Introduction to holography

What is holography?

- deals w/ the fundamental nature of quantum gravity = GR + QM

- needed @ very least to understand black hole / Big Bang singularities

- QG has a natural associated length scale $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\,m$ Planck length
  - QG effects should become strong @ this scale, ...
  - $\simeq$ smallest distance that may be probed $E \gtrsim \frac{\hbar}{d}$, $GE \lesssim d \Rightarrow d \gtrsim \ell_P$ lest a black hole form

- gravity is rather different from the other interactions in nature, which are
  - well-described by relativistic QFT on a fixed backgnd. spacetime ($\eta_{\mu\nu}$)
  - local operators $\mathcal{O}(x)$, causality $[\mathcal{O}(x), \mathcal{O}(y)] = 0$ if $\eta_{\mu\nu}\Delta x^\mu \Delta x^\nu > 0$
  - these QFTs can often be understood as starting w/ classical field theories on a fixed backgnd. & then quantizing them, dealing w/ divergences, etc.

- on the other hand, the framework of classical GR consists of a smooth spt. manifold, equipped w/ a metric that evolves according to Einstein's eqns., which are generally covariant
  - $\exists$ no fixed causal structure $\rightarrow$ microcausality hard to make sense of if $g_{\mu\nu}$ fluctuates (can work perturbatively $g_{\mu\nu} = \underbrace{\bar{g}_{\mu\nu}}_{\text{cls. backgnd.}} + \kappa \underbrace{h_{\mu\nu}}_{\text{pert.}}$ & refer to the lightcone def. by $\bar{g}_{\mu\nu}$)
  - absence of local gauge-invariant operators

- some issues that arise when trying to "quantize" this system ($\exists$ various methods)

  - **canonical quantization** (intro canonical variables & their conjugate momenta)

    - requires 3+1 split (unnatural from the point of view of general covar.)

    - can. vars. γ_{ij} (spatial metric) + momenta π^{ij} + **constraints**

      $H = 0$ + bnd. terms $\mapsto$ closed universes $H|\Psi\rangle = 0$ (not obvious how to recover **dynamics**.)

      very $\neq$! $\rightarrow$ $\hookrightarrow$ infinite worlds (asympt. notion of time)

  - quantize gravity **perturbatively** around a fixed backgnd. (Minkowski). The resulting theory is **non-renormalizable**, but perfectly fine as an **effective field theory** for a **massless spin 2** particle

$$S = \frac{1}{16\pi G}\int d^d x \sqrt{g}\,(R - 2\Lambda + c R^2 + d R^3 + \dots)$$

coeff. suppressed by $\frac{1}{M_s^{\#}}$; $M_s \leq M_{Pl}$

  - this action can perfectly well describe **low-energy** observables $\rightarrow$ expansion in $E/M_{pl/s}$ or in $l_{pl/s}/R$ (R = curvature radius) w/ a given precision.

    take =

  - this expansion obviously breaks down for $E \approx M_{pl}$, or $R \simeq l_p$.

    - such large curvature regimes relevant for **Big Bang** & **black hole singularities**

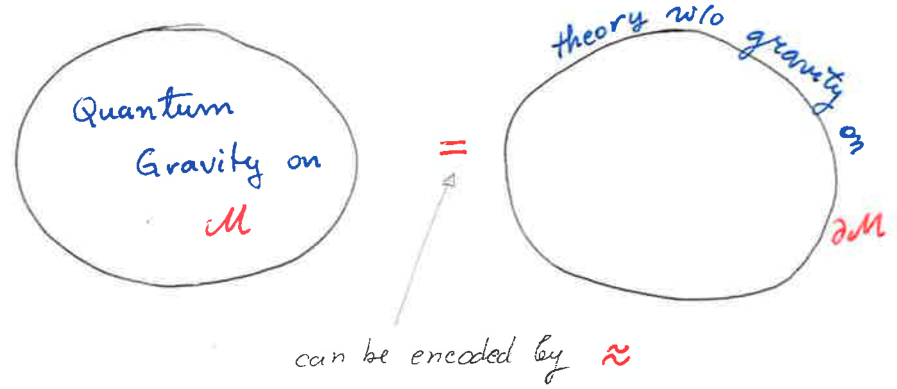
    - there also appears to $\exists$ a **subtle** breakdown of the EFT description in **black hole evaporation** (unexpected, since $R \gg l_p$) $\Rightarrow$ information loss problem

  - **euclidean path integral** methods (seems ill-defined, but interesting semicl. results)

- given $d_{min} = l_p$ @ which spt. can be probed, is the metric a fundamental field, or just a phenomenological description valid just @ large scales? Should one quantize it @ all?

