



BIO-463
Genomics and
bioinformatics

Lecture 8: Bulk gene expression analysis II

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EPFL

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mRNA metabolism and the GHI group

Recap' of week 7

Unsupervised clustering analysis

 Hierarchical agglomerative clustering

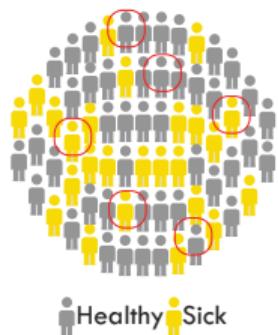
 Principal Component Analysis

 Singular Value Decomposition

Genomics and Health Informatics Group

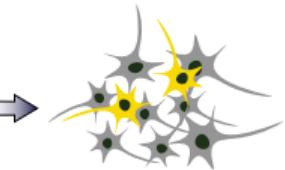
Towards the identification of early mechanisms underlying complex human diseases.

Large Cohort of Individuals

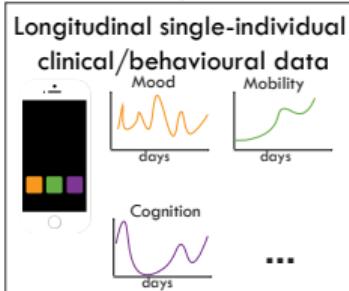
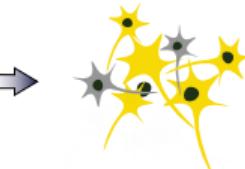


Heterogenous Cell Population

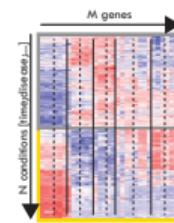
Healthy Controls



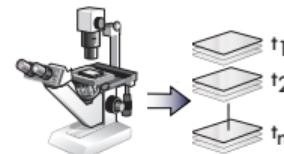
Sick Patients



Longitudinal Bulk RNA-seq

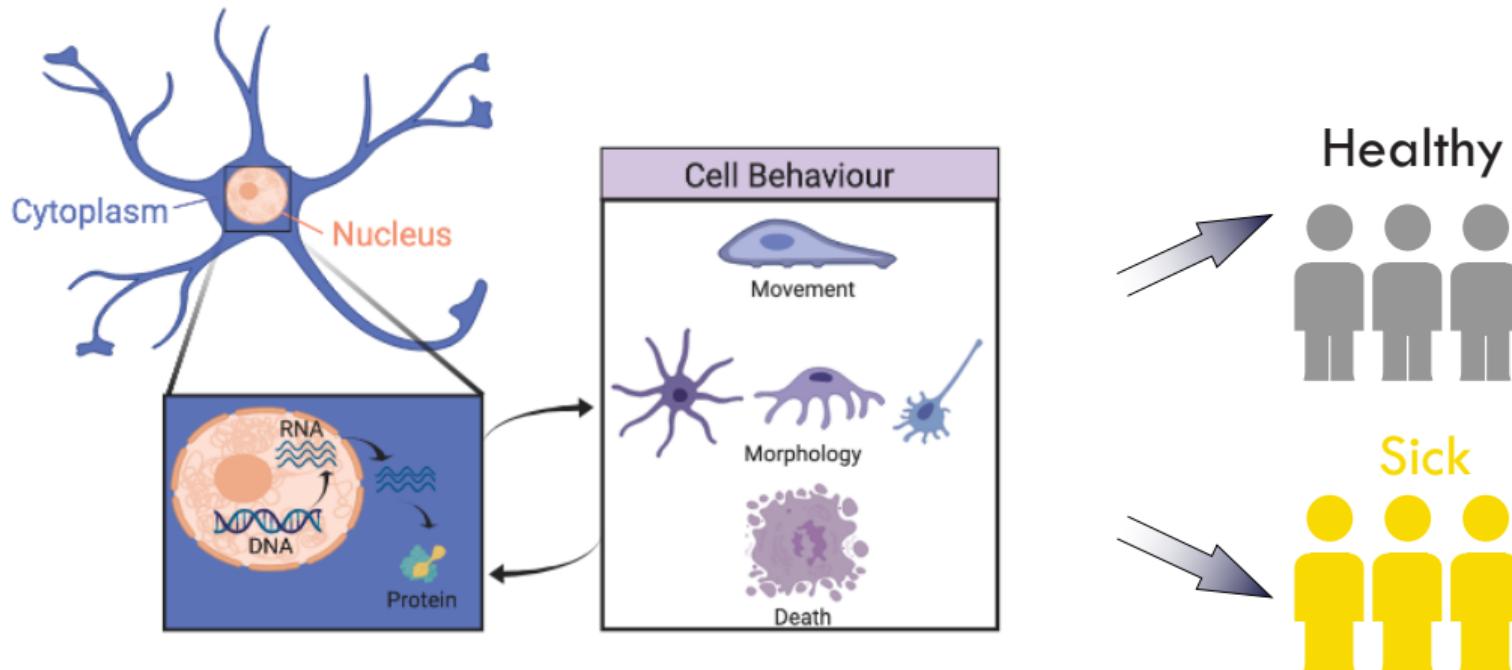


Longitudinal single-cell imaging data



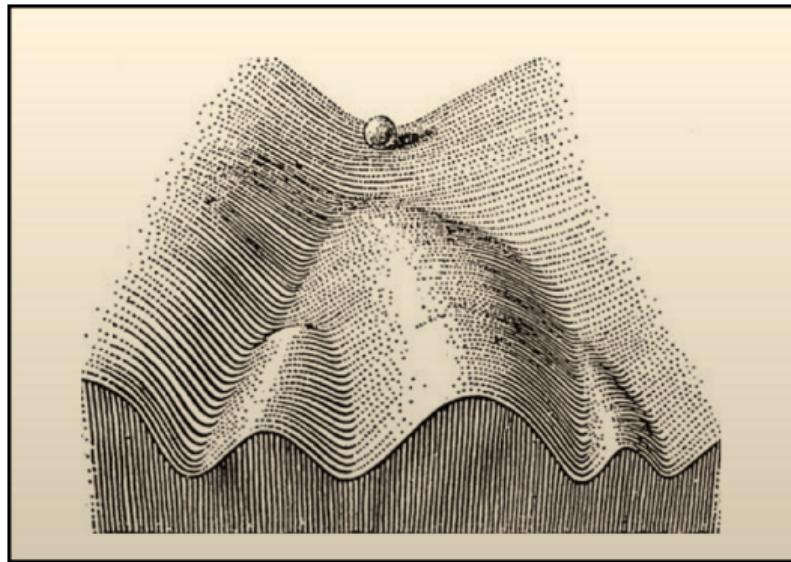
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Which molecular perturbations underly disease cellular states?



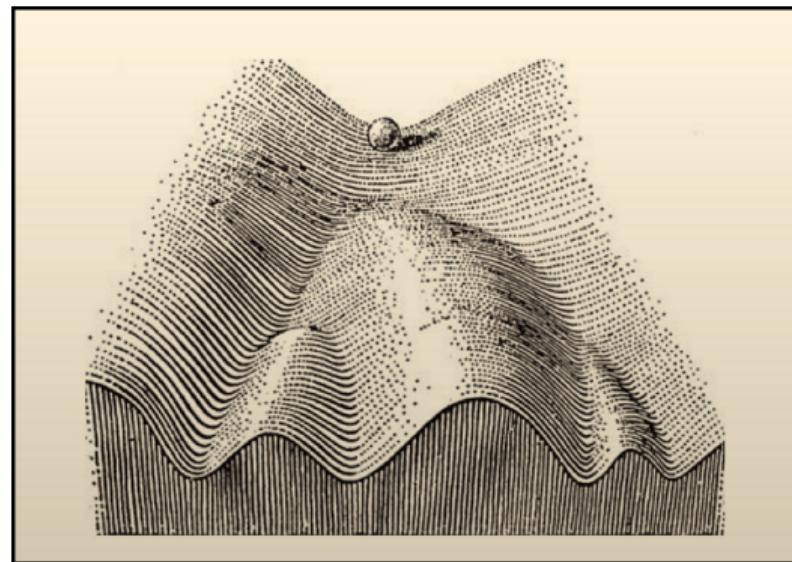
*“Cells are residents of a vast landscape of possible states,
over which they travel during **development** and in **disease**”*

C.H. Waddington



*“Cells are residents of a vast landscape of possible states,
over which they travel during **development** and in **disease**”*

