

**Course:** BIO-341 *Dynamical systems in biology*

**Professor:** Julian Shillcock & Felix Naef

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Note that this document is primarily aimed at being consulted as a Jupyter notebook, the PDF rendering being not optimal.

```
In [5]: #import important libraries
import numpy as np
import matplotlib.pyplot as plt
from ipywidgets import interact
from scipy.integrate import odeint
from IPython.display import set_matplotlib_formats
from matplotlib.markers import MarkerStyle
set_matplotlib_formats('png', 'pdf')
```

## Population dynamics model with a limit cycle

In the course, we studied predator-prey models with a stable spiral, which means that the two populations settle to coexist after the oscillatory transient decay.

Here, we study the following model:

$$\frac{dN}{dt} = a N(K - N) - c P \frac{N}{N + R} \quad (1)$$

$$\frac{dP}{dt} = b P (sN - P) \quad (2)$$

where the predators  $P$  might represent C. Elegans worms feeding on E. Coli bacteria  $N$  (typically used as food in laboratory dishes). All parameters should be taken positive ( $> 0$ ).

**1) Explain the different parameters in the model (a,b, c, R, K, s). What are their units?**

**Hint:** Make the connection with the logistic growth model.

**2) Calculate and plot the nullclines for the following values of the parameters:**

**a = b = 0.01**

**c = 1.**

**K = 200**

**R = 50**

**s = 5**

**Plot the fixed points and qualitatively describe their meanings in terms of populations of predators and preys**

**3) Plot the stability of the fixed point with both  $N, P \neq 0$  in the R-K plane ( for K, R in [5, 500])**

**4) Fix K, and use the same values of the parameters for a,b,c and s as above. Choose 3 values of R for which you have respectively a stable f.p., a stable spiral and a limit cycle.**

**Simulate the trajectories (of N and P) using a python solver (ex: odeint). Plot the trajectories in function of time and the phase portrait (P in function of N) (use subplots) for the three cases with different initial conditions.**

**In the case of the limit cycle, what is the stability of the fixed point?**

```
In [41]: # paramters:
a = 0.01
b = 0.01
c = 1
K = 200
s = 5
tspan = np.linspace(0,100, 10000)
```

**5) Describe in words the behavior of the trajectories in terms of the number of predators and preys for each case.**

**6. (OPTIONAL): Check your answers using euler's method**

Euler's method is the simplest way to solve a differential equation numerically. In order to approximate the solution of :

$$\dot{x} = F(x(t)), x(t_0) = x_0$$

We can write one step of the method as :

$$x(t + dt) \simeq x(t) + dt F(x(t))$$

for a specific timestep size dt.

**a) Implement your own Euler method using Python to solve numerically the following differential equation:**

**b) Simulate the same trajectories than in 4) using the Euler's method.**

```
In [ ]:
```