- whatever the resolution to these puzzles is, it is clear that gravity is **very different** from the other forces of nature

Holography corresp. to an entirely different perspective on QG. than the direct quantization attempts. Its roots lie in black hole thermodynamics a subject aiming to underst. what happens w/ the laws of thermodynamics in presence of black holes. This "thermodynamic reasoning" (which is non-perturbative in GR) led to the idea that



proposed as a fundamental property of Quantum Gravity

- note such a relation implies a high degree of non-locality in QG (the locality seen @ low energies is approximate at best)

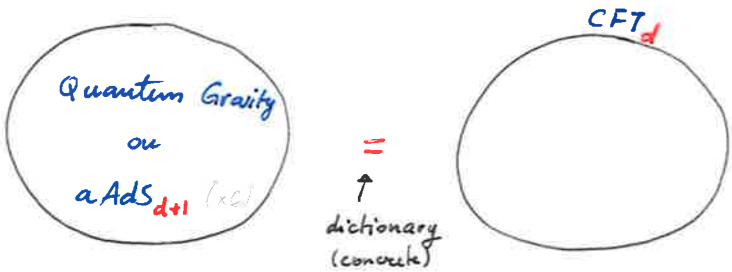
- as we will see, the argument for holography is very compelling & universal; however, it is also quite heuristic & provides no concrete details about how the correspondence is supposed to be realised

- an essential step in holography becoming widely accepted has been its concrete realization in innumerous examples in string theory* w/ a large # of successful non-trivial checks

* string theory: the only consistent theory of QG known to date. While perturbatively defined, many nonperturbative objects inside it have been studied

- in a theory of QG, it only makes sense to fix the asymptotic structure of the metric near ∞, while the metric in the interior is allowed to fluctuate arbitrarily

- the best understood realizations of the holographic correspondence is when the asympt. structure of $\mathcal{M}$ is anti-de Sitter (AdS). This universal str. of the asympt. spt. translates into a universal structure of the dual non-gravitational description = = conformal field theory (CFT) – a very special kind of QFT – in one dim. less



This is the famous AdS/CFT correspondence

- possibly the best-known example of AdS/CFT is the corresp. b/w

$$\boxed{\text{type IIB string th. on } AdS_5 \times S^5 = \mathcal{N}=4\ SU(N) \text{ super Yang-Mills on } \mathbb{R}^{1,3}}$$

but there are many other examples, w/ many other precision checks

- CFTs are nice in that they allow for an axiomatic def^n that is valid even @ strong coupling, and this can be used to establish the hologr. dict to AdS w/o specific reference to string theory or any particular realization

(*)

- ∃ also many non-AdS/non-CFT examples of holographic pairs within string theory, but which have not been similarly collected into universal classes of hologr. correspondences, as for AdS/CFT

- the holographic dictionary is also less well understood in these cases

(*) since CFTs do possess a nonpert. def^n, one may use the corresp. to define QG in AdS

Outline of this course

I. Hints of holography in classical GR

1. The GR Hamiltonian is a pure boundary term on-shell ($\partial M \neq 0$)

   - peek into the structure of the theory
   - energy - as well as all other conserved charges - are naturally defined @ ∂M

2. Black hole thermodynamics suggests that black holes carry an entropy $S = A_H/4G$ $\mapsto$ main argument for holography: universal, but heuristic

II Lightning introduction to string theory & how AdS/CFT is derived

- the string spectrum
- D-branes (non-perturbative objects ← essential to the deriv)
- deriv. of AdS/CFT from two ≠ descr. of D-branes

III The AdS/CFT dictionary

1. CFTs
2. AdS & symmetry match
3. Holographic dictionary : correlation functions
   - finite-temperature states (↔ black holes)
   - entanglement ← emergence of spt. & gravity from the CFT (new perspective on QG)

The Hamiltonian formulation of GR

- we would like to show that the GR Hamiltonian is a pure bnd. term on-shell (in part., $H=0$ for a manifold w/ closed spatial sections, e.g. dS)

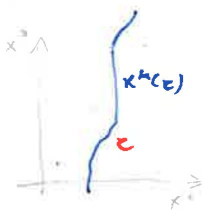
- this analysis will also show how to compute conserved charges in GR (energy, ang. momentum, etc) $\rightarrow$ not a trivial pb.

