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## Quantum Information and Quantum Computing, Problem set 10

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### Problem 1 : A depolarising channel

Take a two-dimensional Hilbert space  $\mathcal{H}$  and a completely positive trace preserving (CPTP) map  $\mathcal{E} : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{H})$  given by

$$\mathcal{E}(\hat{\rho}) := p \frac{\hat{\mathbb{1}}}{2} + (1 - p) \hat{\rho} \quad (1)$$

for  $p \in [0, 1]$ .

This map describes the action of what is known as a depolarizing channel.

- (a) Any quantum channel given by a CPTP map  $\mathcal{E}$  can be written in terms of Kraus operators  $M_a$  as

$$\mathcal{E}(\hat{\rho}) = \sum_a \hat{M}_a \hat{\rho} \hat{M}_a^\dagger \quad (2)$$

where  $\sum_a \hat{M}_a^\dagger \hat{M}_a = \mathbb{1}$ .

Find a Kraus-operator representation for  $\mathcal{E}$ .

*Hint:* You might want to prove (and then use) the identity  $\mathbb{1} = \frac{1}{2}(\hat{\rho} + \hat{\sigma}_x \hat{\rho} \hat{\sigma}_x + \hat{\sigma}_y \hat{\rho} \hat{\sigma}_y + \hat{\sigma}_z \hat{\rho} \hat{\sigma}_z)$ , where  $\sigma_{x,y,z}$  are the Pauli-matrices.

- (b) Any two-dimensional density matrix can be written in the Bloch representation

$$\rho = \frac{1}{2}(\mathbb{1} + \mathbf{r} \cdot \hat{\boldsymbol{\sigma}}) \quad (3)$$

where  $\hat{\boldsymbol{\sigma}} = [\hat{\sigma}_x \ \hat{\sigma}_y \ \hat{\sigma}_z]^T$  is the vector of pauli matrices and  $\mathbf{r} \in \mathbb{R}^3, \|\mathbf{r}\| \leq 1$  is called Bloch vector.

Calculate the action of the depolarizing channel  $\mathcal{E}$  on such a bloch state for a given  $\mathbf{r}$ . How is the radius  $r = \|\mathbf{r}\|$  affected?

*Bonus:* What happens when the depolarizing channel is applied to a pure state?

### Problem 2 : Noise models in Qiskit

In this hands-on exercise we will see how to create custom noise models in Qiskit.

- (a) Learn how to save and plot density matrices in Qiskit using the classical density matrix simulator
- (b) Learn how to configure a noise model in Qiskit and use it to create a bit-flip, a phase-flip and a depolarising channel.

- (c) Apply your noise model to a circuit of choice and plot the final density matrices. What happens to the purity of the matrix?
- (d) Learn how to perform a tomography experiment to evaluate the density matrix of the system on the quantum device