HW6

Spectral density

Consider a free scalar field

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2. \tag{1}$$

Starting from the definition given in the previous homework

$$\rho_O(\mu^2) = \int dr \delta(\mu^2 - M_r^2)|Z_{O,r}| \tag{2}$$

find spectral densities corresponding to operators $\phi(x)$ and $\phi^2(x)$.

Goldstone mode

Using the Ward identity for a symmetry with the spontaneous breaking, shown in class

$$\partial_\mu \langle 0|TJ^\mu(x)\phi(y)|0\rangle = -if\delta^{(4)}(x-y), \tag{3}$$

prove that in the spectrum there is necessarily a massless state. You can proceed as follows:

- Using $\phi(y) = e^{iPy}\phi(0)e^{-iPy}$, and similarly for the current express the following correlator

  $$\langle 0|J^\mu(x)\phi(y)|0\rangle \tag{4}$$

  as a sum over complete set of states $|\vec{p};r\rangle$ (note, we are using only states with zero spin). Use the following notations

  $$\langle 0|J_\mu(0)|\vec{p};r\rangle = -ip_\mu^r f_r, \quad \langle 0|\phi(0)|\vec{p};r\rangle = c_r. \tag{5}$$

- Similarly to what was done for the previous homework, use the CPT transformation

  $$\theta^\dagger\phi(0)\theta = \theta(0), \quad \theta^\dagger J^\mu(0)\theta = -J^\mu(0), \tag{6}$$

  to relate

  $$\langle 0|\phi(y)J^\mu(x)|0\rangle \tag{7}$$

  to the previously computed quantity.

- Convince yourself that the time ordered correlator can be written as

  $$\langle 0|TJ^\mu(x)\phi(y)|0\rangle = \int dr \ldots \partial_x^\mu G^{(m_r)}(x-y) \tag{8}$$

- Lastly, show that only if there is a massless state (meaning that, $m_r = 0$) equation (3) can be satisfied.

Dimensional regularization

1. In dimensional regularization the integral

$$\int \frac{d^d k}{(k^2)^r} = 0 \tag{9}$$

vanishes for arbitrary $r \in \mathbb{R}$. Using this property and the definition

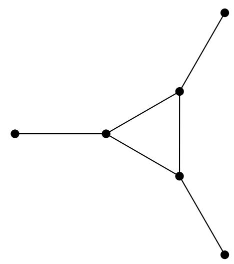
$$I_2 = \int d^d k \frac{1}{k^2 (k+p)^2}, \tag{10}$$

show that

- $$\int d^d k \frac{k_\mu}{k^2 (k+p)^2} = -\frac{p_\mu}{2} I_2, \tag{11}$$

- $$\int d^d k \frac{k_\mu k_\nu}{k^2 (k+p)^2} = \frac{d}{4(d-1)} \left(p_\mu p_\nu - \frac{1}{d} \delta_{\mu\nu} p^2 \right) I_2 \tag{12}$$

2. Using dimensional regularization compute the divergent part of the integral corresponding to the following Feynman diagram in $d = 6 - \varepsilon$ dimensions



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