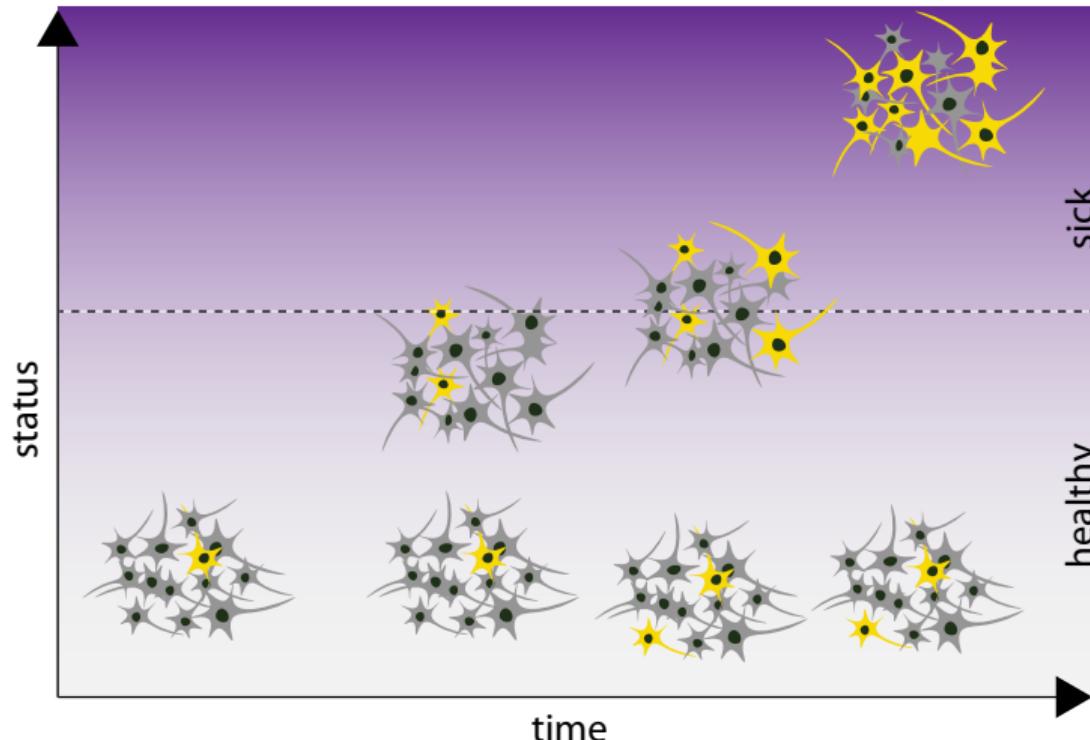
C.H. Waddington



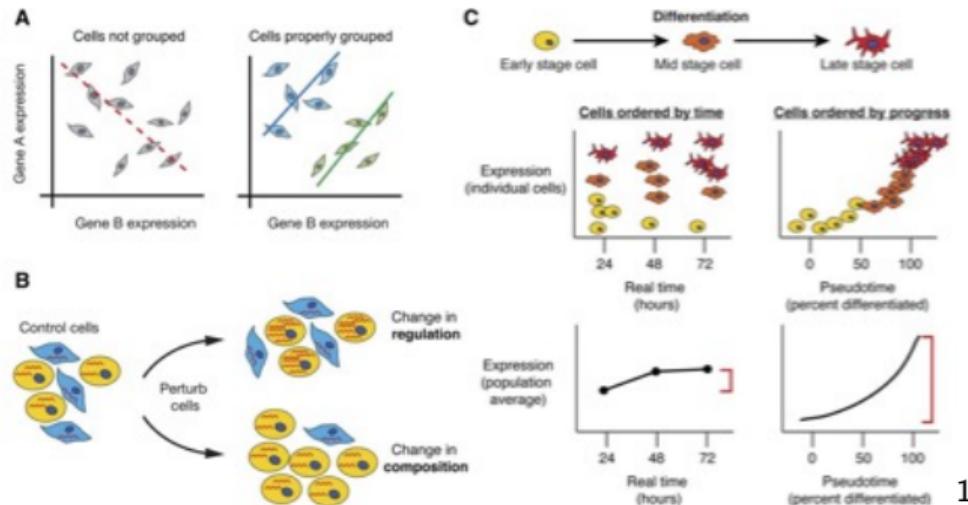
Locating the cells on this landscape == identify cell identity and state. 6/57

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Which molecular perturbations underly disease cellular states?



Simpson's Paradox (Simpson 1951)



Failing to properly compartmentalize the data by cell type leads to a qualitatively incorrect interpretation.

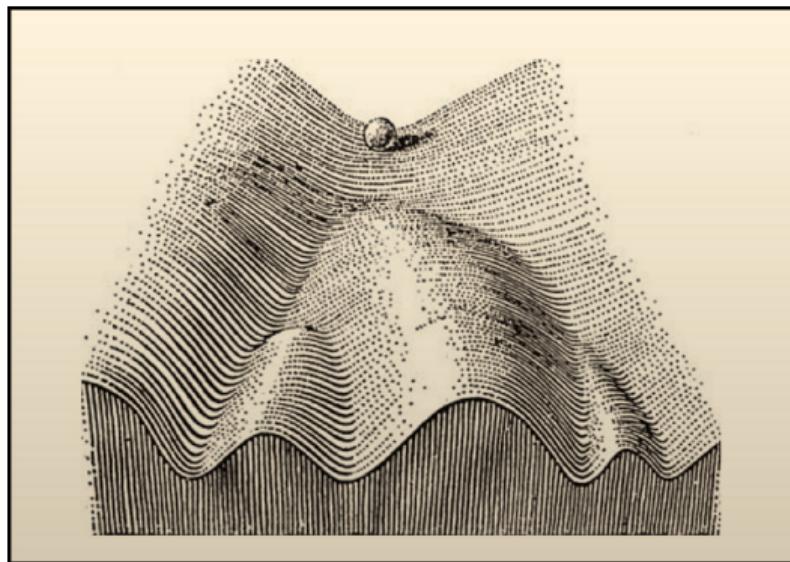
One cell type, several cell states

Cell state can be described as the range of cellular phenotypes arising from the interaction of a defined cell type with its environment.

- ▶ A meaningful definition of cell types must be associated with what cell types do².
- ▶ In response to diverse stimuli, the same cell type can exhibit a range of different phenotypes == states.
- ▶ Several cell states to address the diverse function of cell type.

*“Cells are residents of a vast landscape of possible states,
over which they travel during **development** and in **disease**”*

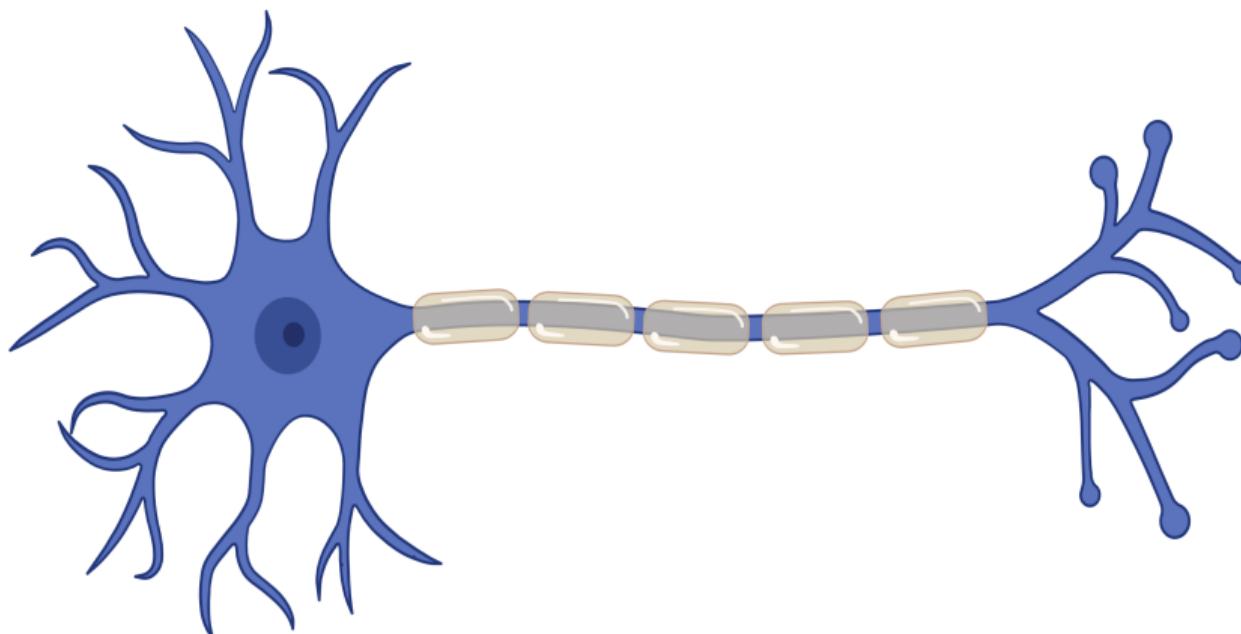
C.H. Waddington



The landscape might indeed change over time!

Decentralised biology of the neurons

Compartment-specific functions



Need to rapidly respond to external cues

Time-scale underlying transcriptional regulation

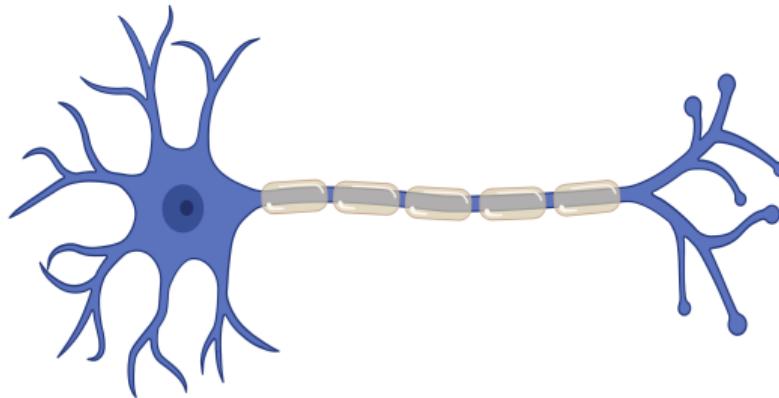
- ▶ For most genes, a new steady-state expression level is established by 120 mn.
- ▶ Estimated median mRNA half-life in human cells is 10 h³.
- ▶ mRNA transcription factors have shorter half-life (< 1 h)⁴

³Yang et al. 2003.

