Quantum Information and Quantum Computing, Problem set 10

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Problem 1 : A depolarising channel

Take a two-dimensional Hilbert space $\mathcal{H}$ and a completely positive trace preserving (CPTP) map $\mathcal{E} : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ given by

$$\mathcal{E}(\hat{\rho}) \coloneqq p\,\frac{\hat{\mathbb{1}}}{2} + (1-p)\,\hat{\rho} \tag{1}$$

for $p \in [0, 1]$.

This map describes the action of what is known as a depolarizing channel.

(a) Any quantum channel given by a CPTP map $\mathcal{E}$ can be can be written in terms of Kraus operators M_a as

$$\mathcal{E}(\hat{\rho}) = \sum_a \hat{M}_a \hat{\rho} M_a^\dagger \tag{2}$$

where $\sum_a \hat{M}_a^\dagger M_a = \mathbb{1}$.

Find a Kraus-operator representation for $\mathcal{E}$.

Hint: You might want to prove (and then use) the identity $\mathbb{1} = \frac{1}{2}(\hat{\rho} + \hat{\sigma}_x\hat{\rho}\hat{\sigma}_x + \hat{\sigma}_y\hat{\rho}\hat{\sigma}_y + \hat{\sigma}_z\hat{\rho}\hat{\sigma}_z)$, where $\sigma_{x,y,z}$ are the Pauli-matrices.

(b) Any two-dimensional density matrix can be written in the Bloch represenation

$$\rho = \frac{1}{2}(\mathbb{1} + \mathbf{r} \cdot \hat{\boldsymbol{\sigma}}) \tag{3}$$

where $\hat{\boldsymbol{\sigma}} = \begin{bmatrix}\hat{\sigma}_x & \hat{\sigma}_y & \hat{\sigma}_z\end{bmatrix}^T$ is the vector of pauli matrices and $\mathbf{r} \in \mathbb{R}^3, \|\mathbf{r}\| \leq 1$ is called Bloch vector.

Calculate the action of the depolarizing channel $\mathcal{E}$ on such a bloch state for a given $\mathbf{r}$. How is the radius $r = \|\mathbf{r}\|$ affected?

Bonus: What happens when the depolarizing channel is applied to a pure state?

Problem 2 : Noise models in Qiskit

In this hands-on exercise we will see how to create custom noise models in Qiskit.

(a) Learn how to save and plot density matrices in Qiskit using the classical density matrix simulator

(b) Learn how to configure a noise model in Qiskit and use it to create a bit-flip, a phase-flip and a depolarising channel.

(c) Apply your noise model to a circuit of choice and plot the final density matrices. What happens to the purity of the matrix?

(d) Learn how to perform a tomography experiment to evaluate the density matrix of the system on the quantum device