Atomic nuclei are quantum bound states of protons, with unit charge, and neutrons, with zero charge. These two particles, collectively called nucleons, are spin-1/2 fermions with nearly equal masses:

$$m_n c^2 = 939.56\,\text{MeV}, \quad m_p c^2 = 938.27\,\text{MeV} \approx 1.67 \times 10^{-27}\,\text{kg}.$$

Nucleons are bound by nuclear forces, which are short-range, sufficiently strong, and attractive enough to overcome the Coulomb repulsion between protons. Compared to the strength of electromagnetic forces, nuclear forces are called strong interactions.

While protons and neutrons differ in terms of their electromagnetic interactions, their strong interactions are essentially the same. This fact, along with their similar masses, justifies the common designation of both particles as "nucleons."

Nuclear reactions

- Our goal is to determine the energy generation rate per unit mass, ε.

- To this end, we first define the reaction rate r_{aX}, along with related quantities such as the lifetime τ of a particle.
- The expression of the product $\langle \sigma v \rangle$ is then derived for a stellar medium, where particle velocities follow a Maxwellian distribution.

- We next introduce the concept of resonant reactions, illustrated by the example of carbon synthesis.

- For non-resonant reactions, we discuss the notion of the *Gamow peak*, which characterizes the most effective energy range for nuclear interactions.

- Finally, we examine the role of the screening effect.

In a nucleus, we denote by N the number of neutrons, by Z the number of protons (the atomic number), and by

$$A = N + Z$$

the total number of nucleons, known as the *mass number*.

A nucleus may be represented in several equivalent ways: (A, Z), ^{A}X, or $^{A}_{Z}X$, where X is the chemical symbol of the element associated with charge Z. Since a neutral atom contains Z electrons, the symbol also identifies the atomic species. For example, ^{4}He denotes the nucleus of helium-4, with $N = 2$ and $Z = 2$, commonly referred to as the α-particle.

The specification of (A, Z) or (N, Z) does not uniquely determine the nuclear state. A given nucleus (A, Z) generally possesses a set of excited states that decay to the ground state through the emission of photons, typically in the form of gamma rays.

Families of nuclei are classified according to specific designations. For instance, **isotopes** are nuclei with the same proton number Z but different neutron number N. Examples include $^{238}_{92}U$ and $^{235}_{92}U$. While the corresponding atoms exhibit nearly identical chemical properties—determined by their Z electrons—their nuclear properties can differ drastically, as in the case of these two uranium isotopes.

Where and when are chemical elements formed?

The Primordial Universe.

A few seconds after the Big Bang, the Universe consisted of a dense, high-energy plasma at extremely high temperature, confined within a rapidly expanding volume. During this epoch, *primordial nucleosynthesis* took place. Following the recombination of quarks into protons and neutrons, light nuclei such as ^{4}He, ^{2}H (deuterium), ^{3}He, and ^{7}Li were synthesized.

Stellar Cores.
In non-degenerate stars, at various stages of their evolution, fusion occurs within chemically complex plasmas at moderate densities and temperatures. These low-energy fusion reactions are the primary source of stellar energy, maintaining hydrostatic equilibrium within the star. The energy produced is eventually released into space in the form of radiation.

Beyond Fusion.
Not all nuclear reactions are fusion processes. In particular, *neutron capture reactions* play a crucial role in the synthesis of elements heavier than iron ($A = 56$).

The reactions we consider are two-body reactions (three-body reactions have a very low probability)