⁴Sharova et al. 2009.

Decentralised biology of the neurons

Compartment-specific functions



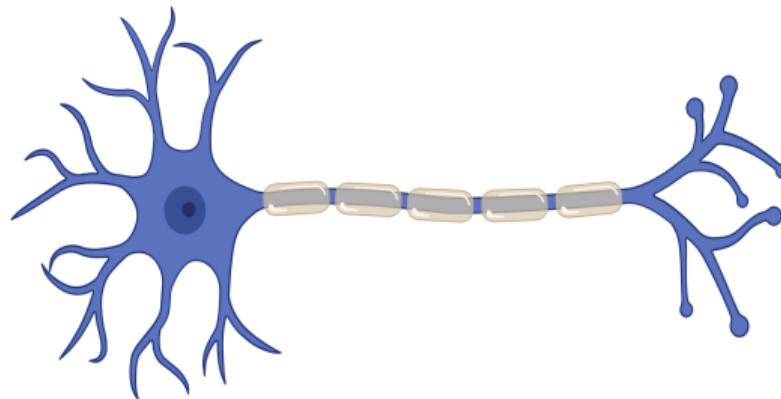
Need to rapidly respond to external cues.



Cannot depend on transcription.

Decentralised biology of the neurons

Compartment-specific functions



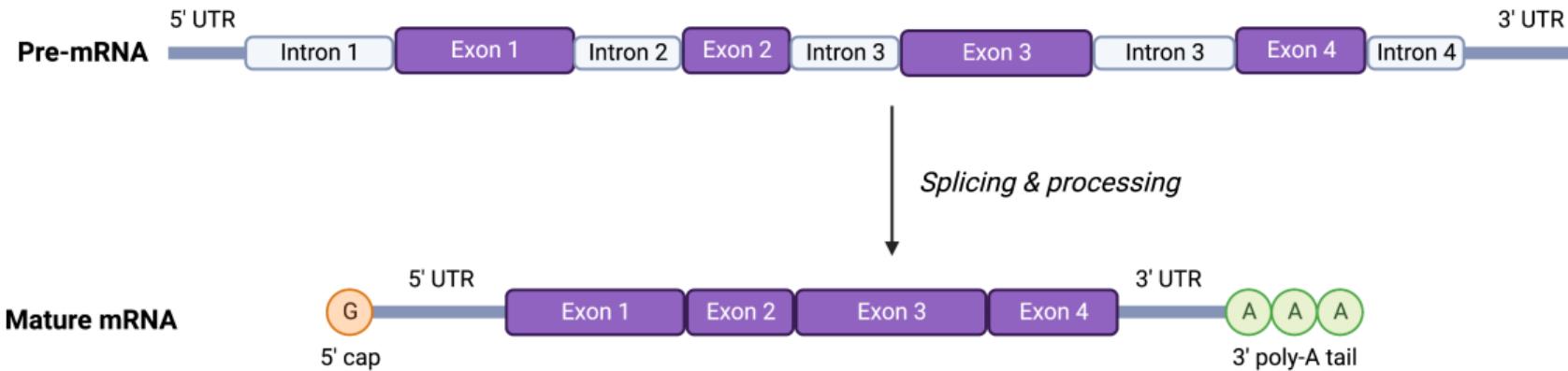
Neurons homeostasis relies on **local translation** through controlled regulation of axonal mRNA **localization, transport, and stability**

Gene expression analysis has been the major focus to precisely define **cellular states** and catalog them in development and disease, **yet**

- ▶ Regulation in gene expression does not enable fast response to environmental cues.
- ▶ Alternatively spliced variants can provide further information and help to refine cell types.⁵

One gene, several isoforms

What about their non-coding portions?



> 90% of the transcriptome does not encode for protein.

Key regulatory functions of 5' UTR, introns, and 3' UTR in health and disease.

Regulation by 5' UTR

*Key regulatory platform for translational efficiency*⁶

- ▶ Critical for ribosome recruitment and ultimately initiation of translation
- ▶ Switching between 5'UTR isoforms is a way to alter protein synthesis rates
- ▶ Can be comprehensively identified using CAGE-seq
- ▶ Impact in neuronal biology⁷ and disease such as cancer⁸

⁶Hinnebusch, Ivanov, and Sonenberg 2016; Jia et al. 2020.

⁷Biever, Donlin-Asp, and Schuman 2019.

⁸Weber et al. 2023.

Regulation by 3' UTR

Key regulatory platform for mRNA localisation, storage and translation

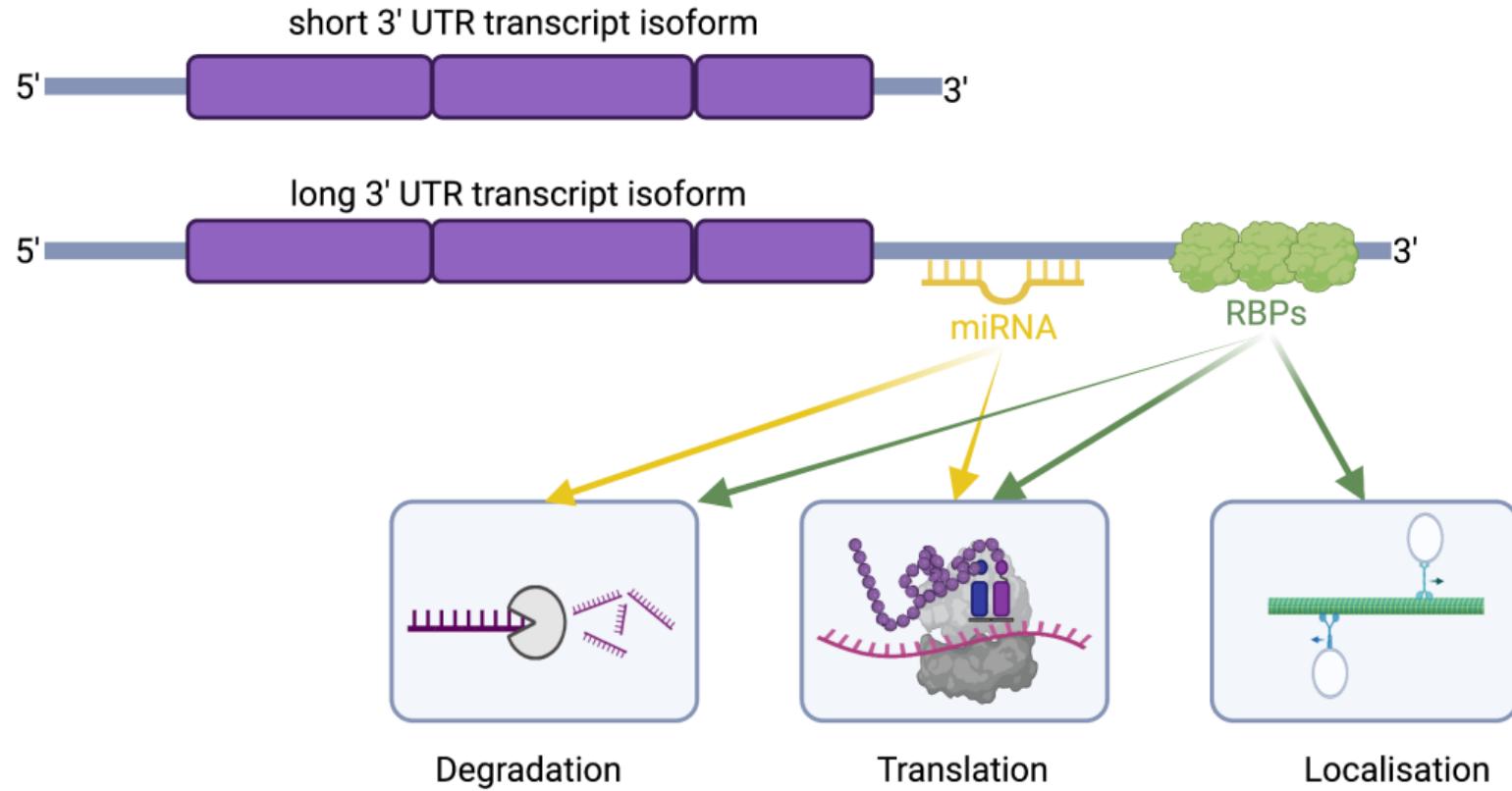
- ▶ Generated by alternative polyadenylation.
- ▶ Tandem 3' UTR: identical protein-coding sequence but different 3' end.
- ▶ APA isoforms are cell-type specific and change on activation of signaling pathway⁹
- ▶ 3' UTR isoforms can undergo cytoplasmic shorting upon external stimuli¹⁰

⁹Mayr 2019.

