Allee Effect Exercise Notes

20/9/24 $\dot{N} = rN - \frac{N}{a}(N-b)^2$ $a, b, r \geq 0$ constants

where are the fixed points? obviously $N^* = 0$ is always one.

consider the relative growth rate $\dot{N}/N$:

$$\frac{\dot{N}}{N} = r - \frac{(N-b)^2}{a}$$

$$\Rightarrow a\left(\frac{\dot{N}}{N}\right) = ar - (N-b)^2$$

i.e. a and r appear only together, so the effect of varying one is the same as varying the other.

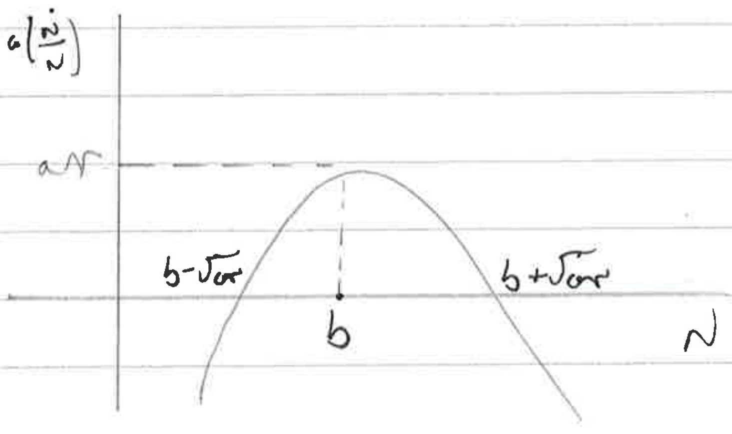
note that multiplying $\frac{\dot{N}}{N}$ by a does NOT change anything qualitatively because $a > 0$

Are there more fixed points?

$$a\left(\frac{\dot{N}}{N}\right) = 0 \Rightarrow (N-b)^2 = ar$$

$$\therefore N^* = b \pm \sqrt{ar}$$

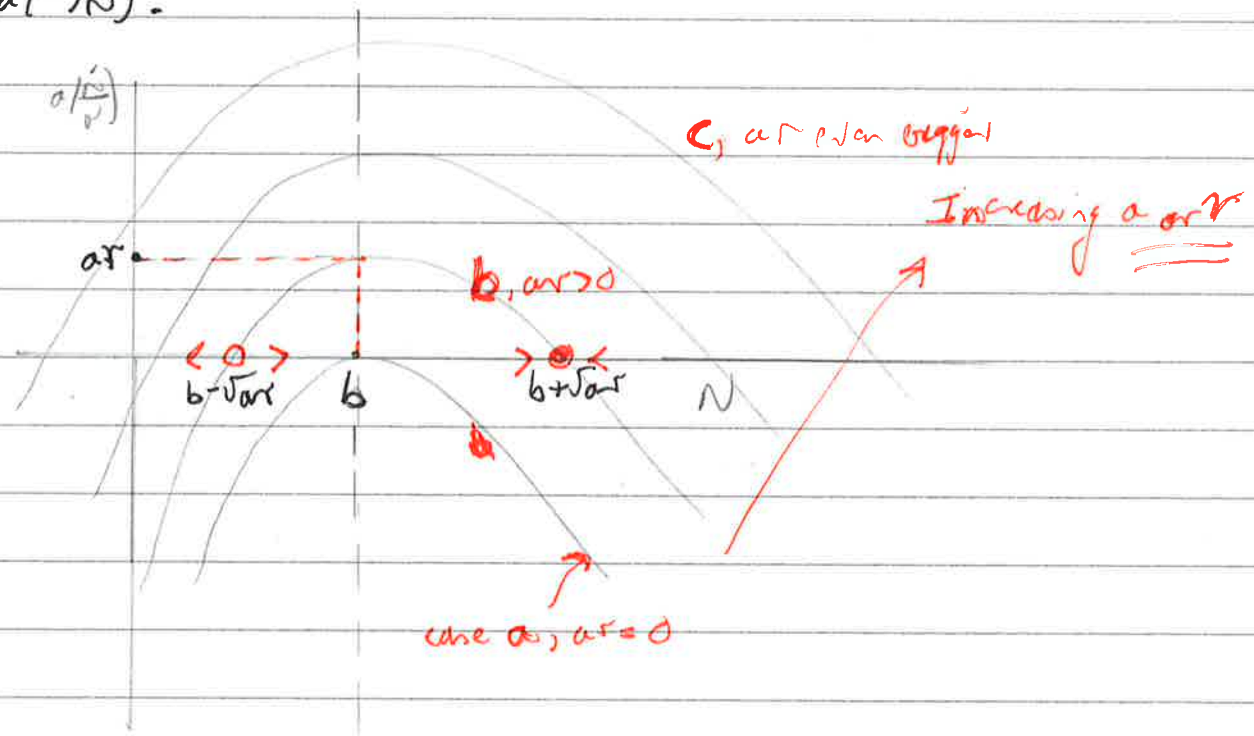
what does $a(\dot{N}/N)$ look like?