The above property of the GR Hamiltonian is related to the time reparam. invariance of GR. We will start w/ a simple toy model for such behaviour, = the relativistic point particle

• The relativistic point particle

- expect Lorentz-invar. dynamics; however, simplest descr. of the motion as $x^i(t)$ in some inertial frame, w/ action $S[x^i(t)] = -m \int dt \sqrt{1-\dot{x}^i \dot{x}_i}$ $\quad \dot{} = \frac{d}{dt}$

- to make Lorentz invar. manifest, intro some arbitrary param τ to label the position on the worldline, now given as $x^\mu(\tau)$. the action is



$$S[x^\mu(\tau)] = -m \int d\tau \sqrt{-\dot{x}^\mu \dot{x}_\mu} \qquad \text{w/} \quad \dot{} = \frac{d}{d\tau} \text{ now}$$

- manifest Lorentz invar: $x'^\mu = \Lambda^\mu{}_\nu x^\nu$ (global symm.)
- reparametrization invar $\quad \tau \to \tau' = \tau'(\tau)$, $\quad x^\mu(\tau) \to x'^\mu(\tau') = x^\mu(\tau)$ $\Big\}$ local symm.

- previous action obtained by gauge-fixing $x^0(\tau) = \tau$ (x^0 no longer dyn.) its time evol. fixed by the gauge cond.

The canonical momenta conjugate to x^μ are

$$p_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = \frac{m \dot{x}_\mu}{\sqrt{-\dot{x}^2}}$$

- they satisfy the constraint $\quad p_\mu p^\mu + m^2 = 0$

- note also one cannot uniquely det. the velocities $\dot{x}^\mu$ from the momenta (arbitrary rescaling $\dot{x}^\mu = \alpha p^\mu$)

- the Hamiltonian $\quad H = p_\mu \dot{x}^\mu - \mathcal{L} = \alpha(p^2 + m^2) = 0 \quad$ on the constraint surface.
  this is a direct conseq. of the reparam. invar. of the action

- the dynamics of the system takes place entirely on the constraint surface $p^2 + m^2 = 0$. Moreover, configs that only differ by a gauge transf. are to be physically identified. In the quantum th., the constraint is taken to annih. physical states $\quad (\hat{p}^2 + m^2)|\psi_{phys}\rangle = 0$

General Relativity

start from Lagr. & construct Ham. in standard way.

- the Einstein-Hilbert action is $\quad S_{EH} = \frac{1}{16\pi G}\int_M d^dx\, R\sqrt{g}$

- however, this action does not have a well-defined variational principle (namely $\delta S \neq 0$ on-shell for generic sols to the e.o.m)

Example : classical point particle

$$S = \int_{t=0,\, q_i}^{T,\, q_f} dt\, \frac{m\dot{q}^2}{2} \quad \text{has a well-def. var. principle}$$

$$\delta S = \int_{q_i,0}^{q_f,T} dt\, m\dot{q}\,\delta\dot{q} = \int_{q_i,0}^{q_f,T} dt\, \partial_t(m\dot{q}\,\delta q) - \int dt\, \underbrace{m\ddot{q}}_{e.o.m}\,\delta q = p\,\delta q\Big|_i^f - \int e.o.m$$

$$\delta S_{on\text{-}shell} = p\,\delta q\Big|_i^f = 0 \quad \text{if } q_{f,i} \text{ fixed} \quad (\text{soln } \exists, \text{ as e.o.m } 2^{nd}\text{order})$$