¹⁰Andreassi et al. 2021.

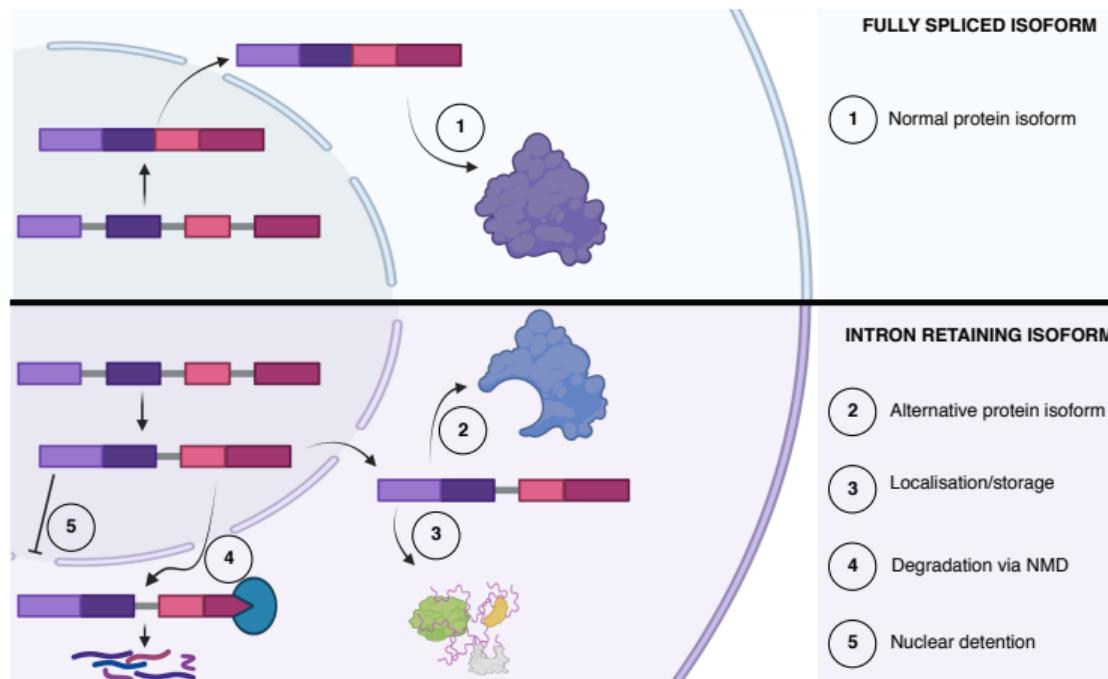
Regulation by 3' UTR

Key regulatory platform for mRNA localisation and translation



Regulation by intronic sequences

Emerging regulatory platform for mRNA and protein localisation



Alternative spliced isoforms enable
mRNA localisation, storage and translation.

And certainly much more!

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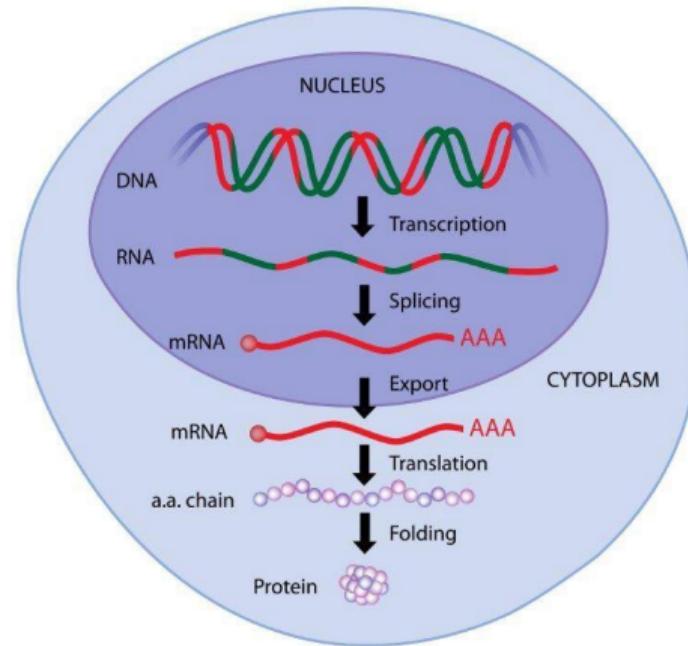
 Hierarchical agglomerative clustering

 Principal Component Analysis

 Singular Value Decomposition

RNA-sequencing data

What are we looking at?



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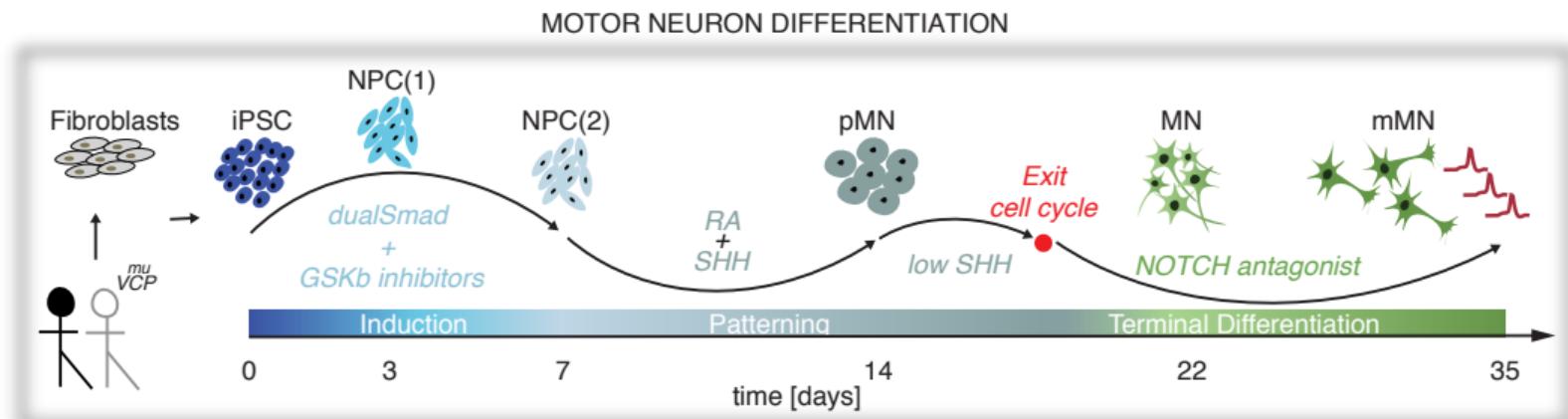
RNA-sequencing

Overview of a standard bioinformatic pipeline

1. Quality control (QC) of the fastq files (FASTQC)
2. Mapping of the reads onto a reference genome and/or transcriptome to obtain a count data table.
3. QC of the library
4. Pre-processing (statistical modelling)
5. Down-stream analysis
 - Unsupervised analysis (clustering)
 - Supervised analysis (DGE, pathway analysis)

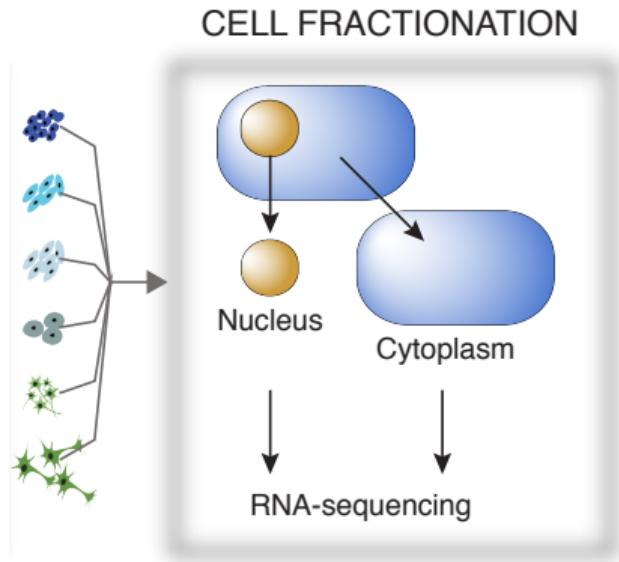
Bulk RNA-sequencing Analysis of Differentiating Motor Neurons

Data-set for practical session



Bulk RNA-sequencing Analysis of Differentiating Motor Neurons

Data-set for practical session



RNA-sequencing

Pre-processing of the read count table

1. Log-transformation
2. Filter-out lowly expressed genes
3. Normalisation

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Compute pair-wise distance between samples

- ▶ Flexible distance metrics between samples.
- ▶ Euclidean, correlation-based (Pearson or Spearman).