note that the peak is always at $N = b$

$$\frac{d(\dot{N}/N)}{dN} = -2(N-b).1 = 0 \therefore N = b.$$

Now keep b constant and let ar decrease and increase; what happens to $d(\dot{N}/N)$?

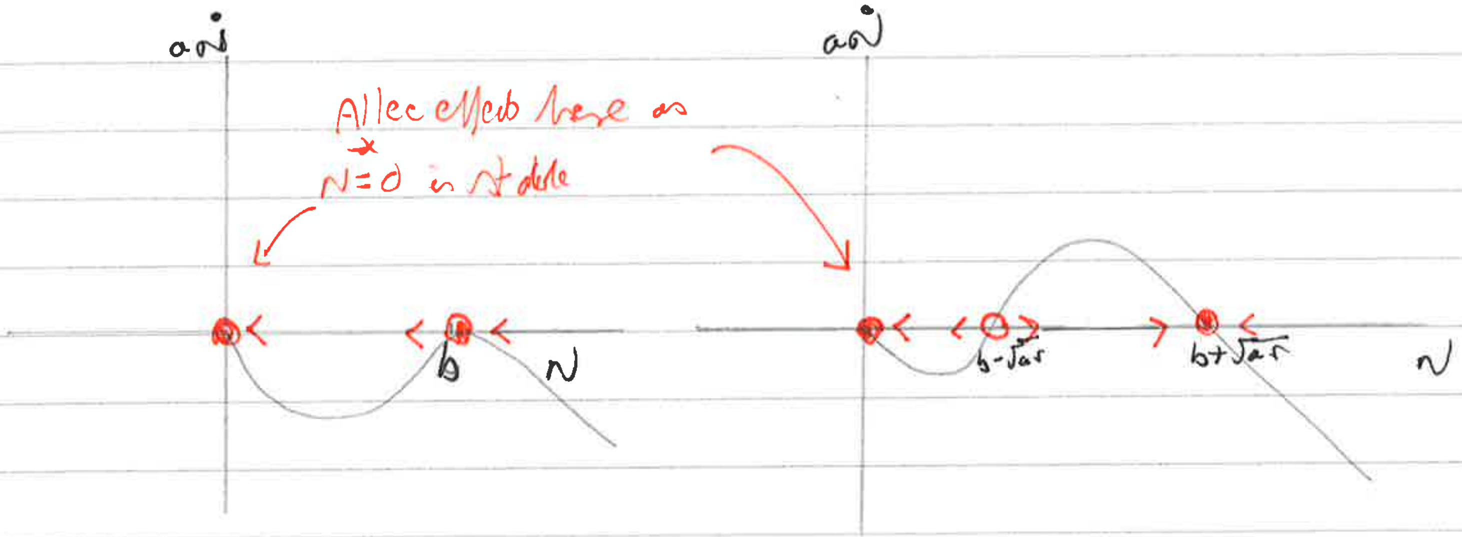


1) Recall that $N^* = 0$ is always a fixed point, but is not shown here.

