

Problem Sheet 13: Entanglement Theory (Part 2)

In this problem sheets, and all others, I highly recommend using Mathematica to deal with any messy algebra.

Class problems

1. *Asymptotic entanglement distillation.* Roughly how many singlets are needed to construct 100 copies of the state $|\phi\rangle = \alpha(2|01\rangle - 3|+0\rangle + 5|22\rangle)$, where α is a normalization constant (that you have to determine) and $|+\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle)$, under LOCC?
2. *Hyperplanes.* In \mathbb{R}^3 determine the hyperplanes defined by the vector \mathbf{v} for the cases a) $\mathbf{v} = (0, 1, 0)$, b) $\mathbf{v} = (0, -2, 0)$ and c) $\mathbf{v} = (1, 1, -1)$. In each case determine the ‘positive’ and ‘negative’ sides of the plane.
3. *Superoperators to detect entanglement.*

Define the maximally entangled state $|\Omega\rangle = |\text{vec}(\mathbb{I})\rangle$ and the Choi state associated with an operator \mathcal{E} as $J(\mathcal{E}) = \mathcal{E} \otimes \mathbb{I}(|\Omega\rangle\langle\Omega|)$.

a) Show that $|\Omega\rangle\langle\Omega|^{T_A} = \text{SWAP}$.

b) Show that $\text{Tr}[A \otimes B \text{ SWAP}] = \text{Tr}[AB]$.

c) Assuming that every positive superoperator \mathcal{E} has a decomposition $\mathcal{E}(X) = \sum_i A_i X B_i^\dagger$ deduce that we can define a ‘dual map’ \mathcal{E}^* such that $\text{Tr}[\mathcal{E}(\rho)B] = \text{Tr}[\rho\mathcal{E}^*(B)]$ for all states ρ and Hermitian operators B .

d) Prove that

$$\text{Tr}[(B \otimes \sigma^T)J(\mathcal{E})] = \text{Tr}[\mathcal{E}(\sigma)B] \quad (1)$$

Let us define the super-operator $\Phi_{\mathcal{J}}(\rho) = \text{Tr}_2[(\mathbb{I} \otimes \rho^T)\mathcal{J}]$

e) Verify (by constructing a couple of examples) that $\Phi_{\mathcal{J}(\mathcal{E})}(\rho) = \mathcal{E}(\rho)$ and $\mathcal{J}(\Phi_{\mathcal{J}(\mathcal{E})}) = \mathcal{J}(\mathcal{E})$. How does this connect to Eq. (1)?

f) What is the connection between SWAP and the Peres-Horodecki criterion?