Agglomerative clustering of the samples

- ▶ Linkage criterion determines the distance between sets of observations as a function of the pairwise distances between observations.
- ▶ Linkage criterion influences shape of the clusters.
- ▶ The definition of shortest distance is what differentiates between the different agglomerative clustering methods.
- ▶ Complete-linkage tends to produce more spherical clusters than single-linkage.
- ▶ Single-linkage tends to produce long thin clusters in which nearby elements of the same cluster have small distances

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Principal Component Analysis

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Unsupervised clustering analysis

Principal Component Analysis

- ▶ To transform a set of possibly correlated variables (genes) into "some more fundamental set of independent variables".
- ▶ To project a dataset from **many correlated** coordinates onto **fewer uncorrelated** coordinates
- ▶ The orthogonal coordinates are called principal components (PCs).
- ▶ PC retain most variability in the data.

Unsupervised clustering analysis

Principal Component Analysis

- ▶ To transform a set of possibly correlated variables (genes) into "some more fundamental set of independent variables".
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- ▶ The orthogonal coordinates are called principal components (PCs).
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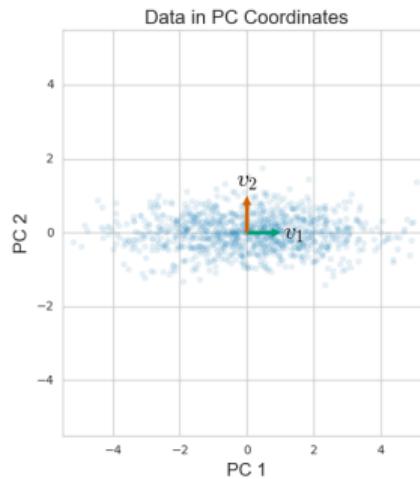
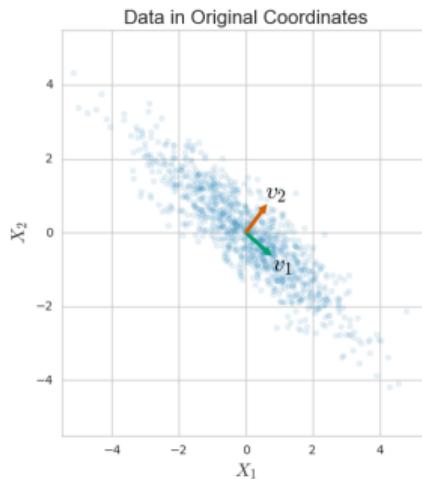


- To reduce the dimensionality of data
- To extract essential information
- To characterize the structure of the data.

Principal Component Analysis

Objective

- ▶ Let $\mathbf{X} \in \mathbb{R}^{n \times d}$ be the data matrix, mean-centered.
- ▶ Find \mathbf{v}_1 such that:
 1. $\|\mathbf{v}_1\| = 1$
 2. Variance of projection onto \mathbf{v}_1 is maximized



Principal Component Analysis

Covariance and projection

- The projection of a vector $\mathbf{x} \in \mathbb{R}^n$ onto \mathbf{v}_1 is given by $\mathbf{v}_1^T \mathbf{x}$.
- Assuming \mathbf{X} is mean-centered, the sample variance of the projection onto \mathbf{v}_1 is:

$$\text{Var}(\mathbf{X}\mathbf{v}_1) = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{v}_1^T \mathbf{x}_i)^2 = \mathbf{v}_1^T \left(\frac{1}{n-1} \mathbf{X}^T \mathbf{X} \right) \mathbf{v}_1$$

- The covariance matrix is defined as:

$$\text{Cov}(\mathbf{X}) = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{x}_i - \boldsymbol{\mu})(\mathbf{x}_i - \boldsymbol{\mu})^T = \frac{1}{n-1} \mathbf{X}^T \mathbf{X}$$

Principal Component Analysis

Solution

The variance of \mathbf{X} projected onto \mathbf{v}_1 must be maximized.

$$\text{Var}(\mathbf{X}\mathbf{v}_1) = \mathbf{v}_1^T \left(\frac{1}{n-1} \mathbf{X}^T \mathbf{X} \right) \mathbf{v}_1 = \mathbf{v}_1^T \text{Cov}(\mathbf{X}) \mathbf{v}_1$$

↓

$$\text{Cov}(\mathbf{X})\mathbf{v}_1 = \lambda_1 \mathbf{v}_1 \quad \Rightarrow \quad \mathbf{v}_1^T \text{Cov}(\mathbf{X}) \mathbf{v}_1 = \lambda_1$$

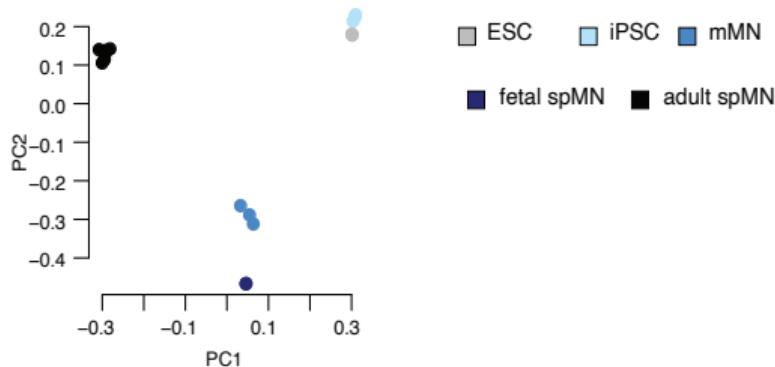
\mathbf{v}_1 and λ_1 are an eigenvector and its associated eigenvalue of the covariance matrix.

↓

The optimal \mathbf{v}_1 is the eigenvector corresponding to the largest eigenvalue of $\text{Cov}(\mathbf{X})$.

$$\text{Cov}(\mathbf{X}) = \frac{1}{n-1} \mathbf{X}^T \mathbf{X} = \mathbf{V} \boldsymbol{\Sigma} \mathbf{V}^T$$

Principal Component Analysis



- ▶ Most biologists focus on the clustering of the samples in PC1 and PC2.
- ▶ This is ok for simple experimental set-up (2 or 3 covariates).
- ▶ However what if more complex experiment?



More subtle but biologically relevant signal might be captured in other components.

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Principal Component Analysis

Singular Value Decomposition

Singular Value Decomposition (SVD)

Let $\mathbf{M} \in \mathbb{R}^{g \times s}$ be a real-valued gene expression matrix, with g genes and s samples ($g \geq s$).

Each entry m_{ij} represents the expression of gene i in sample j .

- ▶ $\vec{g}_i \in \mathbb{R}^s$: expression profile of gene i across samples
- ▶ $\vec{s}_j \in \mathbb{R}^g$: expression profile of sample j across genes

$$\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^T$$

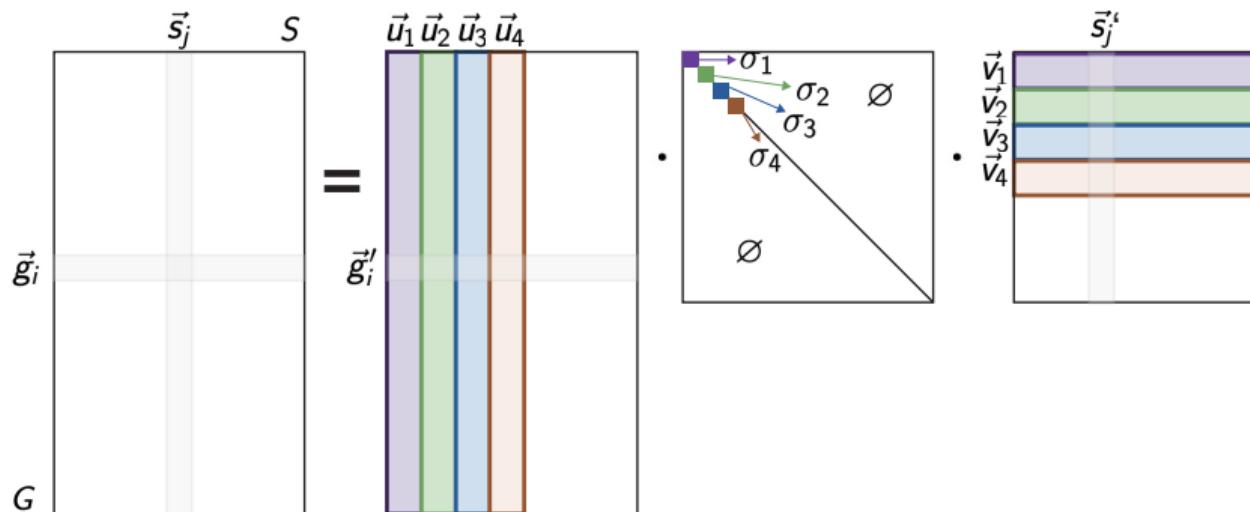
where:

- ▶ $\mathbf{U} \in \mathbb{R}^{g \times s}$: orthonormal basis for genes
- ▶ $\Sigma \in \mathbb{R}^{s \times s}$: diagonal matrix of singular values
- ▶ $\mathbf{V}^T \in \mathbb{R}^{s \times s}$: orthonormal basis for samples

Singular Value Decomposition (SVD) Analysis

Definition

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \mathbf{\Sigma}_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$

