Gauge Theories and the Standard Model

Problem Set 6

Due Tuesday, October 28, in class (BSP 727)

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Exercises: Andrea Luzio, Barak Gabai

Problem 1: Parity and charge conjugation in $W^\pm$ decays

Consider the two-body decays of polarized W^+ and W^- bosons to $e^+\nu_e$ and $e^-\bar{\nu}_e$, respectively. Similarly to $u(p,s)$ and $v(p,s)$ for fermions a massive vector propagates three degrees of freedom described by three polarization vectors. We can choose the basis vectors to describe spin-eigenstates along the z direction. In this case, in the rest frame of the boson ($k = (M_W, \vec{0})$) they can be chosen to be

$$\epsilon^\mu_{W^-}(k, s=+1) = \epsilon^\mu_{W^+}(k, s=+1) = \frac{1}{\sqrt{2}}(0,-1,-i,0) \qquad \text{(transverse } W^+ \text{ or } W^-)\,,$$

$$\epsilon^\mu_{W^-}(k, s=-1) = \epsilon^\mu_{W^+}(k, s=-1) = \frac{1}{\sqrt{2}}(0,1,-i,0) \qquad \text{(transverse } W^+ \text{ or } W^-)\,,$$

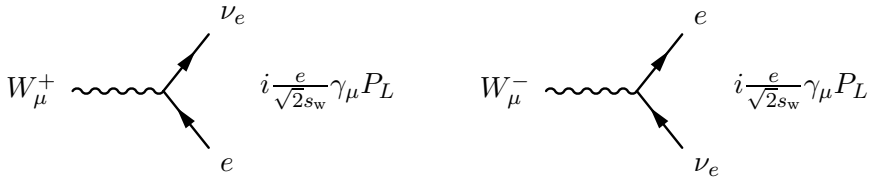
$$\epsilon^\mu_{W^-}(k, 0) = \epsilon^\mu_{W^+}(k, 0) = (0,0,0,1) \qquad \text{(longitudinal } W^+ \text{ or } W^-)\,,$$

(i) Suppose somebody measures for you the six differential decay widths

$$\frac{d\Gamma}{d\cos\theta}(W^+_{T/L} \to e^+\nu_e)\,, \qquad \frac{d\Gamma}{d\cos\theta}(W^-_{T/L} \to e^-\bar{\nu}_e)\,,$$

inclusive in all fermion polarizations. List all seven independent relations among those differential rates that would be fulfilled if C and P were exact symmetries. What are the relations if only the combination CP was a symmetry?

(ii) Using the Feynman rules of the SM



compute the six partial widths neglecting the mass of the electron. Which of the relations from question (i) are not satisfied?

Problem 2: Branching fractions for the decay of the Z boson

(i) Re-derive the Feynman rules for the Standard Model (SM) interaction vertices involving the electroweak gauge bosons and the SM fermions. Ignore the lepton masses and quark masses (and mixings), except for the top and bottom quarks.

(ii) Compute the decay width of the Z boson to SM fermions and find the branching fractions of Z to various two fermion final states.

(iii) Compare your result with the experimental measurements of the Z branching ratios:

Z DECAY MODES

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$e^+ e^-$	[a]	(3.3632±0.0042) %	
Γ_2	$\mu^+ \mu^-$	[a]	(3.3662±0.0066) %	
Γ_3	$\tau^+ \tau^-$	[a]	(3.3696±0.0083) %	
Γ_4	$\ell^+ \ell^-$	[a,b]	(3.3658±0.0023) %	
Γ_5	$\ell^+ \ell^- \ell^+ \ell^-$	[c]	(3.5 ±0.4) $\times 10^{-6}$	S=1.7
Γ_6	invisible	[a]	(20.000 ±0.055) %	
Γ_7	hadrons	[a]	(69.911 ±0.056) %	
Γ_8	$(u\bar{u}+c\bar{c})/2$		(11.6 ±0.6) %	
Γ_9	$(d\bar{d}+s\bar{s}+b\bar{b})/3$		(15.6 ±0.4) %	
Γ_{10}	$c\bar{c}$		(12.03 ±0.21) %	
Γ_{11}	$b\bar{b}$		(15.12 ±0.05) %	