Problem Sheet 6: Quantum Channels (Part 3)

Class problems

1. Consider the other quantum channels from last week's problem sheet. These were

$$\mathcal{E}(\rho) = p_0\rho + p_1 X\rho X + p_2 Y\rho Y + p_3 Z\rho Z \tag{1}$$

the channel associated with the non-normalization Kraus operators

$$A_0 \propto \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad A_1 \propto \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 2 \end{pmatrix} \tag{2}$$

and the channel induced on the system quantum A by the unitary

$$U = \frac{1}{\sqrt{2}} \left(X_A \otimes \mathbb{I}_B + Y_A \otimes X_B \right) . \tag{3}$$

assuming the environment qubit starts in the state $|0\rangle$.

a) Write out the Choi matrices for these three channels.

b) Hence (or otherwise) find a (different) set of Kraus operators to represent the same channels.

c) Consider the operation

$$\mathcal{E}(\rho) = 1/3(\alpha \operatorname{Tr}[\rho]\mathbb{I} + \beta\, \rho^T) .$$

For what values of α and β does this operation preserve the trace (i.e. $\operatorname{Tr}[\rho] = \operatorname{Tr}[\mathcal{E}(\rho)]$)? For what values is it completely positive?

d) For the case where $\mathcal{E}$ represents a genuine quantum channel state a minimal Kraus representation for the channel.

e) Hence state a more general expression for any set of Kraus operators that can represent

2. a) Prove that a linear map $\mathcal{E}$ is completely positive iff $J(\mathcal{E}) = \mathcal{E} \otimes \mathbb{1}(|vec(\mathbb{1})\rangle\langle vec(\mathbb{1})|)$ is positive.

b) Hence show that i. the dephasing channel is completely positive but ii. the transpose operation is not.