· on the other hand $S' = -\int dt \frac{m}{2}\, q\ddot{q}$ does not have a well-def var. principle
(only differs from S by bnd. terms $\propto \frac{d}{dt}(q\dot{q})$)

$$\delta S' = -\frac{m}{2}\int dt\, \delta q\, \ddot{q} - \frac{m}{2}\int dt\, q\, \delta\ddot{q} = -\frac{m}{2}\int dt\, \partial_t(q\,\delta\dot{q}) + \frac{m}{2}\int dt\, \dot{q}\,\delta\dot{q} - \frac{m}{2}\int dt\, \ddot{q}\,\delta q$$

$$= \frac{m}{2}\int dt\, \partial_t(\dot{q}\,\delta q - q\,\delta\dot{q}) - m\int dt\, \underbrace{\ddot{q}}_{e.o.m}\,\delta q = \frac{m}{2}(\dot{q}\,\delta q - q\,\delta\dot{q})\Big|_i^f - \int e.o.m.$$

· e.o.m identical ; however $\delta S' \neq 0$ on-shell b/c we cannot fix both $q, \dot{q}$ @ each endpoint of the interval.

· The problem stems from the fact that, even though the e.o.m are only 2nd order, the Lagrangian contains time deriv. of q higher than first. (Had the e.o.m been higher than 2nd order, then there wouldn't have been a pb. w/ fixing derivatives higher than the first.)

· fix : add a bnd. term $\frac{m}{2} q\dot{q}\Big|_i^f$ to S' (which does not affect the e.o.m) so that

$$\delta S' + \delta\left(\frac{m}{2} q\dot{q}\right) = m\,\dot{q}\,\delta q \quad \checkmark$$

well-def var. p. w/ Dirichlet bnd. cond. (q fixed)

– we could also have added $-\frac{m}{2} q\dot{q}$, case in which

$$\delta S' - \delta\left(\frac{m}{2} q\dot{q}\right) = -m\, q\, \delta\dot{q}$$

& the var. principle is well-def w/ Neumann bnd. cond ($\dot{q}$ fixed)

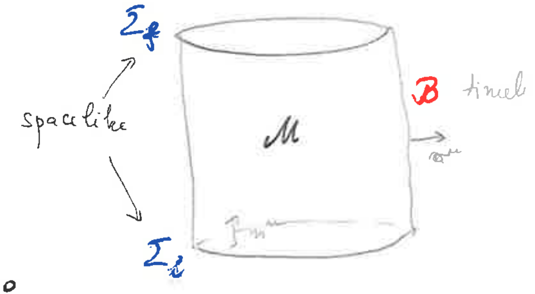
The Einstein-Hilbert action is precisely of the above kind, i.e. contains second derivatives of the metric. & the var. principle w/ fixing the metric on ∂M ($\mathcal{B}$) is not compatible

· this can be fixed by adding the so-called Gibbons-Hawking, York bnd. term, constructed from the extrinsic curvature of the boundary $\mathcal{B}$ inside M

def^n theory　　　　initial/final states

- we will be simultaneously considering timelike and spacelike boundaries, characterized by the sign of the unit normal to the hypersurface

$$n_\mu n^\mu = \varepsilon = \pm 1 \qquad \begin{array}{l} + : \mathcal{B} \\ - : \Sigma_{i,f} \end{array}$$



- we start by intro. the induced metric on $\partial\mathcal{M}$

$$\gamma_{\mu\nu} = g_{\mu\nu} - \varepsilon\, n_\mu n_\nu \qquad \text{w/ prop. } \gamma_{\mu\nu} n^\nu = 0$$

- the extrinsic curvature of the bnd. inside $\mathcal{M}$: how $\gamma_{\mu\nu}$ changes as we move along n^μ

$$K_{\mu\nu} \equiv \frac{1}{2} \mathcal{L}_{n^\mu} \gamma_{\mu\nu} = \frac{1}{2}\left(n^\lambda \nabla_\lambda \gamma_{\mu\nu} + \nabla_\mu n^\lambda \gamma_{\lambda\nu} + \nabla_\nu n^\lambda \gamma_{\lambda\mu} \right)$$

$$= \nabla_{(\mu} n_{\nu)} - \varepsilon\, n^\lambda n_{(\mu} \nabla_\lambda n_{\nu)} = \nabla_\mu n_\nu - \varepsilon\, n_\mu n^\lambda \nabla_\lambda n_\nu$$

e.g. cylinder $dr^2 + r^2 d\varphi^2 + dz^2$, $n_r = 1$, intrinsic now, $K_{\mu\nu} = \frac{1}{2}\partial_r (g_{\mu\nu})$, $K_{\varphi\varphi} = \frac{r}{2}$, $K = \frac{1}{r}$

symmetric

2nd term ... n^μ // geod

where the last equality can be proven using the def^n of the normal: if the hypersurface is defined via $f(x^\mu)$, then $n_\mu \propto \partial_\mu f$.

