Introduction to holography - Lecture III

Last time : grav. collapse & singularity thms.

- the laws of black hole mechanics $\approx$ laws of thermodyn.

  0^{th} : $\kappa = $ const. on $\mathcal{H}$ (stationary $\to$ Killing. hor.)

  1^{st} : $\frac{\kappa}{8\pi G} \delta A = \delta M - \Omega \delta J + \ldots$

  2^{nd} : $\delta A \geq 0$ classically , $3^{rd} \sim \checkmark$

As I already mentioned, these laws were viewed as a mere analogy b/w the mech. of. b.h. & the laws of thermodyn, as the b.h. was believed to have exactly zero temperature : it absorbs everything & emits nothing

- true classically, but not QM : as Hawking famously showed, black holes do emit blackbody radiation at a temperature

$$\boxed{T_H = \frac{\hbar \kappa}{2\pi}} \qquad \text{Hawking temperature}$$

- thus $\kappa \Leftrightarrow$ real. temp. $\Rightarrow A \Leftrightarrow$ "true" entropy $\qquad S = \frac{A}{4G\hbar}$
- since this is a quantum effect, it necessitates a QFT treatment in the b.h. background (no QG, though, since $R_s \gg l_p$)

This time : study an analogous effect in flat space : Unruh effect

- study implications of $S = \frac{A}{4G\hbar}$ for quantum gravity
- but 1^{st}, let me comment a bit more on the 2^{nd} law

Literature: Jacobson gr-qc/0308048
Carroll ch. 9
Mukhanov & Winitzki : Intro to quantum effects in gravity
Townsend 9707012

The Hawking effect

- black holes emit thermal radiation at a temp. $\boxed{T_H = \frac{\kappa\hbar}{2\pi}}$

- main physics : $\neq$ observers have $\neq$ notions of time $\Rightarrow$ $\neq$ Hamiltonians
  what looks like the vacuum state to one obs $\rightarrow$ can contain part. for another

- to see the Hawking effect $\rightarrow$ QFT in a (weakly) curved spt.
  - sufficient to consider a free scalar field

- comments : in a general curved backgnd, no preferred notion of time
  - in a spt. w/ a timelike isometry $\rightarrow$ natural to consider Ham. assoc. w/ this isometry, but not compulsory (see Unruh effect we will discuss shortly).
  - the notion of "particle" does not make sense in a gen. spt ; however for $\omega \gg \frac{1}{R_{curv.}}$, the spt. looks approx. flat, & we can import this notion from Minkowski sp.!
  - if the spt. is asympt. flat, then notion makes sense for all frequencies. (can also call it "field excit").

- in the following, we will discuss the Unruh effect : accelerated obs. in Minkowski spt. sees a thermal flux of part. @ $T = \frac{a\hbar}{2\pi}$

- this is analogous to the Hawking effect, but simpler, a = surface gravity of Rindler horizon

- the general discussion $\rightarrow$ QFT in curved spt, particularized to flat space
  natural setting.

Free scalar in Minkowski spt

- consider a free real scalar field $(\Box - m^2)\varphi(\vec{x},t) = 0$

  Minkowski, but easily gen

- QM: $\phi(\vec{x},t) \to$ operator $\phi^\dagger = \phi$, satisfies K.G. eqn & $[\phi(\vec{x},t), \pi(\vec{x}',t)] = i\hbar\,\delta^{d-1}(\vec{x}-\vec{x}')$

  equal-time comm. rel. $\pi = \dot{\phi}$

- expand $\phi(\vec{x},t)$ in a set of normalizable sols. to the K.G. eqn $f_k = \dfrac{e^{-i\omega_k t + i\vec{k}\vec{x}}}{\sqrt{2\omega_k}}$

$$\phi(\vec{x},t) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left(a_{\vec{k}} \frac{e^{-i\omega_k t + i\vec{k}\vec{x}}}{\sqrt{2\omega_k}} + \underbrace{a^\dagger_{\vec{k}} \frac{e^{i\omega_k t - i\vec{k}\vec{x}}}{\sqrt{2\omega_k}}}_{h.c.} \right)$$

→ ∞ decoupled set of harmonic osc. w/ frequencies $\omega_k = \sqrt{|\vec{k}|^2 + m^2}$

– to see this, one can invert the rel. b/w $a_{\vec{k}}$ & ϕ, & derive the comm. rels of $a_k, a^\dagger_k$ from those of ϕ, $\pi = \dot{\phi}$.

