Introduction to holography - Lecture VI

Last time: quantization of the string

- closed strings ⊃ graviton (@ massless level)

$$S = \frac{1}{2k_0^2} \int d^Dx \sqrt{-G}\, e^{-2\phi} \left(R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + 4 \partial_\mu \phi \partial^\mu \phi \right) + \ldots$$

- valid to leading order in g_s, α'

- open strings ⊃ gauge fields (non-abelian if Chan-Paton factors are included).

$$S = -\frac{1}{4 g_s \alpha'^{\frac{D-4}{2}}} \int d^Dx \, \mathrm{tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

- for the superstring, additional bosonic "RR" p+1-form gauge fields $C_{\mu_1 \ldots \mu_{p+1}}$ (IIA: p even, IIB: p odd)

- the latter turn out to couple to D-branes: extended p+1-dim'l objects that are non-perturbative $\tau_p \sim \frac{1}{g_s \alpha'^{\frac{p+1}{2}}}$

- we initially defined these objects ① as hypersurfaces where open strings w/ Dirichlet bnd. cond. can end

- however, the analysis of massless open string modes indicated these hypersurfaces were dynamical & must be included in ∀ th. w/ open strings

- they are governed by the DBI action $S_{DBI} = -\frac{1}{\alpha'^{\frac{p+1}{2}}} \int d^{p+1}x \, e^{-\phi} \sqrt{\det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} + \ldots \}$ RR coupling
  - descr. tractable @ small 't Hooft coupling $g_s \alpha'^{\frac{p-3}{2}} N$

This time: discuss a 2nd description of D-branes as solitonic sols of the low-eng. eff. action.

- use these 2 ≠ descriptions of D3 branes, for concreteness, to derive an instance of the AdS_5/CFT_4 correspondence from string theory

(2.) D-branes : solitonic sols of the low-eng. eff. action (a.k.a (black) p-branes)

$$S = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{g} \left[e^{-2\phi} (R + 4(\partial\phi)^2) - \frac{2}{(8-p)!} F_{p+2}^2 \right] + \text{other terms we will not need}$$

RR field strength

(careful F_5!)

- extended in $p+1$ directions
- charged under the RR fields
- $1/2$ BPS (supersymmetric) if extremal

· basically, extended generalizations of the Reissner-Nordstrom black hole.

· look for a sol'n w/ $\mathbb{R}^{1,p}$ Poincaré symm. & spherically symm. in the transverse $9-p$ directions

· the sol'n for the (string frame) metric & dilaton is

$$ds^2 = \frac{1}{\sqrt{H(r)}} (-dt^2 + dx_1^2 + \dots + dx_p^2) + \sqrt{H(r)} (dr^2 + r^2 d\Omega^2_{8-p})$$

$$e^\phi = g_s H(r)^{\frac{3-p}{4}} \qquad H(r) = 1 + \# \frac{g_s N \alpha'^{\frac{7-p}{2}}}{r^{7-p}} \qquad C_{0\dots p} = H(r)^{-1}$$

w/ RR charge N $\quad \int_{S^{8-p}} * F_{p+2} = N$ $\quad$ sols of either IIA/IIB depending on whether p is even/odd

· these are extremal p-branes (min mass @ fixed charge, lest naked sing) $\approx$ gnd. states.
(in the non-extremal case, require only $ISO(p) \times SO(9-p)$)
$M \geq \# N$ for no naked sing.

