
Quantum Information and Quantum Computing, Problem set 8

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Problem 1 : The quantum counting algorithm

We wish to write an algorithm to estimate the number M of solutions of a search problem in a database of size $N = 2^n$, using quantum phase estimation (possibly not looking it up on the Nielsen and Chuang or on the Qiskit textbooks!). Suppose you are given the operator corresponding to Grover's search oracle \hat{G} for the given search problem, and that you are also given the code to perform a controlled- \hat{G} gate. For this problem we assume that $|\psi\rangle = \hat{H}^{\otimes n}|0\rangle$ is the constant superposition state of the vectors of the computational basis, and $|\alpha\rangle = (N - M)^{-1/2} \sum_x'' |x\rangle$ and $|\beta\rangle = M^{-1/2} \sum_x' |x\rangle$ are respectively the superpositions of all the non-solutions and of all the solutions in the computational basis.

1. Write the operator \hat{G} as a matrix in the subspace spanned by $|\alpha\rangle$ and $|\beta\rangle$. What are its eigenvalues $e^{i\theta}$ and how are they related to the number of solutions M ?
2. Write a QPE circuit that estimates M using t qubits in the estimate register. How do you prepare the initial state of the n -qubit register so that the QPE always gives a good estimate of M ?
3. Suppose we wish to estimate θ to within 2^{-m} with probability $p = 1 - \epsilon$. Prove that, by setting ϵ small and $m \sim n$, the accuracy on the estimate of M is $O(\sqrt{M})$.

Problem 2 : Code Grover's search algorithm

Explicitly write the quantum circuit for carrying out Grover's search for $f(x) : \{0, 1\}^3 \rightarrow \{0, 1\}$ (always without looking it up!). Assume that $f(x) = 1$ for $x = 101$ and $x = 110$. Write the oracle circuit first, the rest of the \hat{G} operator next. Then write the remaining part of the circuit and execute it on the QASM simulator. Determine first how many times the Grover iteration must be applied.