- this relation can be inverted by introducing the Klein-Gordon inner product on the space of complex sols. to the wave eqn. We keep the discussion general, so as to be able to apply it to a free scalar in a general curved backgnd.

$$\langle f, g \rangle = \frac{i}{\hbar} \int_\Sigma d^{d-1}x \sqrt{\gamma}\, \overset{\text{unit timelike normal}}{n_\mu}\, \underbrace{g^{\mu\nu}(\bar{f}\partial_\nu g - \partial_\nu \bar{f}\, g)}_{\text{KG current } j^\mu}$$

- $\nabla_\mu j^\mu = 0$ if f, g are sols. to the wave eqn $(\Box - m^2) f = 0$.

  ⇒ integral indep. of Σ & of time

- $\overline{\langle f, g \rangle} = \langle g, f \rangle = -\langle \bar{f}, \bar{g} \rangle$ $\qquad \langle f, \bar{f} \rangle = 0$

- not positive definite

- the Minkowski scalar field written above takes the form

$$\phi(\vec{x},t) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left(f_{\vec{k}}(\vec{x},t)\, a_{\vec{k}} + \bar{f}_{\vec{k}}\, a^{+}_{\vec{k}} \right)$$

where $f_{\vec{k}}, \bar{f}_{\vec{k}}$ satisfy $(g_{\mu\nu} = \eta_{\mu\nu})$ $\quad f_{\vec{k}} = \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t + i\vec{k}\cdot\vec{x}}$

$$\langle f_{\vec{k}}, f_{\vec{k}'} \rangle = \frac{i}{\hbar} \int d^{d-1}x \left(\bar{f}_{\vec{k}} \underbrace{\dot{f}_{\vec{k}'}}_{-i\omega_{k'} f_{k'}} - \underbrace{\dot{\bar{f}}_{\vec{k}}}_{i\omega_k \bar{f}_k} f_{\vec{k}'} \right) = \frac{1}{\hbar} \frac{(\omega_k + \omega_{k'})}{2\sqrt{\omega_k \omega_{k'}}} \overbrace{e^{-i(\omega_{k'}-\omega_k)t}} \underbrace{\int d^{d-1}x\, e^{i(\vec{k}'-\vec{k})\cdot\vec{x}}}_{(2\pi)^{d-1}\delta^{(d-1)}(k-k')}$$

$$= \frac{1}{\hbar} (2\pi)^{d-1} \delta^{(d-1)}(k-k')$$

$$\langle f_{\vec{k}}, \bar{f}_{\vec{k}'} \rangle = 0$$

- then $\quad a_{\vec{k}} = \langle f_{\vec{k}}, \phi \rangle \qquad a^{+}_{\vec{k}} = -\langle \bar{f}_{\vec{k}}, \phi \rangle$ $\quad$ time indep.

- using the equal-time canonical comm. rels. $\quad [\phi(\vec{x},t), \phi(\vec{x}',t)] = [\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$

$$[\phi(\vec{x},t), \pi(\vec{x}',t)] = i\hbar\, \delta^{(d-1)}(\vec{x}-\vec{x}'), \qquad \pi = \dot{\phi}$$

we find $\quad [a_{\vec{k}}, a_{\vec{k}'}] = -\langle f_k, \bar{f}_{k'} \rangle = 0 \quad , \quad [a^{+}_{k}, a^{+}_{k'}] = -\langle \bar{f}_k, f_{k'} \rangle = 0.$

$$[a_{\vec{k}}, a^{+}_{k'}] = \langle f_k, f_{k'} \rangle = (2\pi)^{d-1} \delta^{(d-1)}(k-k')$$

$\Rightarrow$ commut. relations of the a_k, a^{+}_k $\Leftrightarrow$ K.G. inner product of the mode functions. They corresp. to standard creation-annih. operators if $\langle f, f \rangle > 0$

- for $\dot{f} = \pm i\omega f \qquad \langle f, f \rangle = \mp \frac{2\omega}{\hbar} \int |f|^2$ $\quad$ need to choose lower sign

$$f_{\vec{k}} = \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t + i\vec{k}\vec{x}}$$

= normalized positive frequency sol'n to the e.o.m.