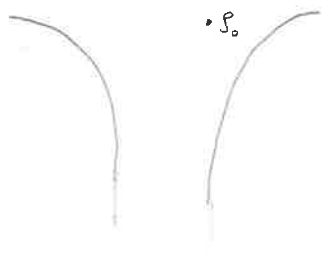
· it is useful to understand the analogy w/ the extremal limit of the RN black hole

$$ds^2_{RN} = -\left(1 - \frac{2GM}{\rho} + \frac{GQ^2}{\rho^2}\right) dt^2 + \frac{d\rho^2}{1 - \frac{2GM}{\rho} + \frac{GQ^2}{\rho^2}} + \rho^2 d\Omega_2^2 \qquad A_0 = \frac{Q}{\rho}$$

horizons @ $\rho_\pm = GM \pm \sqrt{G^2M^2 - GQ^2}$ $\quad$ extremal for $M = Q/\sqrt{G}$, Hawking temp. vanishes

· if b.h. is extremal, then $-g_{tt} = \left(1 - \frac{Q\sqrt{G}}{\underset{\substack{\uparrow \\ r + Q\sqrt{G}}}{\rho}}\right)^2 = \left(\frac{r}{r + Q\sqrt{G}}\right)^2 = \Bigg(\frac{1}{\underbrace{1 + \frac{Q\sqrt{G}}{r}}_{\approx H(r)}}\Bigg)^2$ $\quad$ horizon @ $r = 0$ in these coord.

- note that in the extremal limit, the spt. geom develops a very long throat



$$\underset{\text{finite } \rho}{\text{dist}}(\rho_0, \rho_+) = \int_{\rho_+}^{\rho_0} \frac{d\rho}{\sqrt{\underbrace{1 - \frac{2GM}{\rho} + \frac{GQ^2}{\rho^2}}_{\frac{(\rho-\rho_+)(\rho-\rho_-)}{\rho^2}}}} = 2\ln\left(\frac{\sqrt{\rho-\rho_+} + \sqrt{\rho-\rho_-}}{\sqrt{\rho_+-\rho_-}}\right)\Bigg|_{\rho_+}^{\rho_0} + \sqrt{(\rho-\rho_+)(\rho-\rho_-)}$$

ρ_+

$$\simeq -\ln(\rho_+ - \rho_-) \to \infty \quad \text{as } \rho_- \to \rho_+$$

- the near-horizon geom $(r \ll Q\sqrt{G})$ is $AdS_2 \times S^2$ $\quad ds^2_{RN} \simeq -\frac{r^2}{(Q\sqrt{G})^2} dt^2 + Q^2 G \frac{dr^2}{r^2} + Q^2 G\, d\Omega_2^2$

- also Dp-branes have a horizon @ $r=0$. null if $p<6$, timelike if $p=6$
  - for $p \leq 6$ $\exists$ also curv. sing @ $r_- \to r_+$ (sing. on horizon if $p \neq 3$) smooth if $p=3$
  - for $p<3$, dilaton blows up @ horizon $\Rightarrow$ sol'n not trustworthy
  - $p=3$ extremal sol'n : smooth & can be extended past horizon
    - also $e^\phi = \text{const}$ $\leftarrow$ concentrate on this one

- for $p=3$ $\quad H(r) = 1 + \frac{4\pi\alpha'^2 g_s N}{r^4} \qquad e^\phi = g_s = \text{const}$

- let us briefly comment on the range of validity of the extremal D3 brane sol'n

- note we used classical supergravity, valid provided
  - curvature radius $\gg l_s$ (string (α') corr. are negligible)

    $\downarrow$

    size $S^5 \sim \sqrt[4]{4\pi\alpha'^2 g_s N} \gg \sqrt{\alpha'} \quad \Rightarrow \quad \boxed{g_s N \gg 1}$

    note that from the point of view of the gauge th. descr. of the D-brane this is the regime of very large 't Hooft coupling

  - also need to suppress string loops $\Rightarrow$ $g_s < 1$

$$\Rightarrow \quad \boxed{1 \ll g_s N < N} \quad \Rightarrow \quad \begin{array}{l} N \text{ large} \\ g_s N \text{ large} \end{array}$$

- the need for N large follows directly from $\ell_{S5} = (4\pi\alpha'^2 g_s N)^{1/4} \gg \ell_p = (g_s^2 \alpha'^4)^{\frac{1}{8}}$

  $\Rightarrow N \gg 1$ in order to suppress quantum gravitational effects.