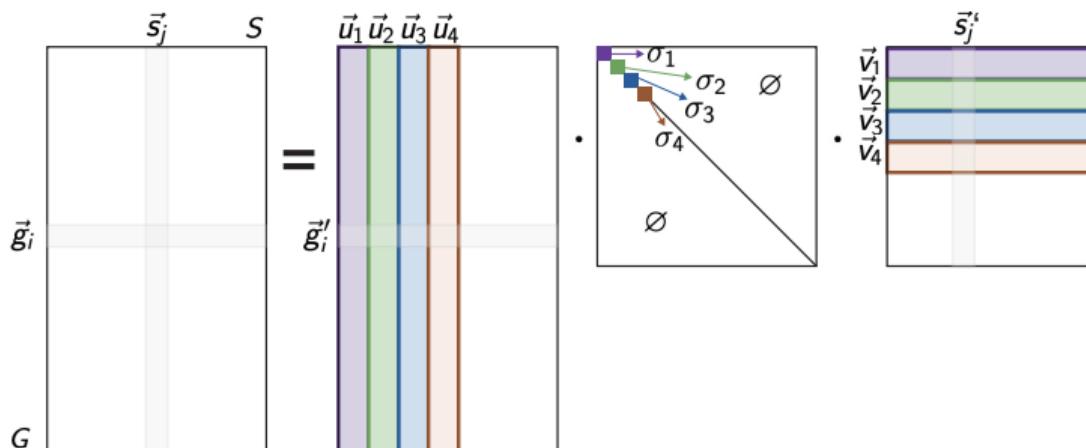


where \vec{s}_j and \vec{g}_i are the expression profiles of **sample j** and **gene i** .

Singular Value Decomposition (SVD)

Definition

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \boldsymbol{\Sigma}_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$

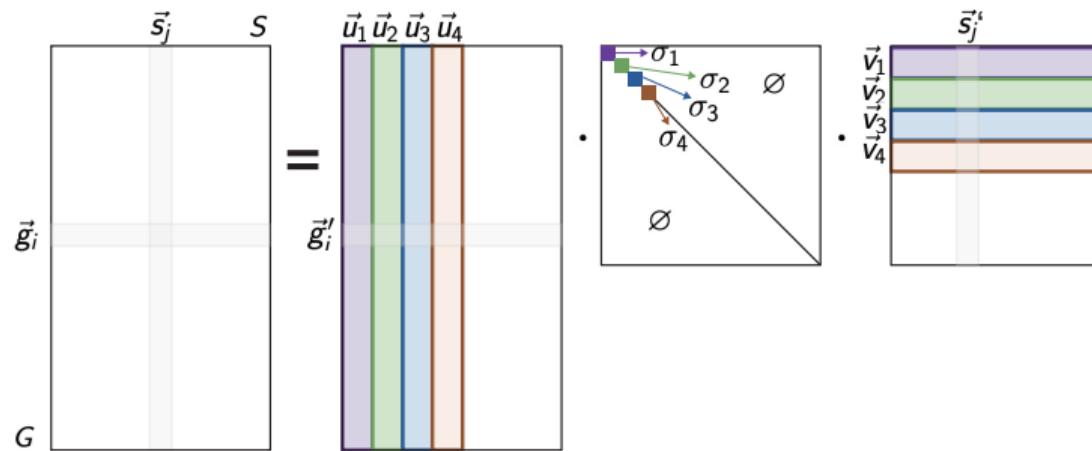


- ▶ the **left** singular vectors $\{\vec{u}_k\}$ form an orthonormal basis of the space of **sample** expression profiles so that $\mathbf{u}_i \cdot \mathbf{u}_j = 1$ for $i = j$ and 0 otherwise.
- ▶ the **right** singular vectors $\{\vec{v}_k\}$ form an orthonormal basis of the space of **gene** expression profiles so that $\mathbf{v}_i \cdot \mathbf{v}_j = 1$ for $i = j$ and 0 otherwise.

Singular Value Decomposition (SVD)

Important results

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \mathbf{\Sigma}_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$



- $M^{(l)} = \sum_{k=1}^l \mathbf{u}_k s_k \mathbf{v}_k^T$ is the closest rank- (l) matrix to M i.e. $M^{(l)}$ minimizes $\sum_{ij} \|m_{ij} - m_{ij}^{(l)}\|^2$

Singular Value Decomposition (SVD)

How to get U , Σ and V

$$M^T M = V \Sigma^2 V^T \quad (5)$$

$$U = X V \Sigma^{-1} \quad (6)$$

Relation Between SVD and PCA

When PCA is performed via the covariance matrix, it is closely related to the SVD of the data matrix \mathbf{M} .

Assuming \mathbf{M} is column-centered:

- ▶ $\mathbf{M}^T \mathbf{M} \propto \text{Cov}(\mathbf{M})$
- ▶ Diagonalizing $\mathbf{M}^T \mathbf{M}$ gives \mathbf{V} , the right singular vectors, which are the principal directions (PCs).
- ▶ The matrix $\mathbf{U}\Sigma$ gives the principal component scores (coordinates of genes in PC space).

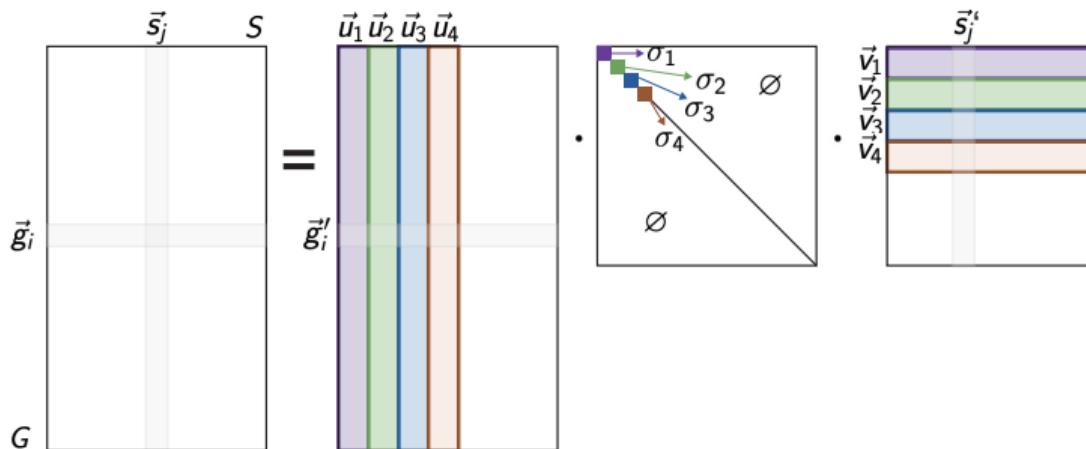
Alternatively, if rows of \mathbf{M} are centered:

- ▶ $\mathbf{M}\mathbf{M}^T \propto \text{Cov}(\text{genes})$
- ▶ The left singular vectors \mathbf{U} then correspond to the principal components of gene expression profiles.

Singular Value Decomposition (SVD)

Definition

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \boldsymbol{\Sigma}_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$

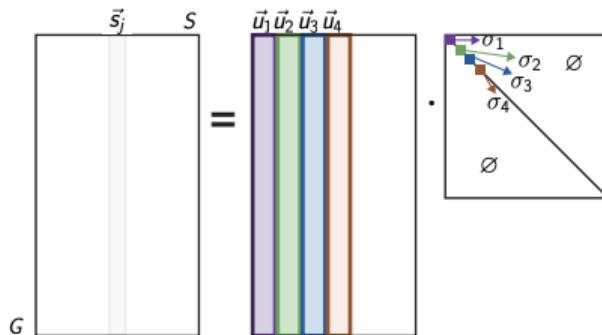


- ▶ The columns vectors $\{\vec{v}_k\}$ with $k \in [1 : S]$ are orthogonal and compose the right singular vectors.
- ▶ Each \vec{v}_k can be thought as a linear combination of $\{\vec{g}_i\}$
- ▶ Each \vec{g}_i can be thought as a linear combination of $\{\vec{v}_k\}$.

