Gauge Theories and the Standard Model

Problem Set 8

Due Tuesday, November 11, in class (BSP 727)

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Problem 1: Extracting $\sin^2\theta_w$ from neutrino-electron scattering

At the end of the 70s it became possible to measure the total cross sections of scattering neutrinos on electrons at rest. It particular, the processes

$$\begin{aligned} \nu_\mu + e &\to \nu_\mu + e \\ \bar\nu_\mu + e &\to \bar\nu_\mu + e \\ \bar\nu_e + e &\to \bar\nu_e + e \end{aligned} \tag{1}$$

were observed. In the rest frame of the initial electron the following total cross sections were measured

$$\begin{aligned} \sigma_{\rm tot}(\nu_\mu e \to \nu_\mu e) &= (1.1 \pm 0.6) \times 10^{-42} \frac{E_\nu}{\rm GeV} {\rm cm}^2 \,, \\ \sigma_{\rm tot}(\bar\nu_\mu e \to \bar\nu_\mu e) &= (2.2 \pm 1.0) \times 10^{-42} \frac{E_{\bar\nu}}{\rm GeV} {\rm cm}^2 \,, \\ \sigma_{\rm tot}(\bar\nu_e e \to \bar\nu_e e) &= (5.0 \pm 1.4) \times 10^{-42} \frac{E_{\bar\nu}}{\rm GeV} {\rm cm}^2 \,, \end{aligned} \tag{2}$$

with E_ν ($E_{\bar\nu}$) the energy of the incoming neutrino (antineutrino). For more details see *Reines et al (Phys.Rev.Lett. 37 (1976)) and Faissner et at (Phys. Rev. Lett. 41 (1978)).*

In this exercise we will work within the Fermi theory and employ the above measurements to extract the (sine of the) weak mixing angle, i.e., $\sin^2\theta_w \equiv s_w^2$.

(i) Within the Fermi theory, derive for for the processes in Eq. (1) the relevant Feynman rules, draw the corresponding diagrams, and write the corresponding amplitudes. Make sure you understand the origin of the amplitudes also in terms of amplitudes with Z and W bosons.

Hint 1: Remember that when two diagrams contribute the ordering of the fermion spinors may induce minus signs.

Hint 2: To simplify one amplitude you can use an identity in the space of gamma matrices called a Fierz relation

$$(\gamma_\mu P_L)_{\alpha\beta} \otimes (\gamma^\mu P_L)_{\gamma\delta} = -(\gamma_\mu P_L)_{\alpha\delta} \otimes (\gamma_\mu P_L)_{\gamma\beta} \,.$$

If you are not familiar with Fierz relations see, e.g., Nishi Am.J.Phys. 73 (2005).

(ii) Assume that neutrinos have only one helicity, i.e., they are left-handed. Therefore, to compute the unpolarized matrix-element square of the three processes of Eq. (1) average only over the initial electron spin and sum over the final-state fermion spins. You should find that

$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}(\nu_\mu e \to \nu_\mu e)|^2 = 16G_{\text{F}}^2(s^2(g_V^e + g_A^e)^2 + t^2(g_V^e - g_A^e)^2)\,,$$
$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)|^2 = 16G_{\text{F}}^2(s^2(g_V^e - g_A^e)^2 + t^2(g_V^e + g_A^e)^2)\,,$$
$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}(\bar{\nu}_e e \to \bar{\nu}_e e)|^2 = 16G_{\text{F}}^2(s^2(g_V^e - g_A^e)^2 + t^2(1 + g_V^e + g_A^e)^2)\,,$$

where s is the usual square of the sum of initial fermion momenta, and t is defined as the square of the final-state *neutrino* momentum minus the initial-state *electron* momentum.

Hint: To avoid recomputing similar traces it is useful to derive that

$$\begin{aligned}\text{Tr}[\not{p}_1\gamma^\mu(1-\gamma_5)\not{p}_2\gamma^\nu(1-\gamma_5)]\text{Tr}[\not{p}_3\gamma_\mu(X+Y\gamma_5)\not{p}_4\gamma_\nu(X+Y\gamma_5)] &= \\ &= 64\left((X-Y)^2 p_1\!\cdot\! p_3\; p_2\!\cdot\! p_4 + (X+Y)^2 p_1\!\cdot\! p_4\; p_2\!\cdot\! p_3\right)\end{aligned}$$

(iii) Compute the total cross sections in the rest-frame of the initial electron. Combine your computation with the experimental results of Eq. (2) to plot in the plane of $g_V^e - g_A^e$ the regions preferred by the three experiments. Is it possible to unambiguously determine the signs of g_A^e and g_V^e? Can you extract s_w^2 if you assume that the SM is the right description? *Hint: To combine the measurements construct a χ^2-function with all three observations and their uncertainties.*