Gauge Theories and the Standard Model

Problem Set 9

Due Tuesday, November 18, in class (BSP 727)

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Exercises: Andrea Luzio, Barak Gabai

Problem: Extracting $\sin^2\theta_w$ from lepton data (Part 2)

In the last problem set we discovered that neutrino-scattering data provided information to test the Standard Model, and even extract s_w^2. However, these data could not eliminate a two-fold ambiguity in the g_A^e–g_V^e plane. To solve this ambiguity new observables need to be employed. Here, we will use $e^+e^- \to \mu^+\mu^-$ measurements from MARK-J at PETRA. The MARK-J collaboration provides measurements of the observable

$$A = \frac{N^- - N^+}{N^- + N^+}$$

where N^- (N^+) is the number of events with $\mu^-(\mu^+)$ observed in the solid-angle range $50^\circ \leq \theta \leq 80^\circ$, with θ the angle between the incoming e^- and the outgoing $\mu^-(\mu^+)$. The collaboration reports the measurements

$$\begin{aligned} A &= -0.07 \pm 0.02 \qquad && \text{for } \sqrt{s} = 30\,\text{GeV} \\ A &= -0.1 \pm 0.03 \qquad && \text{for } \sqrt{s} = 35\,\text{GeV}\,. \end{aligned}$$

In addition it has provided measurements of the total cross-section of $e^+e^- \to \mu^+\mu^-$ for various $\sqrt{s}$'s. For this exercise we will use

$$\begin{aligned} \sigma_{\text{tot}} &= (96 \pm 4)\,\text{pb} \qquad && \text{for } \sqrt{s} = 30\,\text{GeV}\,, \\ \sigma_{\text{tot}} &= (71 \pm 6)\,\text{pb} \qquad && \text{for } \sqrt{s} = 35\,\text{GeV}\,. \end{aligned}$$

(i) In order to test the Standard Model, we will compute within the theory in which the Z boson has generic vector and axial-vector couplings such that the relevant Feynman rules are $-ie\gamma^\mu$ and $i\frac{e}{c_W s_W}\gamma^\mu(g_V - g_A\gamma^5)$ for the photon and Z-boson couplings, respectively, and with $s_w \equiv \sin\theta_w$ and $c_w \equiv \cos\theta_w$. Here we have assumed that g_V and g_A are universal, i.e., same for electrons and muons. What is the value of g_V and g_A in the Standard Model? Show that this definition of g_V and g_A agrees with the definition of g_V and g_A in the Fermi theory in the lecture.

(ii) Compute the unpolarised, differential cross section for $e^+e^- \to \mu^+\mu^-$ as a function of the s and t parameters. Neglect the fermion masses, but not the Z boson width, i.e., use the Breit-Wigner propagator for the Z boson. In the center-of-mass frame show that

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left(A(1+\cos^2\theta) + B\cos\theta\right)$$

with

$$A = 1 - 8\sqrt{2}\frac{G_{\rm F} s}{e^2}{\rm Re}[\mathcal{K}(s)] g_V^2 + 32 G_{\rm F}^2 s^2 |\mathcal{K}(s)|^2 (g_A^2 + g_V^2)^2$$
$$B = -16\sqrt{2}\frac{G_{\rm F} s}{e^2}{\rm Re}[\mathcal{K}(s)] g_A^2 + (16)^2 G_{\rm F}^2 s^2 |\mathcal{K}(s)|^2 g_A^2 g_V^2 \,,$$

and

$$\mathcal{K}(s) \equiv \frac{M_Z^2}{M_Z^2 - s - i M_Z \Gamma_Z} \,.$$

What would be the corresponding result if, instead, you had computed directly in the Fermi theory?

(iii) Now, do take the limit in which the Fermi theory is a good effective theory and combine the three neutrino-scattering measurements of the previos problem set and the four $ee \to \mu\mu$ measurements above by constructing a χ^2 function that depends on the observations and (g_A, g_V). In the plane of $g_A - g_V$, show the preferred 68.27% Confidence Level (CL) regions, i.e., the 1σ regions, of the different measurements and their combination. Is there still a two-fold ambiguity?
Hint: since there are two independent parameters (g_A and g_V) the preferred 68.27% *CL regions are those for which (g_A, g_V) satisfy $\chi^2(g_A, g_V) - \chi^2_{min} \leq 2.3$.*

(iv) Finally, assume the Standard model is correct and use the χ^2 function that you constructed to extract s_w^2 and its 1σ uncertainty. Compare it to the PDG value.