- since the Hamiltonian is $\propto$ number operator $a^\dagger_{\vec{k}} a_{\vec{k}}$ (up to an ∞ ct. shift)

$$\& \quad [N, a^\dagger] = a^\dagger \qquad [N, a] = -a$$

→ to build the Hilbert sp. of the th, intro the vacuum st.

$$a_{\vec{k}} |0\rangle = 0 \qquad \forall \vec{k}$$

– Fock space of st. $a^\dagger_{k_1} \dots a^\dagger_{k_n} |0\rangle$ excited states w n particles

The free quantum scalar in curved spt.

- the above disc. can be easily generalised to a free scalar prop. in a curved spt.

- $(\Box_g - m^2)\Phi = 0$

- as discussed, in curved spt $\nexists$ an a priori preferred notion of time & positive freq. modes w.r.t. it

- so, we will simply decompose the scalar field in a (some) basis of complex sol's to the e.o.m

$$\Phi(x) = \overset{\text{continuous}}{\sum_i} a_i f_i(x) + a_i^\dagger \bar{f}_i(x)$$

choice normalis.

w/ the property that $\langle f_i, f_j \rangle = \delta_{ij}$ $\quad \langle f_i, \bar{f}_j \rangle = 0 \quad \forall f_i, \bar{f}_j$

$(= -\langle \bar{f}_i, \bar{f}_j \rangle)$

≈ decomp. of the space of complex sols into a positive norm subsp. & its complex conjugate.

- given such a decomposition, we can simply define annihilation operators

$$a_i = \langle f_i, \phi \rangle \qquad a_i^\dagger = -\langle \bar{f}_i, \phi \rangle$$

(from which it follows ϕ can be decomp. as above)

- the norm. prop. of the f_i $\Rightarrow$ $a_i, a_i^\dagger$ satisfy standard creation-annihilation comm. rels. $[a_i, a_j^\dagger] = \langle f_i, f_j \rangle = \delta_{ij}$

- the "vacuum" state can be def. as $a_i |0_f\rangle = 0$

- "excited st" $a_i^\dagger \cdots a_p^\dagger |0_f\rangle$ Fock sp. of states.

However, the basis f_i of sols to the KG eqn is by no means unique.

$\Rightarrow$ many $\neq$ notions of "vacua" possible (in general, no preferred one)

- if $\exists$ another basis of complex sols $g_i = \sum_j \overset{\langle g_i, f_j\rangle}{\alpha_{ij}} f_j + \overset{-\langle g_i, \bar{f}_j\rangle}{\beta_{ij}} \bar{f}_j$

then g_i has same norm. prop. if
$$\begin{cases} \sum_k \alpha_{ik} \alpha^*_{jk} - \beta_{ik} \beta^*_{jk} = \delta_{ij} \\ \sum_k \alpha_{ik} \beta_{jk} - \beta_{ik} \alpha_{jk} = 0. \end{cases}$$

- the creation-annih. ops assoc. w/ the g_i basis $b_i, b_i^\dagger$ can be defined as above, & we can decomp.

$$\phi = \sum_i b_i g_i + b_i^\dagger \bar{g}_i \qquad = \sum_i a_i f_i + a_i^\dagger \bar{f}_i$$

$\Rightarrow$ the two sets of creation-annih. ops. are rel. as

$$a_i = \sum_j \alpha_{ji} b_j + \beta^*_{ji} b_j^\dagger \qquad b_i = \sum_j \alpha^*_{ij} a_j - \beta^*_{ij} a_j^\dagger$$

Bogoljubov transf.