- on the other hand, $g_s N = \left(\frac{\ell_{S5}}{\ell_s}\right)^4$ $\gg 1$ supergravity valid; $\lesssim 1$ highly stringy, but not QG.

- for $p \neq 3$ (non-conformal D-branes), the dilaton runs, and so the sugra descr. is only valid for a certain range of r

Summary : D-branes are well-described by gauge theory for $E \ll \frac{1}{\sqrt{\alpha'}}$, w/ perturbative calculations valid for $g_s N \ll 1$

- they are well-described as supergravity sols for $g_s N \gg 1$, w $N \gg 1$

- the (highly non-trivial) identification b/w the two descriptions follows from supersymmetry (both ½ BPS), the RR charges they both carry & duality

Two complementary descriptions : open/closed duality

open (pl where open strings can end) | closed source of closed strings | open string loop / closed string exchange (tree

Deriving the AdS_5/CFT_4 correspondence

- consider type IIB string th. in flat 10d Minkowski sp + N ∥ D3-branes sitting v. close to each other & lying along $x^0, \dots, x^3$

- string th. in this backgnd. contains 2 types of perturbative excit:
  - closed strings (excit. of empty spt)
  - open strings (excit. of the D-branes)

  generically coupled

- if we now consider this system @ low energies $E \ll \frac{1}{\sqrt{\alpha'}}$ ⇒ only massless states will be excited → write eff. action for them

$$S = \underbrace{S_{bulk/closed}}_{\substack{10d\ \text{sugra} \\ +\text{higher d-corr}}} + \underbrace{S_{brane/open}}_{\substack{\mathcal{N}=4\ U(N)\ \text{SYM} \\ +\text{corr}\ 4d}} + \underbrace{S_{int}}_{\substack{\text{coupling} \\ \text{bulk-brane} \\ (\text{DBI})}}$$

Wilsonian eff. action obt. by int. out massive d.o.f (string scale).

- to better understand what happens in the low-eng. limit $E \ll \frac{1}{\sqrt{\alpha'}}$, one may keep E fixed & send $\alpha' \to 0$ w/ g_s, N fixed. Then
  - $S_{bulk} = S_{\text{free gravitons in 10d}}$, obt. by writing $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_{10}}\, h_{\mu\nu}$ & noting that $\sqrt{G_{10}} = g_s \alpha'^2 \to 0$ in this limit. Also Higher-deriv → 0.
  - $S_{brane} = S_{\mathcal{N}=4\ SYM\ U(N)}$. All higher deriv. couplings → 0
  - $S_{int} \to 0$

- thus, in this limit we obtain two decoupled systems
  - free gravitons in $\mathbb{R}^{1,9}$
  - $\mathcal{N}=4$ SYM on the branes

- note we worked in the open string picture (!) for the D-branes

- now, consider the same limit, but viewing the D3-branes as supergravity sols [2]

$$ds^2 = \frac{1}{\sqrt{H(r)}}\left(-dt^2 + dx_1^2 + \dots + dx_3^2\right) + \sqrt{H(r)}\left(dr^2 + r^2 d\Omega_5^2\right) \qquad e^{\phi} = g_s$$

$$F_5 = (1 + *_{10})\, dt \wedge dx^1 \wedge \dots \wedge dx^3 \wedge d(H^{-1}) \qquad H(r) = 1 + \frac{4\pi g_s N \alpha'^2}{r^4}$$

