Quantum Information and Quantum Computing, Problem set 6

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Problem 1 : Quantum phase estimation

We want to study the quantum phase estimation algorithm further. In particular we want to answer two questions. First, what happens if the eigenvalue cannot be expressed exactly with t qubits. Second, can we use the algorithm to estimate the eigenvector from the knowledge of the eigenvalue. We will always assume here that the eigenvalue we are considering is not degenerate. Let us start with the first question.

1. Assume that the phase φ of the eigenvalue $e^{2\pi i\varphi}$ is such that $2^t\varphi$ is not an integer. Define the integer b as being the best approximation of $2^t\varphi$ from below, and $\delta = \varphi - b/2^t$. Compute the amplitudes α_k of the final state $|\psi\rangle = \sum_k \alpha_k|k\rangle$ of the algorithm.

2. Assume that we want to obtain as a result of the final measurement a value m which is close to b, i.e. $|m-b| \leq e$ where e is a positive integer. Compute an upper bound to the probability $p(|m-b| > e)$.

3. Use the previous result to show that, in order to estimate φ with accuracy $e = 2^{t-n} - 1$ ($t > n$), with probability $p = 1 - \epsilon$, one must set $t \geq n + \log\left(2 + \frac{1}{2\epsilon}\right)$.

We now assume that the eigenvalue φ is known

4. Suppose first that $2^t\varphi$ can be expressed exactly as a t-bit integer. Assume that the input quantum state in the second register is $|\psi_{in}\rangle$. Compute the probability of measuring $2^t\varphi$ on the first register (which results in the second register being projected on the eigenstate $|\varphi\rangle$ associated to the eigenvalue φ).

5. Suppose now that $2^t\varphi$ can't be expressed exactly as a t-bit integer. If $\lfloor 2^t\varphi \rfloor$ (which denotes the integer part of is $2^t\varphi$) measured on the first register, what will be the final state $|\psi_{out}\rangle$ of the second register? How much will it differ from $|\varphi\rangle$?

6. Draw your conclusions about how to choose the input quantum state so to optimize the performance of the algorithm.