Bogolyubov coeff

- the upshot of this discussion is that the exp. value of the # qp. assoc. w/ the g_i modes in the vacuum defined w.r.t. the f_i ones is $\neq 0$

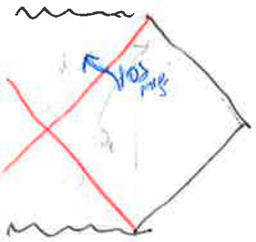
$$\langle 0_f | b_i^{+} b_i | 0_f \rangle = \sum_{jk} \langle 0_f | (\alpha_{ij} a_j^{+} - \beta_{ij} a_j)(\alpha^*_{ik} a_k - \beta^*_{ik} a_k^{+}) | 0_f \rangle$$

$$= \sum_j \beta_{ij} \beta^*_{ij} \geq 0$$

$\Rightarrow$ if the $\beta_{ij} \neq 0$, then the $|0_f\rangle$ vacuum appears to carry field excit. from the p.d.v. of the g Fock space.

Back to the Hawking effect

- what lies @ the heart of the Hawking effect is that the state of the quantum fields on a b.h. backgnd should be $\ni$ an infalling observer through the event horizon feels nothing part. happening ($\sim |0\rangle_{\text{infalling}}$, in part no high. freq. modes in this frame should be present). This state looks like a thermal state to the observer using Schwarzschild time to define his +/- frequency modes (accelerated observers).



This main physical effect can be illustrated via a simpler computation in Rindler space (patch of 2d Minkowski seen by accelerated observers). To see why, expand the Schwarzschild metric near the horizon

$$ds^2 = \left(1 - \frac{2M}{r}\right)\left(-dt^2 + dr_*^2\right) + r^2 d\Omega_2^2 \qquad dr_* = \frac{dr}{1 - \frac{2M}{r}}$$

$$r = 2M + \rho^2 \quad \Rightarrow \quad ds^2 = \frac{\rho^2}{2M}\left(-dt^2 + \left(2\rho d\rho \cdot \frac{2M}{\rho^2}\right)^2\right)$$

Rindler space.

$$= 8M\, d\rho^2 - \frac{\rho^2 dt^2}{2M} = 8M\left(d\rho^2 - \rho^2 \frac{dt^2}{16M^2}\right)$$

• also, a free scalar in the Schw. backgnd. can be decomp. in spherical harmonics

$$\Phi(t,r,\theta,\varphi) = \sum_{\ell,m} \frac{\varphi_{\ell m}(t,r)}{r} Y_{\ell m}(\theta,\varphi)$$

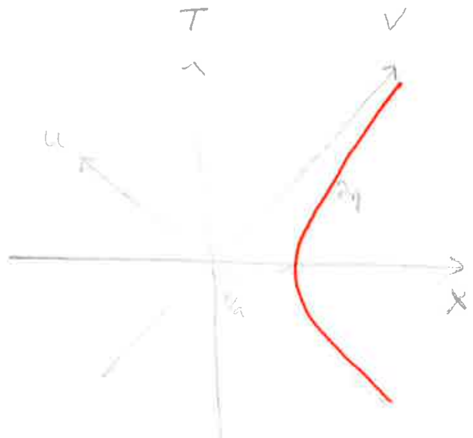
$$(\Box - m^2)\Phi = 0 \quad \Rightarrow \quad (\partial_t^2 - \partial_{r_*}^2 + \underbrace{V_{\ell m}}_{\left(1-\frac{2M}{r}\right)\left(\frac{2M}{r^3} + \frac{\ell(\ell+1)}{r^2} + m^2\right)})\varphi_{\ell m} = 0$$

$\rightarrow 0$ near the hor.

$\approx$ massless scalar in 2d.

• so, we can consider a toy model = massless scalar in 2d Rindler sp.