- the trace of the extrinsic curvature is :

$$K \equiv \gamma^{\mu\nu} K_{\mu\nu} = g^{\mu\nu} K_{\mu\nu} = \nabla_\mu n^\mu \qquad (\text{since } K_{\mu\nu} n^\nu = 0)$$

$n_\mu = \dfrac{\varepsilon\, \partial_\mu S}{\sqrt{g^{\mu\nu} \partial_\mu S \partial_\nu S}}$

$d\sigma^\mu \propto g^{\mu S}$

$\propto \varepsilon\, n^\mu$

- it can be shown that the action

GHY bnd. term

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^d x \sqrt{g}\, R + \frac{\varepsilon}{8\pi G} \int d^{d-1} x \sqrt{\gamma}\, K$$

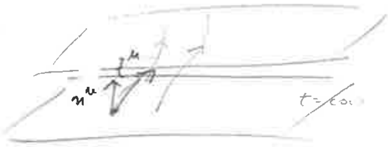
does have a well-def var. principle for Dirichlet bnd. cond

there is one such Gibbons-Hawking-York bnd. term for each timelike ($\varepsilon = 1$) & spacelike ($\varepsilon = -1$) component of $\partial\mathcal{M}$.

see. e.g. M. Blau's lecture notes on GR

- we would now like to pass to the Hamiltonian formalism. This necessarily introduces a split of the coord. x^μ into "time" & "space" – not natural from p.d.v. of general covariance.

- to start, we *foliate* the spacetime $\mathcal{M}$ by a family of spacelike hypersurf. Σ_t, param by a global time function t (so the *normal* to Σ_t $n_\mu \sim \partial_\mu t$) & some vector field t^μ that will be defining the "flow of time", norm. $\ni$ $t^\mu \partial_\mu t = 1$.



- the vector t^μ may be decomposed into $\parallel$ & $\perp$ parts w.r.t n^μ

$$t^\mu = N n^\mu + N^\mu \quad , N^\mu n_\mu = 0 \quad \Rightarrow \quad N = -t^\mu n_\mu \, , \; N_\mu = t_\mu - \overbrace{N}^{-t^\nu n_\nu} n_\mu = \underbrace{(g_{\mu\nu} + n_\mu n_\nu)}_{h_{\mu\nu}} t^\nu$$

- the spt. metric may then be param. as

$$ds^2 = -\underbrace{N^2}_{\text{lapse}} dt^2 + \underbrace{h_{ij}}_{\text{induced metric}} (dx^i + \underbrace{N^i}_{\text{shift}} dt)(dx^j + N^j dt)$$

(note that $n_\mu \propto \partial_\mu t \Rightarrow n_\mu \propto \{1, 0 \ldots\}$ in these coord
$n_\mu N^\mu = 0 \Rightarrow N^0 = 0$)

- the functions N, N^i are *arbitrary* (arbitrary choice of slicing)

- decomposing the Lagrangian according to this "3+1" split, we find

$$R = \underbrace{\bar{R}[h_{ij}]}_{\text{Ricci sc. constr from } h_{ij}} \overbrace{-}^{+\epsilon} (K^2 - K^{\mu\nu} K_{\mu\nu}) \overbrace{-2}^{+2\epsilon} \nabla_\mu (n^\nu \nabla_\nu n^\mu - n^\mu \nabla_\nu n^\nu)$$

so
$$S_{EH} = \frac{1}{16\pi G} \Big[\int d^d x \sqrt{h}\, N (\bar{R} - K^2 + K_{\mu\nu} K^{\mu\nu}) \overbrace{+2}^{2\epsilon \times (+\epsilon)} \int d^{d-1} x \sqrt{h}\, \underbrace{n_\mu (n^\nu \nabla_\nu n^\mu - n^\mu \nabla_\nu n^\nu)}_{K}$$

intro $x^\mu = (t, y^a)$ $\quad E^\mu_a = \frac{\partial x^\mu}{\partial y^a}$
$E^t_i = 0, \; E^j_i = \delta^j_i$