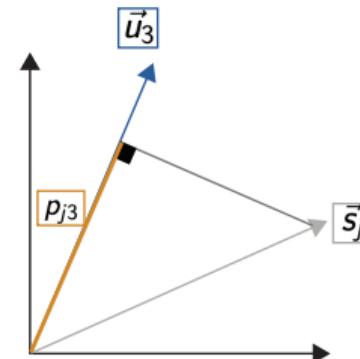
Singular Value Decomposition (SVD)

How individual samples relate to singular vectors

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \mathbf{\Sigma}_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$



$$p_{j3} = \vec{s}_j \cdot \vec{u}_3$$

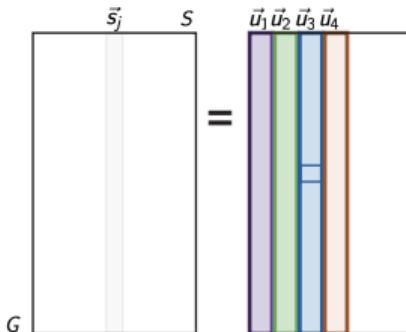


- ▶ The projection of \vec{s}_j onto the k^{th} left singular vector \vec{u}_k is p_{jk}

Singular Value Decomposition (SVD)

How individual samples relate to singular vectors

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \Sigma_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$

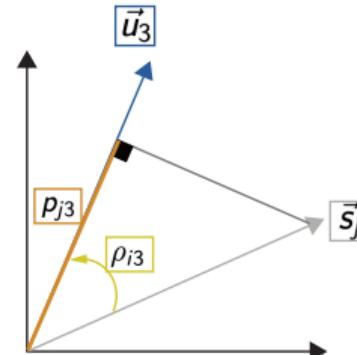


PROJECTION

$$p_{j3} = \vec{s}_j \cdot \vec{u}_3 = (V\Sigma)_{j3} \propto v_{j3}$$

CORRELATION

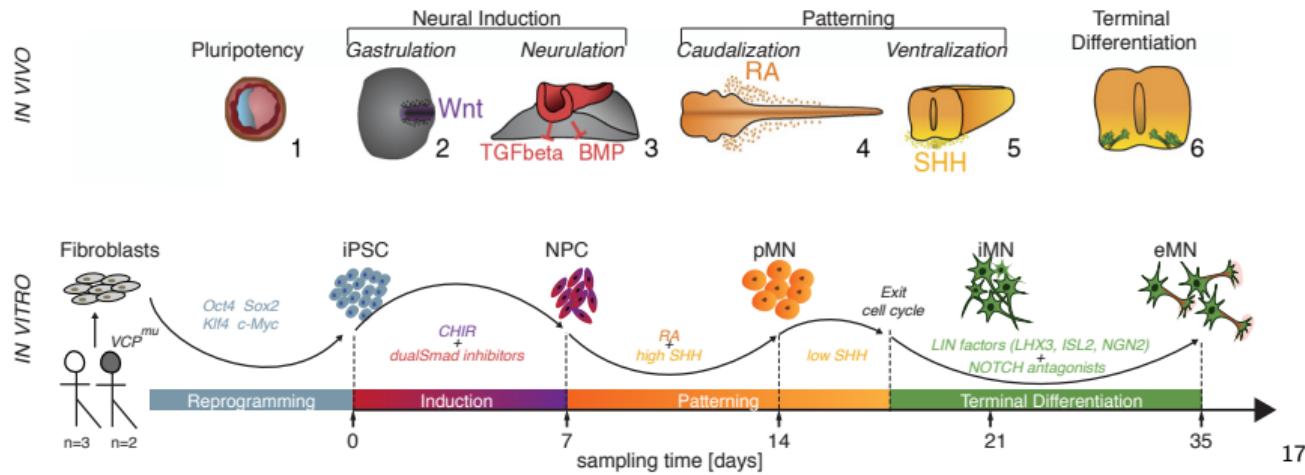
$$\rho_{j3} = \frac{\vec{s}_j \cdot \vec{u}_3}{\|\vec{s}_j\| \cdot \|\vec{u}_3\|} = \frac{p_{j3}}{\sqrt{\sum_{k'} (p_{jk'})^2}}$$



- ▶ The projection of \vec{s}_j onto the k^{th} left singular vector \vec{u}_k is p_{jk}
- ▶ $p_{jk} = \vec{s}_j \cdot \vec{u}_k = (M^T U)_{ik} = (U^T M)_{ik} = (V\Sigma)_{jk} \propto v_{jk}$
- ▶ v_{jk} represents how much a sample contributes to the left singular vector \vec{u}_k .

Singular Value Decomposition (SVD)

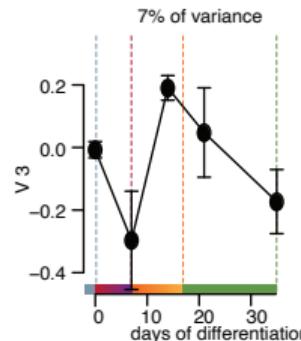
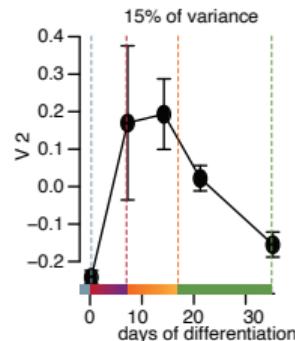
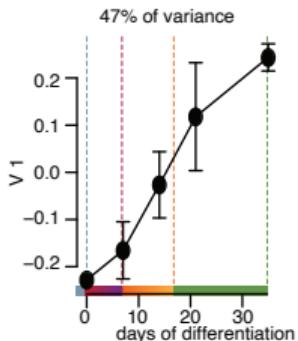
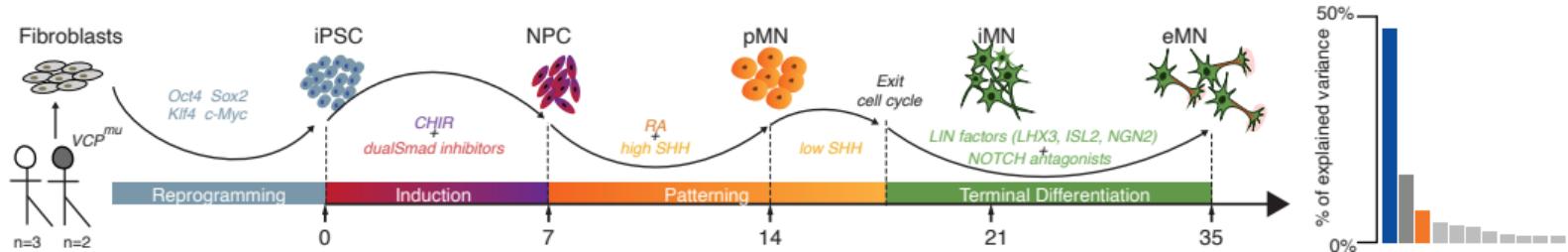
The analysis of the samples loadings onto v_k



¹⁷ Adapted from "Modeling ALS with motor neurons derived from human induced pluripotent stem cells" by Samuel Sances, Lucie I Bruijn, Siddharthan Chandran et al. 2016, Nat Neurosci, 19(4). Copyright 2016 Nature America, Inc. All rights reserved.

Singular Value Decomposition (SVD)

The analysis of the samples loadings onto v_k



Singular Value Decomposition (SVD)

How individual genes relate to singular vectors

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \boldsymbol{\Sigma}_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$

\vec{g}_i is a column vector of size G representing a gene. It is decomposed into:

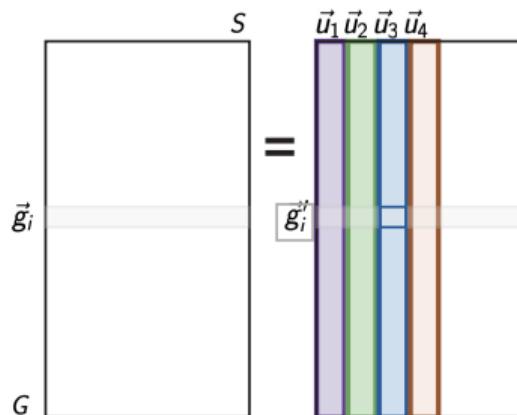
$$\vec{g}_i = \vec{u}_1 \vec{u}_2 \vec{u}_3 \vec{u}_4 \cdot \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \sigma_4 \end{bmatrix} \cdot \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{bmatrix}$$

The diagram illustrates the decomposition in a 2D coordinate system. The original gene vector \vec{g}_i is shown as a grey arrow originating from the origin. It is projected onto the first singular vector \vec{u}_3 (blue arrow), which is the first column of \mathbf{U} . The error vector \vec{v}_3 (blue arrow) is the difference between the original vector and its projection onto \vec{u}_3 .

Singular Value Decomposition (SVD)

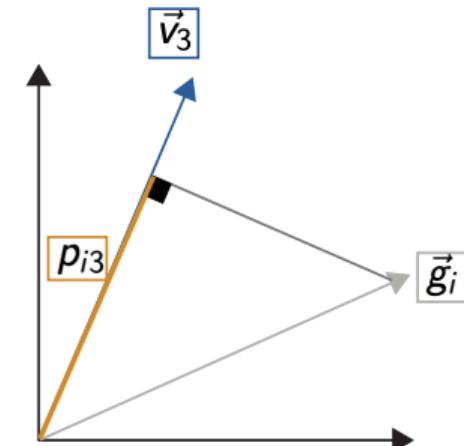
How individual genes relate to singular vectors

$$\mathbf{M}_{[G \times S]} = \mathbf{U}_{[G \times S]} \cdot \mathbf{\Sigma}_{[S \times S]} \cdot \mathbf{V}^T_{[S \times S]}$$