The Unruh effect



• remember Rindler space is a patch of 2d Minkowski, as seen by an unif. accelerated obs (a)

• his worldline is $T(\tau) = \frac{1}{a}\sinh a\tau$

$X(\tau) = \frac{1}{a}\cosh a\tau$

& one can compute $a^\mu = \frac{D^2 x^\mu}{d\tau^2} = \frac{d^2 x^\mu}{d\tau^2} = (a\sinh a\tau,\ a\cosh a\tau)$

$\Rightarrow a^\mu a_\mu = a^2 = \text{const.}$

$$X^2 = T^2 + \frac{1}{a^2}$$

• can choose coord. on Minkowski sp. $|T| < X$ ← adapted to accelerated motion

$$T = \frac{1}{a} e^{a\xi} \sinh(a\eta) \qquad X = \frac{1}{a} e^{a\xi}\cosh(a\eta), \quad \eta,\xi \in (-\infty,\infty)$$

(previous obs has $\eta = \tau$ & $\xi = 0$)

obs w/ ≠ acc α have $\tau_\alpha = \frac{a}{\alpha}\eta$ & $\xi_\alpha = \frac{1}{a}\ln\frac{a}{\alpha}$

- in these coord, the Minkowski metric takes the form

$$ds^2 = e^{2a\xi}\left(-d\eta^2 + d\xi^2\right)$$

- the surface gravity of the assoc. Rindler horizon is $\kappa = a$

- we would now like to show that the Minkowski vacuum looks like an excited state from the p.d.v. of the accelerated observer

- as advertised, we will look at a massless 2d scalar field & compute the corresp. Bogoliubov coeff.

- intro null Minkowski coord $U, V = T \mp X$; Rindler coord $\tilde{u}, \tilde{v} = \eta \mp \xi$ $\Big\}$ $U = -\frac{1}{a} e^{-\tilde{u}a} < 0$, $V = \frac{1}{a} e^{\tilde{v}a} > 0$.

- the wave eqn. in these coord. are $\partial_U \partial_V \phi = 0$; $\partial_{\tilde{u}} \partial_{\tilde{v}} \phi = 0$,
w/ sols $\underbrace{f(U)}_{RM} + \underbrace{g(V)}_{LM}$, $\underbrace{f(\tilde{u})}_{RM} + \underbrace{g(\tilde{v})}_{LM}$ right / left-moving $k \doteq \pm\omega$

- in the $X > |T|$ patch, the scalar field can be written as

$$\phi = \int_0^\infty \frac{d\omega}{2\pi} \frac{1}{\sqrt{2\omega}} \left(e^{-i\omega U} a_\omega + e^{i\omega U} a_\omega^+ \right) + LM \qquad \text{Minkowski modes.}$$

or

$$\phi = \int_0^\infty \frac{d\Omega}{2\pi} \frac{1}{\sqrt{2\Omega}} \left(e^{-i\Omega\tilde{u}} b_\Omega + e^{i\Omega\tilde{u}} b_\Omega^+ \right) + LM \qquad \text{Rindler modes.}$$

w/ both sets a_ω, b_Ω satisfying the standard comm. rels.

$$[a_\omega, a^+_{\omega'}] = \delta(\omega-\omega') \qquad [b_\Omega, b^+_{\Omega'}] = \delta(\Omega-\Omega')$$

- the Minkowski vacuum is defined as

$$a_\omega |0_M\rangle = 0 \qquad + LM \qquad \text{global vac. st. in Minkowski}$$

- the Rindler vacuum corresp. to $b_\Omega |0_R\rangle = 0$. no part. detected by acc. obs. (but singular state)

- the two sets of ops. are related via a Bogolyubov transf

$$b_\Omega = \int d\omega \left(\alpha^*_{\Omega\omega} a_\omega - \beta^*_{\Omega\omega} a^\dagger_\omega \right)$$

w/ the normaliz. cond. $\int d\omega \left(\alpha_{\Omega\omega} \alpha^*_{\Omega'\omega} - \beta_{\Omega\omega} \beta^*_{\Omega'\omega} \right) = \delta(\Omega - \Omega')$

- not possible to invert b/c Rindler modes are defined only in 1/4 Mink. sp.