$-2\epsilon K$

- for *spacelike* boundaries

$$S_{EH} + S_{GH} = \frac{1}{16\pi G} \int dt\, d^{d-1}x\, N \sqrt{h} \left(\bar{R}[h] - K^2 + K_{\mu\nu} K^{\mu\nu} \right)$$

w/
$$K_{ij} = \frac{1}{2N} \left(\dot{h}_{ij} - \bar{\nabla}_i N_j - \bar{\nabla}_j N_i \right) = E^\mu_i E^\nu_j K_{\mu\nu}$$

$$\mathcal{L}_{ADM} = \bar{R} + \underbrace{G^{ijkl}}_{\frac{1}{2}(h^{ik}h^{jl} + h^{il}h^{jk} - 2h^{ij}h^{kl})} K_{ij} K_{kl}$$

(residual symm. after fixing diff + Weyl). conserved.

- note this Lagrangian does not contain $\dot{N}$ or $\dot{N}_i$ => N, N^i act as Lagrange multipliers => constraints.

- @ the level of the Lagr., one finds the N, N_i e.o.m. yield $\underbrace{\bar{R}[h] - K_{\mu\nu}K^{\mu\nu} + K^2}_{\mathcal{H}} = 0$ and $\bar{\nabla}_i\,(K^{ij} - K\,h^{ij}) = 0$

- another consequence is that the momenta canonically conjugate to N, N^i vanish identically $\pi_N = \pi_{N_i} = 0$

- the canonical vars. are thus h_{ij} & $\pi^{ij} = \dfrac{\partial \mathcal{L}_{ADM}}{\partial \dot{h}_{ij}} = \sqrt{h}\,(K^{ij} - K\,h^{ij})$ (ignoring $1/16\pi G$ factor)

- the Hamiltonian density is

$\mathcal{H}$ = Hamiltonian constraint ($\perp$ defs to Σ)

$$\mathcal{H}_{ADM} = \pi^{ij}\dot{h}_{ij} - \mathcal{L}_{ADM} = N\sqrt{h}\,\overbrace{\left(\frac{\pi_{ij}\pi^{ij}}{h} - \frac{\pi^2}{(d-2)h} - \bar{R}[h]\right)} + 2\pi^{ij}\,\bar{\nabla}_i N_j$$

$$= N\sqrt{h}\,\mathcal{H} - 2N_j\,\underbrace{\bar{\nabla}_i\pi^{ij}}_{\mathcal{H}^j} + 2\bar{\nabla}_i(\pi^{ij}N_j)$$

$\mathcal{H}^j$ = momentum constraint (spatial diffs)

same as constr. above

- more standardly, the constraints $\mathcal{H} = 0$, $\mathcal{H}^i = 0$ are obtained in the Hamiltonian formalism, by requiring that the constraints $\pi_N = \pi_{N_i} = 0$ hold @ all times, => $\{H_{ADM}, \pi_{N}, \pi_{N_i}\} = 0$ => $\mathcal{H} = 0$, $\mathcal{H}_i = 0$

- thus (up to bnd. terms), $H_{ADM} = N\mathcal{H} + N^i\mathcal{H}_i = 0$ when the constr. are satisfied

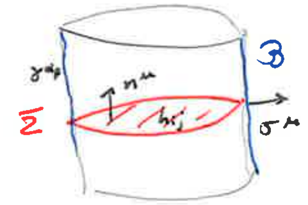
- h_{ij}, π^{ij} → too many vars; however, since $\mathcal{H}$ is highly non-linear in h & π => very hard to solve

- in an initial value formulation of GR (specify h_{ij} $\dot{h}_{ij}$ on initial surface) they are constraints on the initial data (preserved by time evolution)

- if ∃ matter => constraints modified by adding $T_{\mu\nu}n^\mu n^\nu$, $T_{\mu\nu}n^\mu E^\nu_i$:

- let us now turn to computing the energy of a given spacetime. Clearly, the answer can only be nonzero if $\partial\Sigma \neq 0$, or $\mathcal{M}$ has a non-triv timelike bnd. $\mathcal{B}$.