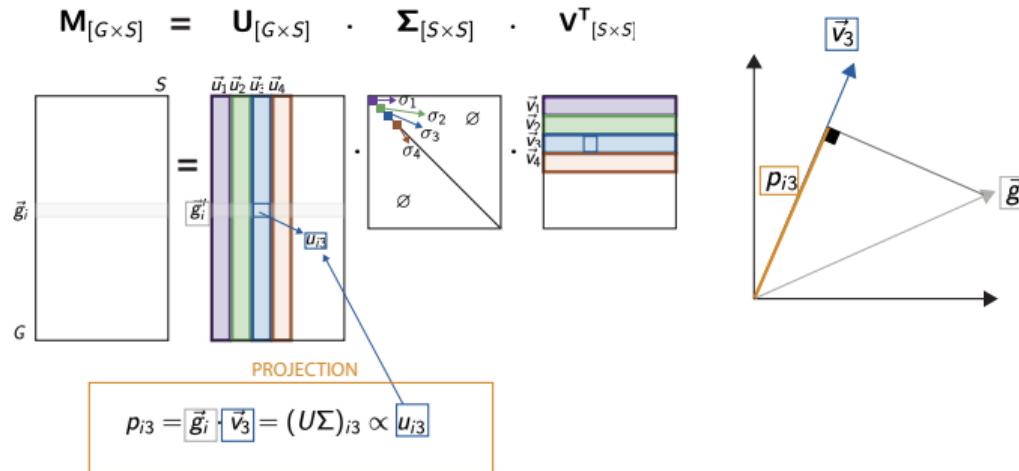
PROJECTION

$$p_{i3} = \vec{g}_i \cdot \vec{v}_3$$



Singular Value Decomposition (SVD)

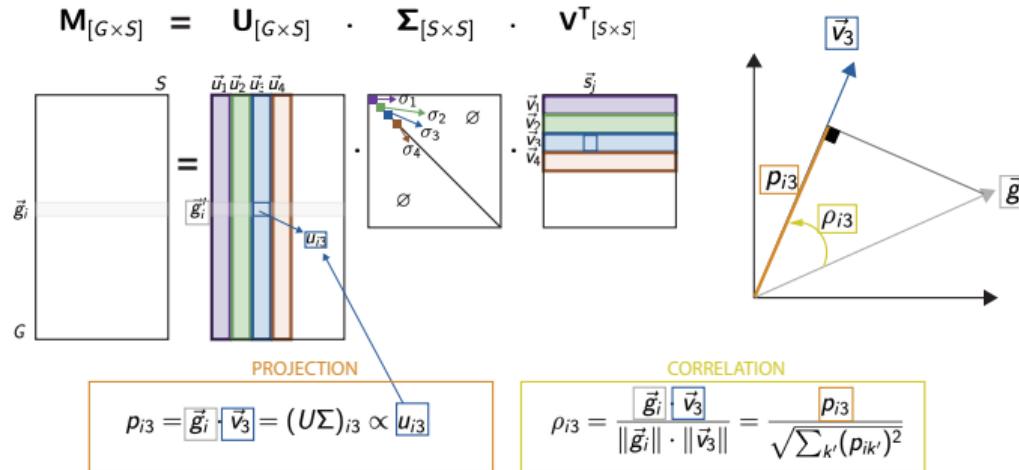
How individual genes relate to singular vectors



- ▶ The projection of \vec{g}_i onto the k^{th} *right* singular vector \vec{v}_k is p_{ik}
- ▶ $p_{ik} = \vec{g}_i \cdot \vec{v}_k = (MV)_{ik} = (U\Sigma)_{ik} \propto u_{ik}$
- ▶ u_{ik} represents how much a gene contributes to the *right* singular vector \vec{v}_k .

Singular Value Decomposition (SVD)

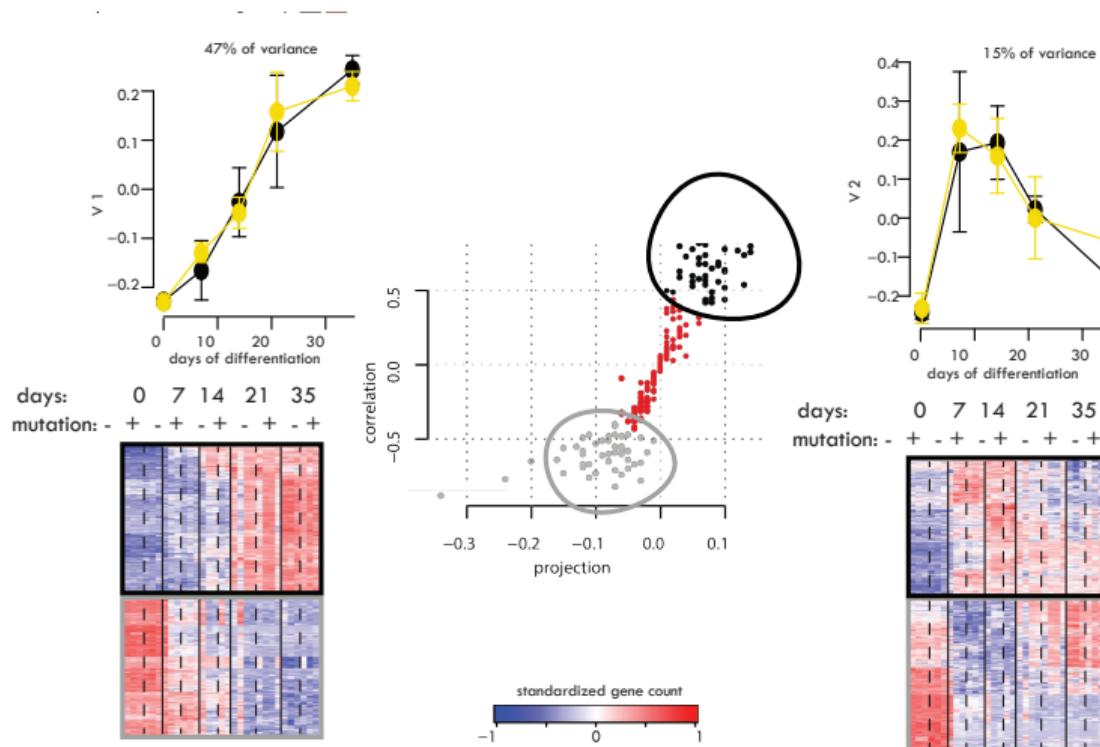
How individual genes relate to singular vectors



- ▶ The projection of \vec{g}_i onto the k^{th} *right* singular vector \vec{v}_k is p_{ik}
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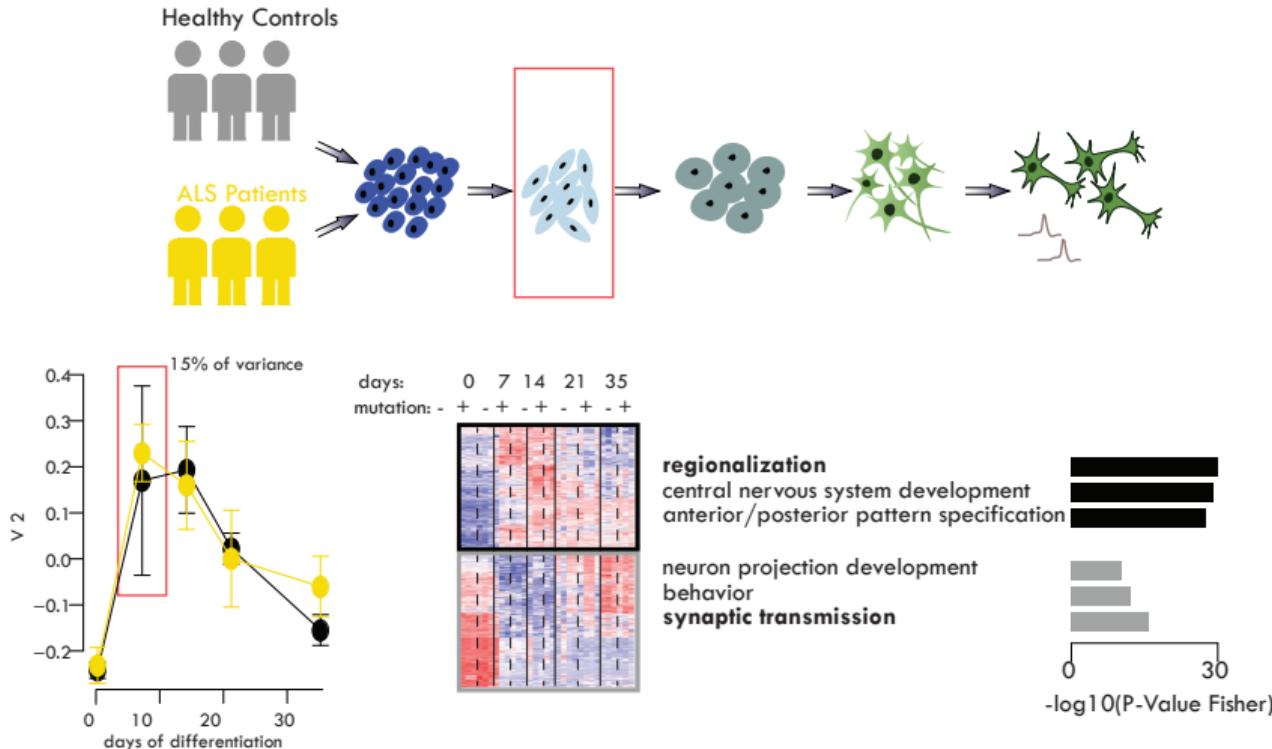
Singular Value Decomposition (SVD)

Extract most contributing and correlating genes



Singular Value Decomposition (SVD)

Biological Pathway Enrichment Analysis



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