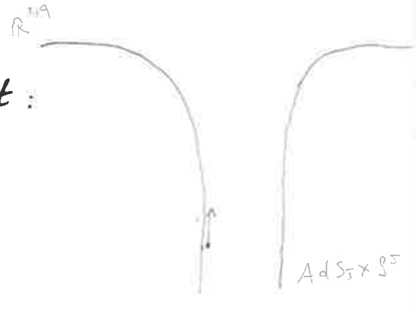
- note that, due to the warp factor, the energy of an object measured by an obs. @ r vs. an obs. @ ∞ are related by $E_\infty \sqrt{-g_{tt}(\infty)} = E(r) \underbrace{\sqrt{-g_{tt}(r)}}_{\text{redshift factor}}$

$$E_\infty = \frac{E(r)}{\sqrt[4]{H(r)}} \to 0 \quad \text{as} \quad r \to 0.$$

- thus, the same object, if brought closer & closer to $r=0$, appears to have lower & lower energy from the point of view of an obs. @ ∞

- taking the low-energy limit mentioned above from this point of view $\Rightarrow$ 2 types of low-energy excitations:
  - long wavelength modes propagating in the bulk
  - modes very close to $r=0$. There, the geom. is $H(r) \approx \frac{R^4}{r^4}$ w/ $R^4 \equiv 4\pi g_s N \alpha'^2$

$$ds^2 = \underbrace{\frac{r^2}{R^2}\left(-dt^2 + dx_1^2 + \dots + dx_3^2\right) + R^2\frac{dr^2}{r^2}}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{S^5} \qquad \text{w/ radius } R$$



- these 2 types of modes decouple in the low-energy limit:
  - for modes in the throat, hard to climb out
  - for modes w/ long wavelength / small ω, cannot "see" the throat region ($\sim$ size R): abs. cross section $\sigma \sim \omega^3 R^8$ for ω small

- thus, we note that from both points of view, we obtain two decoupled systems in the low-energy limit, one of which is free gravity in flat sp. $\Rightarrow$ natural to identify the other 2.

$$\boxed{\mathcal{N}=4 \;\; SU(N) \;\; SYM \text{ on } \mathbb{R}^{1,3} \;\; = \;\; \text{type IIB string th. on } AdS_5 \times S^5}$$

U(1) decouples

$r \approx 0$ region of D3 sol'n $\qquad \forall\, g_s, N$

- to see we obtain full type IIB string th., & not just supergravity, want to take the $\alpha' \to 0$ limit ∋ the energies in the throat are fixed in string units (i.e., we should be able to consider arbitrary excited string states from the near-horizon point of view). Thus, we should take $\underset{\text{throat}}{E(r)}\sqrt{\alpha'}$ = fixed as $\alpha' \to 0$.

the energy @ ∞ $\quad E_\infty = \dfrac{E(r)}{\sqrt[4]{H(r)}} \approx \dfrac{E(r)\cdot r}{\sqrt{\alpha'}}$ = eng. measured in field. th = fixed.

$\Rightarrow$ we should take $\alpha' \to 0$ w/ $\dfrac{r}{\alpha'} \equiv u$ fixed

- in terms of the u coord, the metric is $ds^2 = \alpha'\left[\dfrac{u^2}{\sqrt{4\pi g_s N}}(-dt^2 + \dots + dx_3^2) + \sqrt{4\pi g_s N}\left(\dfrac{du^2}{u^2} + d\Omega_5^2\right)\right]$

- notice that, while it may naively appear that the string th. side dep. on g_s, N and α', while the SYM side only dep on $g_{YM}^2 = g_s$ & N, α' only actually appears in relative quantities ∋ $\alpha'/(\text{curvature radius})^2$. Notice α' as written above actually drops out of the sugra action

- it is convenient to effectively set the curvature radius $(= R_{S^5})$ to 1 by letting $ds^2 = R^2 d\tilde{s}^2$

in these conventions, $\alpha' \sim \underbrace{\dfrac{1}{\sqrt{g_s N}}}_{\text{controls stringy corr.}} \sim \dfrac{1}{\sqrt{\lambda}}$ and $G_N \sim g_s^2 \alpha'^4 \sim \underbrace{\dfrac{1}{N^2}}_{\text{controls QG corr.}}$

