basic SIR Model

- First key insight: $R_0 = \beta / \gamma$

Modeling Infectious Diseases
A Basic SIR Model

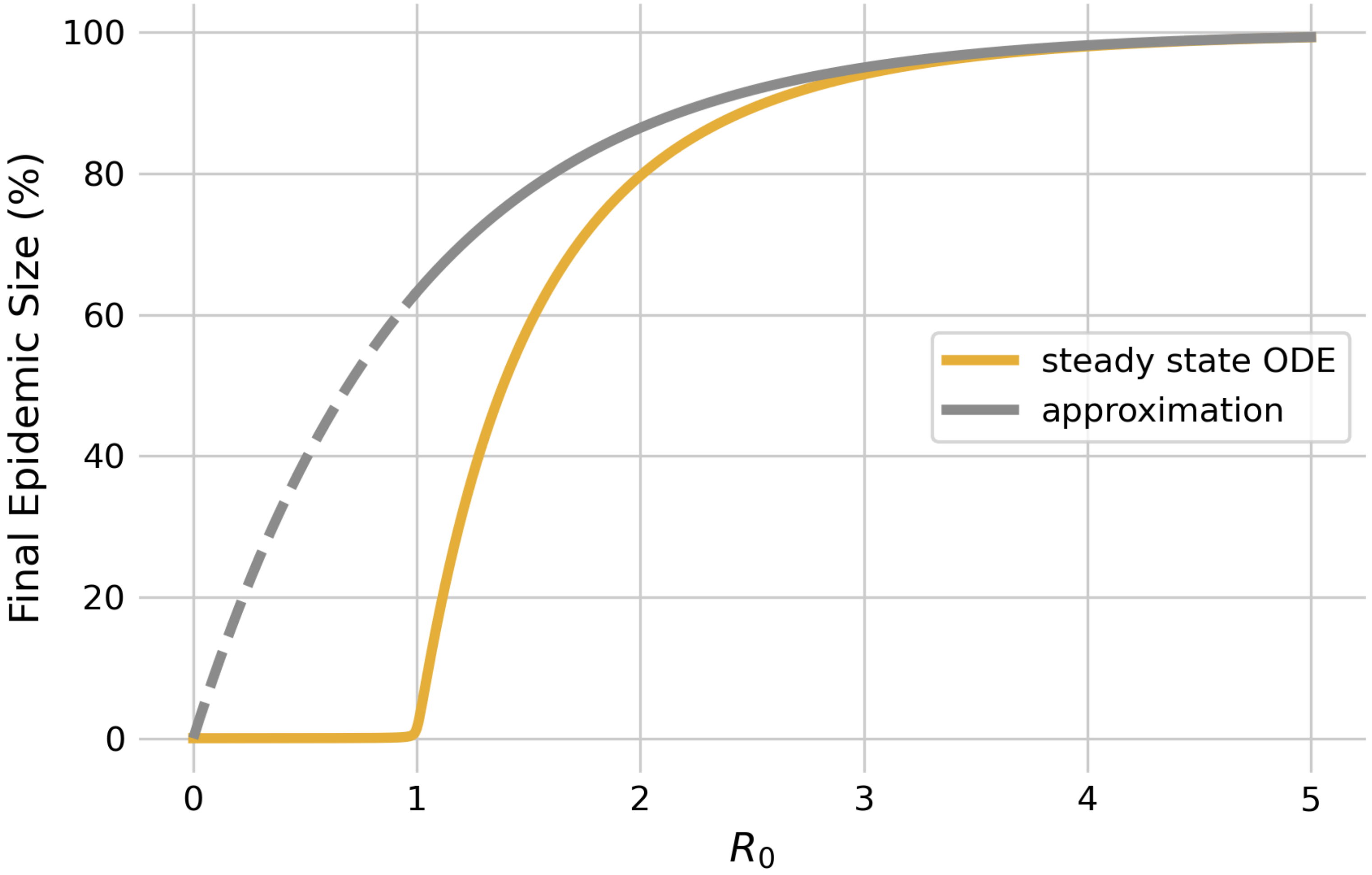
- Second key insight: outbreak is not over when $R_e \leqq 1$