2) Note that the higher fixed point ($N^* = b + \sqrt{ar}$) is always stable and the lower one ($N^* = b - \sqrt{ar}$) is always unstable.

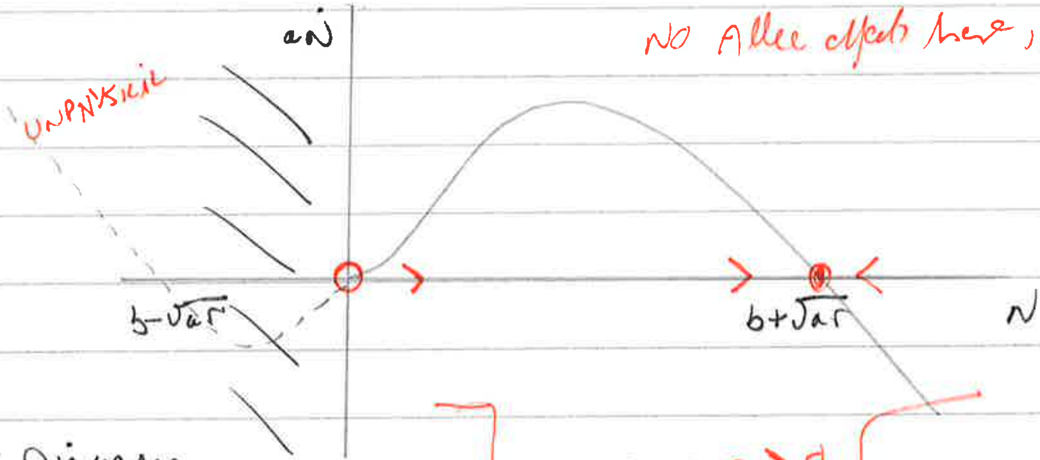
3) The lower fixed point becomes unphysical when $b < \sqrt{ar}$ because then $N^* < 0$, and N is a population and must be greater or equal to zero.

What does $a\dot{N}$ look like for different values of a keeping b, r fixed? We can multiply the curves in the above plot by N and plot them, knowing that $a\dot{N} = 0$ whenever it crosses the x axis.



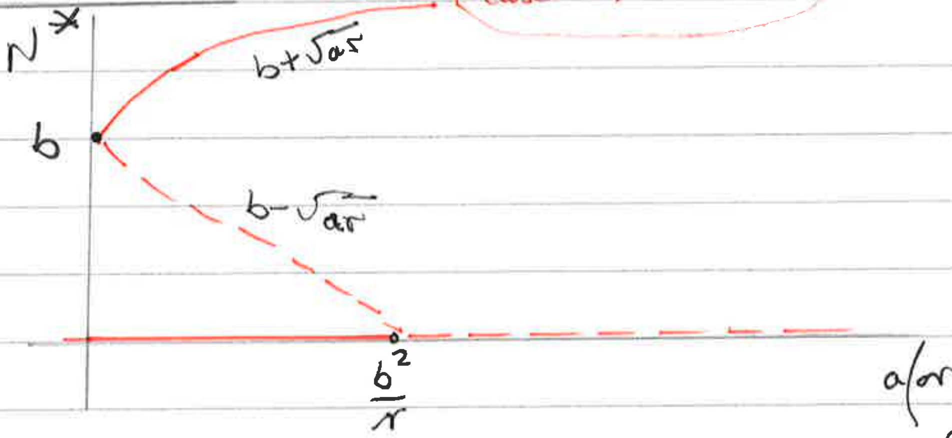
case a, $ar = 0$ case b $ar > 0$

No Allee effect here, $N^* = 0$ is unstable



case c) $ar > 0$

BIFURCATION DIAGRAM



a (or r, in which case FP is at $\frac{b^2}{a}$)