Gauge Theories and the Standard Model

Problem Set 4

Due Tuesday, October 7, in class (BSP 727)

Lecture: Marc Riembau
Exercises: Andrea Luzio

Problem 1: Shift symmetries

Consider the theory of a real scalar that respects a discrete Z_2 symmetry $\phi(x) \to -\phi(x)$ and exhibits spontaneous symmetry breaking, i.e., ϕ obtains a vacuum expectation value, v. Work with the shifted coordinate/field, $\chi(x)$, $\phi(x) = v + \chi(x)$. Without employing the field $\phi(x)$, show that:

- (i) The most general $d \leq 4$ potential for $\chi(x)$ that is invariant under a shift symmetry, $\chi(x) \to -2v - \chi(x)$, is indeed the same as the one you obtained in the lecture by expanding the ϕ potential. For now, ignore terms that are linear in χ, i.e., exclude tadpole terms.

- (ii) Allow now for tadpoles. If you had to go through the same classification as before, would you find the same Lagrangian? If not, then does this mean that the tadpole theory can have different observational consequences than the no-tadpole theory? Hint: Remember that tadpoles can be removed by a field redefinition.

- (iii) Is there a $d = 5$ operator that respects the shift symmetry? Can you add simultaneously a $d = 5$ and a $d = 6$ operator without breaking the shift symmetry?