- as already discussed, QG effects are suppressed for N large

  - sugra valid, in addition, for $R \gg \sqrt{\alpha'}$ $\Rightarrow$ $\underbrace{g_s N}_{g_{YM}^2 N \text{ 't Hooft coupling of SYM}}$ large

  - perturbative SYM descr. valid for $g_{YM}^2 N \ll 1$

- the ranges of validity of these two descriptions are perfectly incompatible

- we say AdS/CFT is a strong/weak coupling duality : the two theories are (conjecturally) the same, but when one side is weakly coupled (i.e., easy to compute) the other side is strongly coupled ⇒ i.) hard to prove the conj.

  ii.) useful : can use sugra to perform strongly-coupled comput. in CFT.

- why is this a conjecture & not a proof? — the string side was not treated non-pert. Can distinguish ≠ strengths of the corresp.

  - weakest : SYM described by gravity @ large N & $g_s N = \lambda$, but the full string th ≠ field theory (α', or $1/\lambda$, corrections may not agree).

  - medium : SYM = str. th. $\forall\, \lambda = g_s N$, but only for $N \to \infty$ (g_s, or, $\frac{1}{N}$ corr. may not agree)

  - strong form (most interesting) : true @ $\forall\, g_s, N$ & can use CFT @ $\underbrace{\text{finite } N}_{G_N \text{ finite}}$ to define what we mean by QG in AdS

    proper QG w/ fluctuating geom (only fix $AdS_5 \times S^5$ asympt.),

- if the CFT is used to define the gravity th, then the AdS radial dir & grav. are emergent from the CFT point of view. What is their meaning in the CFT?

  - traditionally, $\underset{AdS}{\underbrace{r}}$ has been associated w/ the energy scale in the CFT

$$ds^2_{AdS} = r^2 \left(-dt^2 + d\vec{x}^{\,2} \right) + \frac{dr^2}{r^2}$$

to see this, note that $E_{FT} = E(r \simeq 1) = \sqrt{-g_{tt}}\, E_{proper}(r) \longrightarrow \infty$ as $r \to \infty$

 r

$$\text{size}_{FT} = |\Delta \vec{x}\,| = \frac{1}{r}\,(\text{proper size}) \to 0 \quad \text{as} \quad r \to \infty$$

thus, short distances / high energies in CFT ⟷ large r in gravity

known as the UV / IR correspondence   in part, UV divergences in FT ⟺ IR div. in gravity.

- Generalizations : the above decoupling arg. can be run also for other brane systems in string / M-theory (11d. th = strong-coupling limit of type IIA)

- other proposed dualities b/w CFT_d & string / M-theory (large N).

  - D1-D5 ⟷ IIB on $AdS_3 \times S^3 \times T^4$ ⟷ CFT_2
  - M5 / M2 ⟷ $AdS_7 \times S^4$ or $AdS_4 \times S^7$ backgnds in M-th.

- ∃ also decoupling limits that yield non-conformal or non-local theories

  - Dp for $p \neq 3$ ⟷ complicated backgnd. →
  - NS5 - branes ⟷ 7d asympt. flat spt $\times S^3$ w/ linear dilaton
  - Dp - branes in non-trivial backgnd B-field → non-commutative def. of above SYM th. ⟶ string th. in funny backgnds.

- all these are explicit examples of holographic dualities b/w gravitational theories (string. th. on some backgnd) & non-gravitational ones living in one (non-compact) dimension less

- all these are non-trivial realizations of the holographic principle within the string. th. context.

- How to test this duality ? - check symmetries, Hilbert sp on the two sides agree
  - Need a dictionary !

- since the two sides of the corresp. are supposed to be the same

$$\boxed{Z_{\text{gravity}}[\text{bnd. cond.}] = Z_{\text{"FT"}}[\text{operator sources}]}$$

- in a th. of gravity, it doesn't make sense to fix the backgnd, but can fix it only asympt. & $\sum$ all geom. w/ these fixed bnd. cond. In part, topology change is allowed.