1.0
0.8
0.6
0.4
0.2
0.0
Fraction
S
I
R
R_e
2.5
2.0
1.5
1.0
0.5
R_e
0
20
40
60
80
100
120
140
Time in days

Modeling Infectious Diseases
A Basic SIR Model

- Third key insight: not everybody gets infected

$$S(\infty) = 1 - e^{-R_0}$$


100
80
60
40
20
0
Final Epidemic Size (%)
0
1
2
3
4
5
R_0
steady state ODE
approximation

Modeling Infectious Diseases
Mitigation

- "Flatten the curve"

Modeling Infectious Diseases

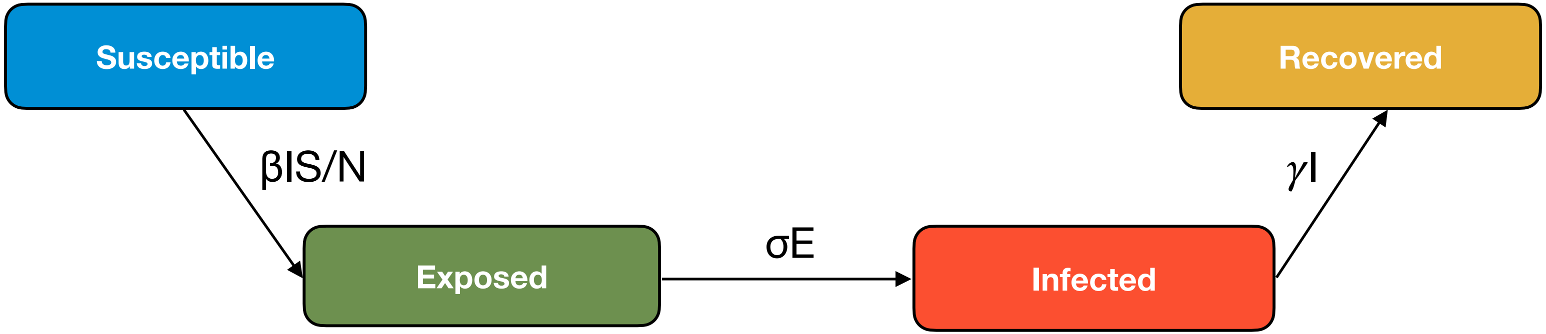
Mitigation

- Vaccination and herd immunity

$$c > 1 - (\frac{1}{R_0})$$

Modeling Infectious Diseases

SEIR Model



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Modeling Infectious Diseases

SEIR Model

$$\frac{dS}{dt} = -\beta I \frac{S}{N}$$

$$\frac{dE}{dt} = \beta I \frac{S}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

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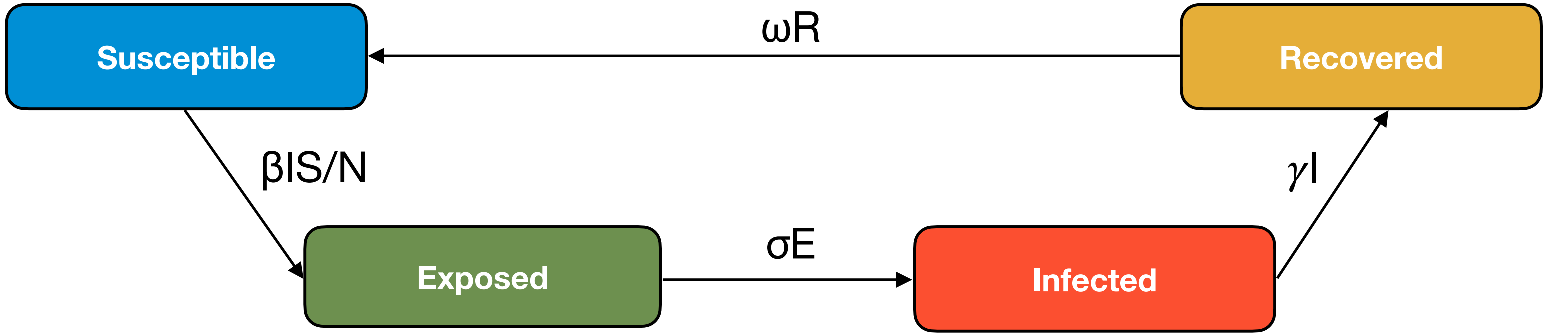
Modeling Infectious Diseases
SEIRS Model



