Non-equilibrium dynamics

Exercise Sheet 11

1. Consider a 1D translation invariant tight binding model:

$$\hat{\mathcal{H}} = \sum_k \varepsilon_k \hat{\mu}_k^\dagger \hat{\mu}_k, \tag{1}$$

where $\hat{\mu}_k^\dagger$ are momentum-space fermions of energy dispersion $\varepsilon_k = -U \cos k$. Take as an initial wave function the state with a single (real space) fermion at site 0:

$$|\Psi(t=0)\rangle = \hat{a}_0^\dagger |\text{vac}\rangle . \tag{2}$$

The particle will propagate away from the initial site $l = 0$ in a superposition of all momentum modes. Since the momentum quasiparticles do not interact with each other, the wavefront of the density is expected to propagate balistically with a velocity given closely by $v_g \approx \max |\partial\varepsilon_k/\partial k|$.

Answer the following questions in the infinite length limit $L \to \infty$:

(a) Calculate the overlap $\langle\Psi(t)|\Psi(0)\rangle$ as an integral and relate it to the Bessel functions:

$$J_n(u) \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(nq - u\sin q)} dq. \tag{3}$$

(b) Show that the fermion density $\hat{n}_l(t)$ along the chain over time can be written as $\hat{n}_l(t) = J_l^2(Ut)$. Check that the initial state $\hat{n}_l(0) = \delta_{l0}$ is satisfied.

(c) Given the previous result, what should the sum $\sum_l J_l^2(t)$ be?

(d) Plot the particle density $\hat{n}_l(t)$ in the (l, t) plane and check for a "light-cone" behavior bounded by a line of velocity v_g.

Suggestion: The script ex1.py will create such a plot already. Non-integer values can be taken for l to obtain a smoother plot.

(e) Add now a next nearest neighbor coupling term to the model and calculate $\hat{n}_l(t)$ by numerical integration in the (l, t) plane. Calculate v_g and check if it agrees with the numerical results.

Suggestion: Complete the script ex1.py. Reduce the density of points to be calculated as the numerical integration is more intensive.

2. (Optional) Using the exact diagonalization code you prepared last week to solve the Ising model with transverse and longitudinal fields, calculate and plot the average of $\hat{\sigma}^x$, $\hat{\sigma}^y$, and $\hat{\sigma}^z$ along the ring for all eigenstates as a function of their energy density ε. Compare different limits (h_x dominated, h_z dominated, disorder dominated...). How do they scale as a function of the system size?

Suggestion: Take $L \leq 12$. In Python, `n.bit_count()` counts the number of 1 bits of integer n.