- plugging this into the scalar field, we obtain

$$\phi_I = \int_0^\infty \frac{d\Omega}{2\pi} \frac{1}{\sqrt{2\Omega}} \left(e^{-i\Omega\tilde{u}} \int d\omega \left(\alpha^*_{\Omega\omega} a_\omega - \beta^*_{\Omega\omega} a^\dagger_\omega \right) + e^{i\Omega\tilde{u}} \int d\omega \left(\alpha_{\Omega\omega} a^\dagger_\omega - \beta_{\Omega\omega} a_\omega \right) \right)$$

reg. I

$$= \int_0^\infty \frac{d\omega}{2\pi} \frac{1}{\sqrt{2\omega}} \left(e^{-i\omega u} a_\omega + e^{i\omega u} a^\dagger_\omega \right)$$

$\int_{-\infty}^{\infty} d\tilde{u}\, e^{i(\Omega-\Omega')\tilde{u}} = 2\pi\delta(\Omega-\Omega')$

$$\Rightarrow \quad \frac{1}{\sqrt{\omega}} e^{-i\omega u} = \int \frac{d\Omega'}{\sqrt{\Omega'}} \left(\alpha^*_{\Omega'\omega} e^{-i\Omega'\tilde{u}} - \beta_{\Omega'\omega} e^{i\Omega'\tilde{u}} \right) \times e^{\pm i\Omega\tilde{u}}$$

& int over $\tilde{u} \in (-\infty, \infty)$

$$2\pi\, \alpha^*_{\Omega\omega} = \frac{\sqrt{\Omega}}{\sqrt{\omega}} \int_{-\infty}^{\infty} d\tilde{u}\, e^{i\Omega\tilde{u} - i\omega u} = \sqrt{\frac{\Omega}{\omega}} \int_{-\infty}^{0} (-)\frac{du}{au} (-au)^{-\frac{i\Omega}{a}} e^{-i\omega u}$$

$$2\pi\, \beta_{\Omega\omega} = -\sqrt{\frac{\Omega}{\omega}} \int_{-\infty}^{\infty} d\tilde{u}\, e^{-i\Omega\tilde{u} - i\omega u} = -\sqrt{\frac{\Omega}{\omega}} \int_{-\infty}^{0} du\, (-au)^{\frac{i\Omega}{a} - 1} e^{-i\omega u}$$

- These int. are of the form $\int_0^\infty dz\, z^{\frac{i\Omega}{a} - 1} e^{\mp i\omega z/a}$ (for α, β).

similar to Γ function $b^{-s}\Gamma(s) = \int_0^\infty dz\, z^{s-1} e^{-bz}$ w/ $s = \frac{i\Omega}{a}$ & $b = \pm\frac{i\omega}{a}$

need $\mathrm{Re}\, b = \varepsilon > 0$ for convergence. $b^{-s} = e^{-s \ln b} = e^{-\frac{i\Omega}{a} \ln\left(\varepsilon \pm \frac{i\omega}{a}\right)}$, $\ln\left|\frac{\omega}{a}\right| \pm \frac{i\pi}{2}$

$$b^{-s} = e^{-\frac{i\Omega}{a} \ln\left|\frac{\omega}{a}\right| \pm \frac{\pi\Omega}{2a}}$$

- thus, we find that $|\alpha_{\Omega\omega}|^2 = e^{\frac{2\pi\Omega}{a}} |\beta_{\Omega\omega}|^2$

- sanity check: $|\beta_{\Omega\omega}|^2 \to 0$ as $a \to 0$

- according to our previous considerations, the exp. value of the b-particle # op. (Rindler) in the a-vacuum state (Minkowski) is

$$\langle 0_M | N_\Omega | 0_M \rangle = \int_0^\infty d\omega \, |\beta_{\omega\Omega}|^2$$

mean # of part. of freq. Ω found by the accelerated observer

- using the norm. cond for $\Omega' = \Omega$

$$\int d\omega \left(|\alpha_{\Omega\omega}|^2 - |\beta_{\Omega\omega}|^2 \right) = \delta(\Omega - \Omega) = \text{Vol. spt} \cdot V$$

$$= \int d\omega \left(e^{\frac{2\pi\Omega}{a}} - 1 \right) |\beta_{\Omega\omega}|^2 = V$$

$$\Rightarrow \langle 0_M | N_\Omega | 0_M \rangle = \frac{V}{e^{\frac{2\pi\Omega}{a}} - 1}$$

$\Rightarrow$ mean density of part. w/ freq Ω is

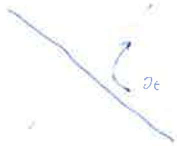
$$\boxed{n_\Omega = \frac{\langle N_\Omega \rangle}{V} = \frac{1}{e^{\frac{2\pi\Omega}{a}} - 1}}$$

Bose-Einstein distr. w/ $\boxed{T = \frac{a\hbar}{2\pi}}$

- thus, the acc. obs. sees a thermal bath of particles @ $T = \frac{a\hbar}{2\pi}$.

