Entanglement entropy

Exercise Sheet 2

Consider the Su–Schrieffer–Heeger (SSH) model, an open fermionic chain of length L (even) with no interactions but with dimerized hopping terms:

$$\hat{\mathcal{H}} = -\sum_{l=1}^{L-1} t_l (\hat{a}_l^\dagger \hat{a}_{l+1} + \hat{a}_{l+1}^\dagger \hat{a}_l), \qquad t_l = \begin{cases} t_o, & l \text{ odd} \\ t_e, & l \text{ even} \end{cases}, \tag{1}$$

where t_o and t_e are non-negative. We want to calculate the entanglement entropy S of the ground state at half-filling (total fermion number is $L/2$), on the set A of sites from 1 to u.

1. What is the entropy S as a function of u in the limits $t_o = 0$ and $t_e = 0$?
2. Prepare a script that will determine all the quasiparticle excitations of the SSH model and their energies.

   Suggestion: Write the Hamiltonian in the form $\hat{\mathcal{H}} = \sum_{i,j} \hat{a}_i^\dagger M_{ij} \hat{a}_j$, then define the matrix M in the python script provided (ex.py).
3. Expand the script to calculate the matrix C_{ij} in the ground state, then diagonalize this matrix to finally calculate S.

   Suggestion: Write the C_{ij} matrix elements in terms of the quasiparticle correlators $\langle \hat{a}_k^\dagger \hat{a}_{k'} \rangle$ and complete the appropriate function in ex.py.
4. Analyze the scaling of S as a function of the subsystem size u and the total length L, considering three cases for the couplings: $t_o \gtrsim t_e$, $t_o = t_e$ and $t_o \lesssim t_e$.

   Suggestion: Use these parameters, approximately:

$$t_o, t_e \in \{1, 0.98, 0.96\}, \quad L = 2^n, n = 4, \ldots, 10, \quad u = 1, 2, 3, L/2. \tag{2}$$

5. (Optional) Calculate the spectrum s_i and analyze their distribution in the three cases above for $u = L/2$.
6. Take now the set of parameters in Eq. (2) and increase the difference between t_e and t_o (for example, use $t_e = 1/2, t_o = 1$ and $t_e = 1, t_o = 1/2$). What do you observe?