- for this, we need to evaluate the boundary terms in the Hamiltonian. We denote by σ^μ the unit normal to $\mathcal{B}$, & assume that $n^\mu \sigma_\mu = 0$



- two sources of bnd. terms : 1) $$S^{(\mathcal{B})}_{GHY} = \frac{1}{8\pi G}\int_{\mathcal{B}} \overbrace{d^{d-1}x}^{dt\, d^{d-2}x_{\mathcal{B}}} \sqrt{\gamma}\; K_{\mathcal{B}}$$ $K_{\mathcal{B}} = \nabla_\mu\sigma^\mu - \sigma^\mu\sigma^\nu\nabla_\mu\sigma_\nu$

 constr. from $\gamma_{\alpha\beta} = g_{\alpha\beta} - \sigma_\alpha\sigma_\beta$

 2) when writing the EH Lagrangian in Gauss - Codacci form, $\exists$ a bnd term $-2\nabla_\mu(n^\nu\nabla_\nu n^\mu - n^\mu \nabla_\nu n^\nu)$ Its contr. on $\Sigma_{i,f}$ canceled that of GHY. However, @ the timelike bnd. $\mathcal{B}$ it contributes as

orthogonality

$$\Delta S_{ADM} = \frac{1}{16\pi G}(-2)\int_{\mathcal{B}} d^{d-1}x\,\sqrt{\gamma}\;\sigma_\mu\,(n^\nu\nabla_\nu n^\mu - \cancel{n^\mu\nabla_\nu n^\nu})$$

$$= \frac{1}{8\pi G}\int_{\mathcal{B}} d^{d-1}x\,\sqrt{\gamma}\; n^\mu n^\nu \nabla_\nu \sigma_\mu$$

- their sum is thus $$\Delta S = \frac{1}{8\pi G}\int_{\mathcal{B}} d^{d-1}x\,\sqrt{\gamma}\;\underbrace{\sigma^{\mu\nu}\nabla_\mu\sigma_\nu}_{k_{\partial\Sigma}}$$

$\sigma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu - \sigma_\mu\sigma_\nu$ is the induced metric on $\partial\Sigma$

$\sigma_{\mu\nu} n^\mu = \sigma_{\mu\nu}\sigma^\mu = 0$ $\qquad \sqrt{\gamma} = N\sqrt{\sigma}$

- the contr. of this term to the ADM Hamiltonian is $\oplus$ bnd. term when obt. mom. constr.

$$H_{ADM} = \text{constraints} - \frac{1}{8\pi G}\int_{\partial\Sigma} d^{d-2}x\,\sqrt{\sigma}\; N\, k_{\partial\Sigma} + \frac{1}{8\pi G}\int_{\partial\Sigma} d^{d-2}x\; N_i \pi^{ij}\sigma_j$$

- the bnd terms repr the value of the Hamilt. evaluated on an on-shell config that satisfies the constraints

- in the asymptotically flat case, for asympt. static sols $N=1$ & $N_i=0$ then, the energy is

$$E = -\frac{1}{8\pi G}\int_{\partial\Sigma} d^{d-2}x\,\sqrt{\sigma}\,k_{\partial\Sigma}$$

- taking $\partial\Sigma$ to be an asympt. large sphere of radius r_0, the expr. for the candidate ADM energy is $\lim_{r_0\to\infty} -\frac{1}{8\pi G}\oint_{S^2_{r_0}} d^{d-2}x\,\sqrt{\sigma}\,k_{r_0}$

- this is however divergent even for flat sp. $d\bar{s}^2 = dr^2 + r^2 d\Omega_2^2$ $\quad \underline{3d}$

$k_{ab} = \frac{1}{2}\cdot\partial_r r^2 \bar{\sigma}_{ab}$

- energy differences should nonetheless make sense. We thus finally arrived @ the def^n of the ADM energy

$k = \frac{2}{r}$ $\quad \sqrt{\sigma} = r^2$

$$\boxed{E_{ADM} = -\frac{1}{8\pi G}\lim_{r_0\to\infty}\oint_{S^{d-2}_{r_0}} d^{d-2}x\,\sqrt{\sigma}\left(k_{S^{d-2}} - k^{(\eta)}_{S^{d-2}}\right)}$$

- this expression was obtained via a very direct evaluation of the GR Hamiltonian

- we will, however, find a somewhat different expr. useful when dealing w/ $\neq$ asymptotics