This temperature diverges as the obs. approaches the horizon $(a \to \infty)$ & $\to 0$ if obs is @ ∞.

The Hawking effect (analogy)



Rindler	Schwarzschild
acc. obs.	obs following orbits of ∂_t (schw. time) @ fixed r.
Minkowski spt	Kruskal diagram
Minkowski vacuum	infalling obs vacuum (eternal b.h)
acc. a	surface gravity $\kappa = \frac{1}{4GM}$ $\quad T_H = \frac{\hbar}{8\pi GM}$
Rindler vac.	"Boulware" vac (no part. det. by schw obs → singular on future (past hor.).
	* Unruh vac → b.h. from collapse. backreaction (evaporates) ⇒ BH info paradox

- greybody factors: prop. from hor. to ∞

Comments :

- the fact that the b.h. emits thermal rad $\Rightarrow$ it evaporates has led to the famous **black hole information problem** : if. b.h. is formed by collapse of matter in a pure quantum state , then evolution appears pure $\to$ mixed state , contradicting Q.M.

  – biggest clash b/w GR , QM (modern phrasing in terms of entanglement)

- in QM, state of quantum system is. repr. by state vector $|\psi\rangle \in \mathcal{H}$ **pure state**
- more generally , the system can be in a **statistical ensemble** of different state vectors. This situation (characteristic for subsystems of a quantum system) is characterised by a **density matrix**

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \qquad \rho = \rho^+ \ , \ Tr\rho = 1$$

p_i: prob. that system in state $|\psi_i\rangle$.

  – pure state $\rho = |\psi\rangle\langle\psi|$ in some basis, have $\rho^2 = \rho$, $\forall$ else mixed

  – can distinguish the two by computing the **von Neumann entropy** $S = -Tr\rho\ln\rho = \sum_i p_i \ln p_i$

- consider splitting a system $|\psi\rangle$ into subsystem A & its complement $\bar{A}$ $\Rightarrow \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$

  - the **reduced density matrix** $\rho_A = Tr_{\bar{A}} |\psi\rangle\langle\psi|$ defines state subsystem A.
  - the **entanglement entropy** of A w/ the rest $S_A = -Tr_A\, \rho_A \ln \rho_A$.

- the fact that the Rindler obs. detects thermal rad. can be understood also in terms of entanglement b/w ~~the~~ L & R modes.

- in your p.s. , you will find rel b/w Minkowski annih. ops & the left/right Rindler modes

$$a|0_M\rangle \propto \left(b_R e^{\frac{\pi\Omega}{2a}} - b_L^+ e^{-\frac{\pi\Omega}{2a}}\right)|0_M\rangle = 0 \quad \Rightarrow b_R|0_M\rangle = e^{-\frac{\pi\Omega}{a}} b_L^+ |0_M\rangle$$

$$\frac{\partial}{\partial b_R^+}$$

$$|0_M\rangle = e^{e^{-\frac{\pi\Omega}{a}} b_L^+ b_R^+} \underbrace{|0_R\rangle}_{\text{left+right}} = \sum_m \frac{1}{m!} e^{-\frac{\pi\Omega m}{a}} (b_L^+ b_R^+)^m |0_R\rangle = \sum_m \overbrace{e^{-\underbrace{\frac{\pi}{a}}_{\beta/2} E_m}}^{e^{-\beta/2 E_m}} |m_L\rangle|m_R\rangle$$

TFD state

$$\rho_R = Tr_L |0_M\rangle\langle 0_M| = Tr_L\, e^{-\beta E} |E_L\rangle|E_R\rangle\langle E_L|\langle E_R| = \underbrace{e^{-\beta E} |E_R\rangle\langle E_R|}_{\text{thermal } \rho}$$

due to fact that Rindler obs. only sees $\frac{1}{2}$ Min