HW3

Explicit Wave Function

Compute explicitly the wave functions in the helicity basis for the following particles:

- A massive spin 1, interpolated by a vector field $A^\mu(x)$

$$\varepsilon^\mu(\vec{p},\lambda) = \langle 0|A^\mu(0)|\vec{p},\lambda\rangle, \tag{1}$$

  with normalisation $\varepsilon^\dagger_\mu \varepsilon^\mu = -1$.

- A massive spin 1/2 particle and its anti-particle, mediated by the Dirac (4-component) spinor

$$u(\vec{p},\lambda) = \langle 0|\Psi(0)|\vec{p},\lambda;r\rangle, \quad \text{and} \quad v(\vec{p},\lambda) = \langle 0|\Psi^*(0)|\vec{p},\lambda;\bar{r}\rangle, \tag{2}$$

  with normalization $u^\dagger u = v^\dagger v = 2E$, where states with r and $\bar{r}$ (corresponding to particle and antiparticle) are related via charge conjugation

$$\mathcal{C}|\vec{p},\lambda;r\rangle = |\vec{p},\lambda;\bar{r}\rangle \tag{3}$$

  whose action on the Dirac spinor is given by

$$\Psi^c(x) \equiv U(\mathcal{C}^\dagger)\Psi(x)U(\mathcal{C}) = -i\gamma^2\Psi^*(x). \tag{4}$$

  You may want to also use parity transformation for particles at rest

$$\mathcal{P}|\bar{p},\lambda;r\rangle = |\bar{p},\lambda;r\rangle, \tag{5}$$

  and for the Dirac field

$$U(\mathcal{P}^\dagger)\Psi(t,\vec{x})U(\mathcal{P}) = \gamma^0\Psi(t,-\vec{x}). \tag{6}$$

Results

You should find

$$u_\lambda = \begin{bmatrix} \omega_{-2\lambda}\chi_{2\lambda} \\ \omega_{2\lambda}\chi_{2\lambda} \end{bmatrix}, \qquad v_\lambda = \begin{bmatrix} 2\lambda\omega_{2\lambda}\chi_{-2\lambda} \\ -2\lambda\omega_{-2\lambda}\chi_{-2\lambda} \end{bmatrix},$$

where $\omega_\pm = \sqrt{E \pm |\vec{p}|}$ and

$$\chi_+ = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{bmatrix}, \qquad \chi_- = \begin{bmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{bmatrix},$$

$$\varepsilon^\mu_+ = -(\varepsilon^\mu_-)^* = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 0 \\ -\cos\theta\cos\phi + i\sin\phi \\ -i\cos\phi - \cos\theta\sin\phi \\ \sin\theta \end{bmatrix}, \qquad \varepsilon^\mu_0 = \frac{1}{m}\begin{bmatrix} p \\ \frac{E}{p}\vec{p} \end{bmatrix}.$$