Modeling Infectious Diseases

SEIRS Model

$$\frac{dS}{dt} = \omega R - \beta I \frac{S}{N}$$

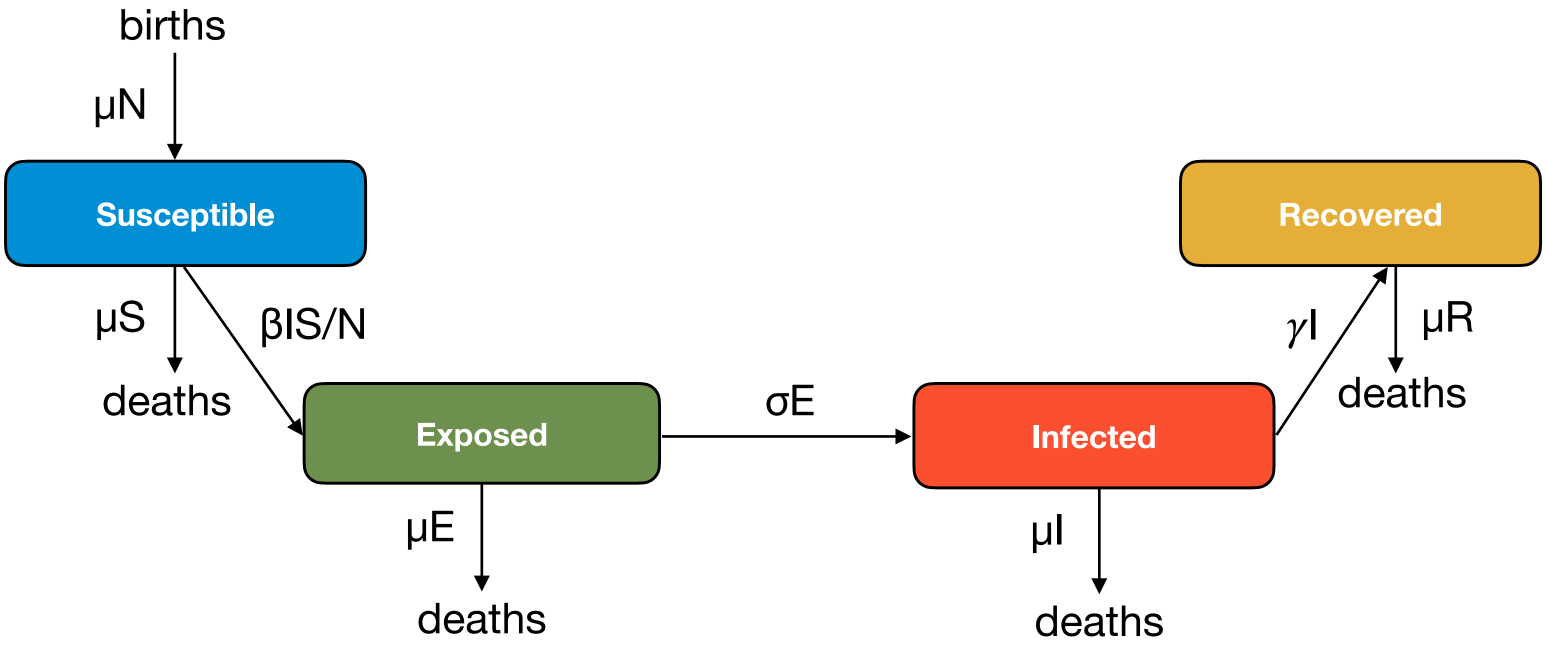
$$\frac{dE}{dt} = \beta I \frac{S}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I - \omega R$$

Modeling Infectious Diseases

An Open Epidemic



Modeling Infectious Diseases
An Open Epidemic

$$\frac{dS}{dt} = \mu N - \beta I \frac{S}{N} - \mu S$$

$$\frac{dE}{dt} = \beta I \frac{S}{N} - \sigma E - \mu E$$

$$\frac{dI}{dt} = \sigma E - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

