Gauge Theories and the Standard Model

Problem Set 10

Due Tuesday, November 25, in class (BSP 727)

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Problem 1: Breit–Wigner Propagator of the Z-Boson

In unitary gauge the Breit–Wigner propagator for the Z boson reads

$$\Delta_{\mu\nu}(p) = -\frac{i}{p^2 - M_Z^2 + iM_Z\Gamma_Z}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}\right),$$

with Γ_Z the width of the Z boson. The important physical property of the Breit–Wigner propagator is that it has no poles for any value of p^2 on the real axis. As shown in the lecture for the case of a scalar field, the Breit–Wigner propagator is obtained by including loop corrections to the tree-level propagator and resumming them to all orders by means of a geometric series.

For the case of the Z boson the series takes the form

$$\Delta^Z_{\mu\nu}(p) = \frac{iK_{\mu\nu}}{p^2 - M_Z^2 + i\epsilon} + \frac{iK_{\mu\rho}}{p^2 - M_Z^2 + i\epsilon}\, i[\Sigma_{ZZ}(p)]^{\rho\sigma}\, \frac{iK_{\sigma\nu}}{p^2 - M_Z^2 + i\epsilon} + \dots$$

where

$$K_{\mu\nu} \equiv -\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}\right).$$

In this exercise we explicitly derive the Breit–Wigner propagator for the transverse part of the Z-boson propagator by evaluating the relevant one-loop correction in $\Sigma_{ZZ}(p)$ and resumming the series.

We consider the generic case in which the photon and the Z boson couple to a fermion f, with Feynman rules:

Photon–fermion vertex: $iQ_f e\,\gamma^\mu$
Z–fermion vertex: $i\,e/(s_w\,c_w)\,\gamma^\mu(g_V - g_A\gamma^5)$.

(i) Compute the two-body width $Z \to f\bar{f}$. Neglect the fermion mass.

(ii) We are interested only in the transverse part of the propagator, i.e. the part proportional to $\eta_{\mu\nu} - p_\mu p_\nu/p^2$. Decompose $K_{\mu\nu}$ and $\Sigma_{ZZ,\mu\nu}(p)$ into transverse and longitudinal parts (K_T, K_L) and ($\Sigma_{ZZ,T}$, $\Sigma_{ZZ,L}$). Show that only the transverse pieces contribute to the transverse part of the propagator, and resum all contributions using the geometric series.

Up to trivial normalisation factors in K_T and $\Sigma_{ZZ,T}$, you should find

$$\Delta^{Z,T}_{\mu\nu}(p) = \frac{-i}{p^2 - M_Z^2 + K_T\Sigma_{ZZ,T}}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right).$$

From this equation you see that if $\Sigma_{ZZ,T}$ has an imaginary part, the propagator does not have a pole on the real axis.

(Hint: in sheet 3 we used a transverse projector to decompose the tree-level propagators. Use a similar methodology.)

(iii) Begin the evaluation of $\Sigma_{ZZ,T}$ at one loop. $\Sigma_{ZZ,T}$ is the transverse part of the amputated two-point function of the Z-boson self-energy. Draw the corresponding one-loop diagram and write the corresponding amplitude. Neglect the fermion mass and evaluate all traces.

(iv) At this stage you should be left with three different tensor loop integrals I, I^μ and $I^{\mu\nu}$. Decompose them into independent Lorentz structures and use the relation

$$k \cdot p = \frac{1}{2}\left[(k+p)^2 + i\epsilon - (k^2 + i\epsilon) - p^2\right]$$

to express them in terms of two scalar integrals:

$$A_0 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon}, \qquad B_0 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + i\epsilon)((k+p)^2 + i\epsilon)}.$$

This is the Passarino–Veltman reduction. The result should be

$$\Sigma_{ZZ,T} = i\left(f_A(p^2)A_0 + f_B(p^2)B_0\right),$$

with $f_{A,B}(p^2)$ real. A_0 is scaleless and vanishes in dimensional regularisation, so it cannot produce a finite imaginary part. However, if $B_0(p^2)$ has a non-vanishing real part as $\epsilon \to 0$, then $\Sigma_{ZZ,T}$ has an imaginary part.

(v) Evaluate $B_0(p^2)$ and show that it has a real part as $\epsilon \to 0$. Using Feynman parametrisation,

$$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{[xA + (1-x)B]^2},$$

rewrite the integral as

$$B_0 = \int \frac{d^4\ell}{(2\pi)^4} \int_0^1 dx \, \frac{1}{(\ell^2 - x(1-x)p^2 + i\epsilon)^2}.$$

To find the real part of B_0 as $\epsilon \to 0$, rotate to Euclidean space ($dl_0 = i\, dl_{E,0},\ k^2 \to -k_E^2$). Do you find a real part for any value of p^2? If you had kept the fermion masses, would this still be the case?

(Hint: the relation

$$\int_0^\infty \frac{t\, dt}{(t + \Delta - i\epsilon)^2} = \left[\frac{-t}{t + \Delta - i\epsilon}\right]_0^\infty + \int_0^\infty \frac{dt}{t + \Delta - i\epsilon}$$

may be useful.)

(vi) Use your result for B_0 to find the imaginary part of $\Sigma_{ZZ,T}$ and obtain the transverse propagator of the Z from part (ii). Compare your result to the Breit–Wigner propagator with Γ_Z the two-body width from part (i).

(vii) In part (ii) we defined and resummed the contributions from the Z self-energy Σ_{ZZ}. However, in the presence of electromagnetism the mixed self-energy Σ_{ZA} does not vanish. Is the Breit–Wigner propagator for the Z then still correct or do we need to modify it?