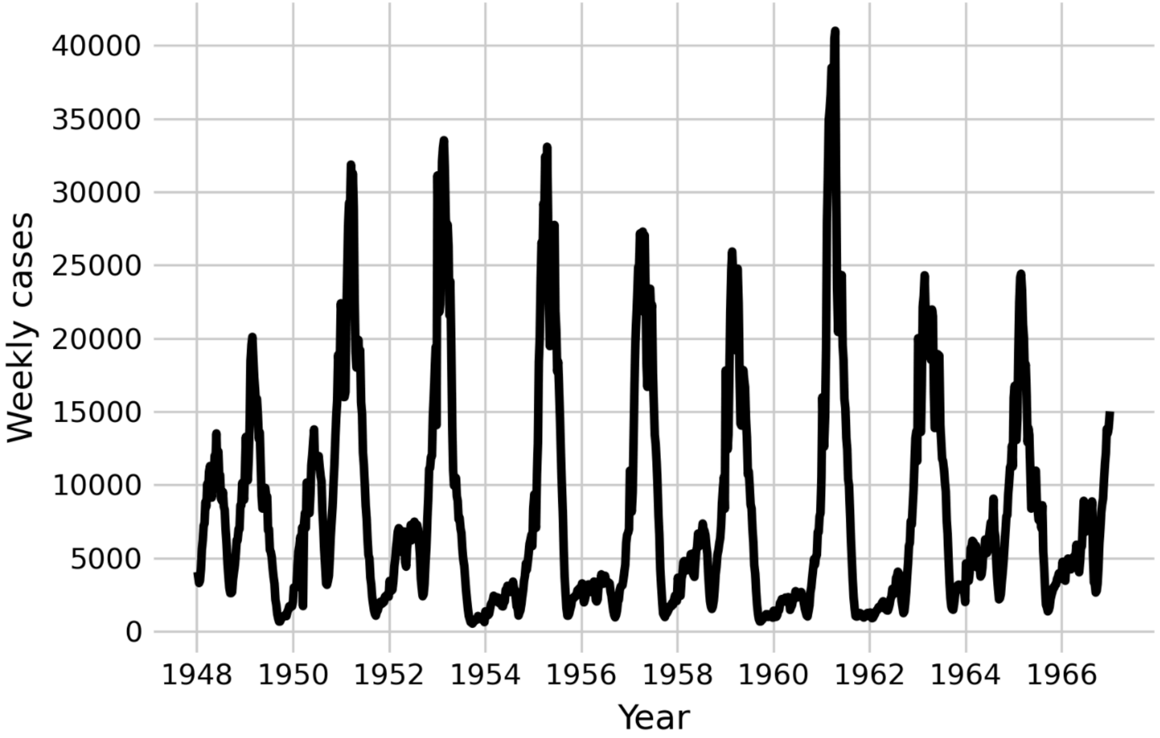
Modeling Infectious Diseases

An Open Epidemic With Seasonality

$$\beta \longrightarrow \beta(1 + \beta_S \, cos(2\pi t))$$

Modeling Infectious Diseases

An Open Epidemic With Seasonality



Modeling Infectious Diseases
Stochastic Models

- So far, all models were deterministic - run them again and they will produce exactly the same, predetermined result.

- Stochastic models integrate randomness - some decision are made by “rolling a dice” -> no two simulations will be identical.

Modeling Infectious Diseases
Gillespie Algorithm

- Stochastic simulation method:
  1. generates random times for each event
  2. updates states based on state transition probabilities

Modeling Infectious Diseases
Gillespie Algorithm

$$\frac{dS}{dt} = \mu N - \beta I \frac{S}{N} - \mu S$$

$$\frac{dE}{dt} = \beta I \frac{S}{N} - \sigma E - \mu E$$

$$\frac{dI}{dt} = \sigma E - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

Transition	Rate
$S \to S+1$	μN
$S \to S-1$	$\beta I \frac{S}{N} + \mu S$
$E \to E+1$	$\beta I \frac{S}{N}$
$E \to E-1$	$\sigma E + \mu E$
$I \to I+1$	σE
$I \to I-1$	$\gamma I + \mu I$
$R \to R+1$	γI
$R \to R-1$	μR

Modeling Infectious Diseases
Gillespie Algorithm

- Standard version:
  1. Calculate time to next event
  2. Choose the next transition, according to probabilities
- “tau leaping”:
  1. Fix regular time step tau
  2. Calculate all transitions that happen in that timestep