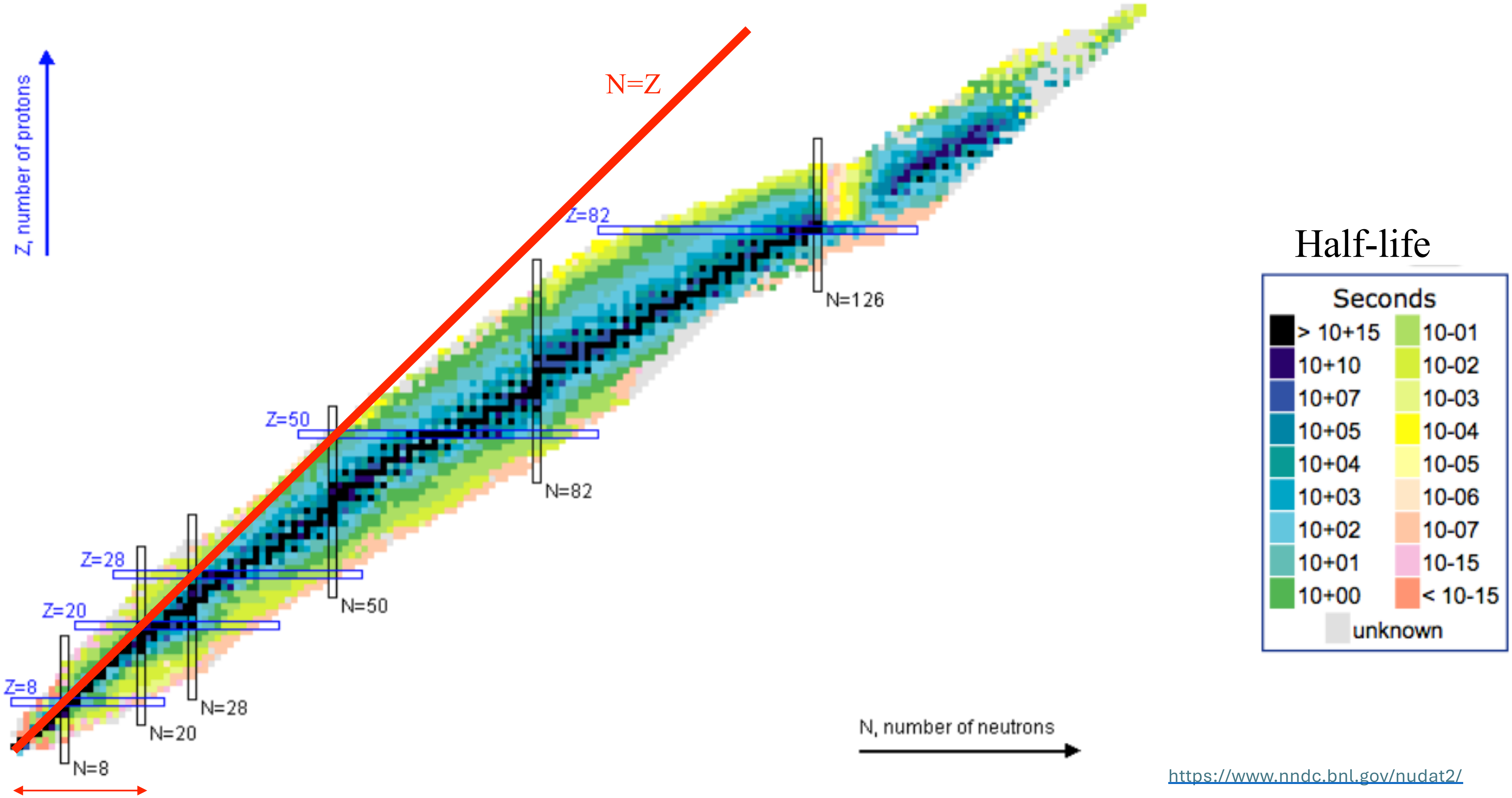
$$X + a \to Y + b$$

X is a nucleus (A, Z)
a is a particle, which can be X itself or another nucleus

Fusion leads to Y (A', Z'), the resulting nucleus, and another particle (another nucleus, a nucleon, or a photon), which may not be produced

The stability of a nucleus is influenced by its number of protons and neutrons.
Nuclei with even numbers of protons and/or neutrons are most likely to be stable.
Nuclei with magic numbers (N or Z = 2, 8, 20, 28, 50, 82, 126) are especially stable.

Segrè Diagram



https://www.nndc.bnl.gov/nudat2/

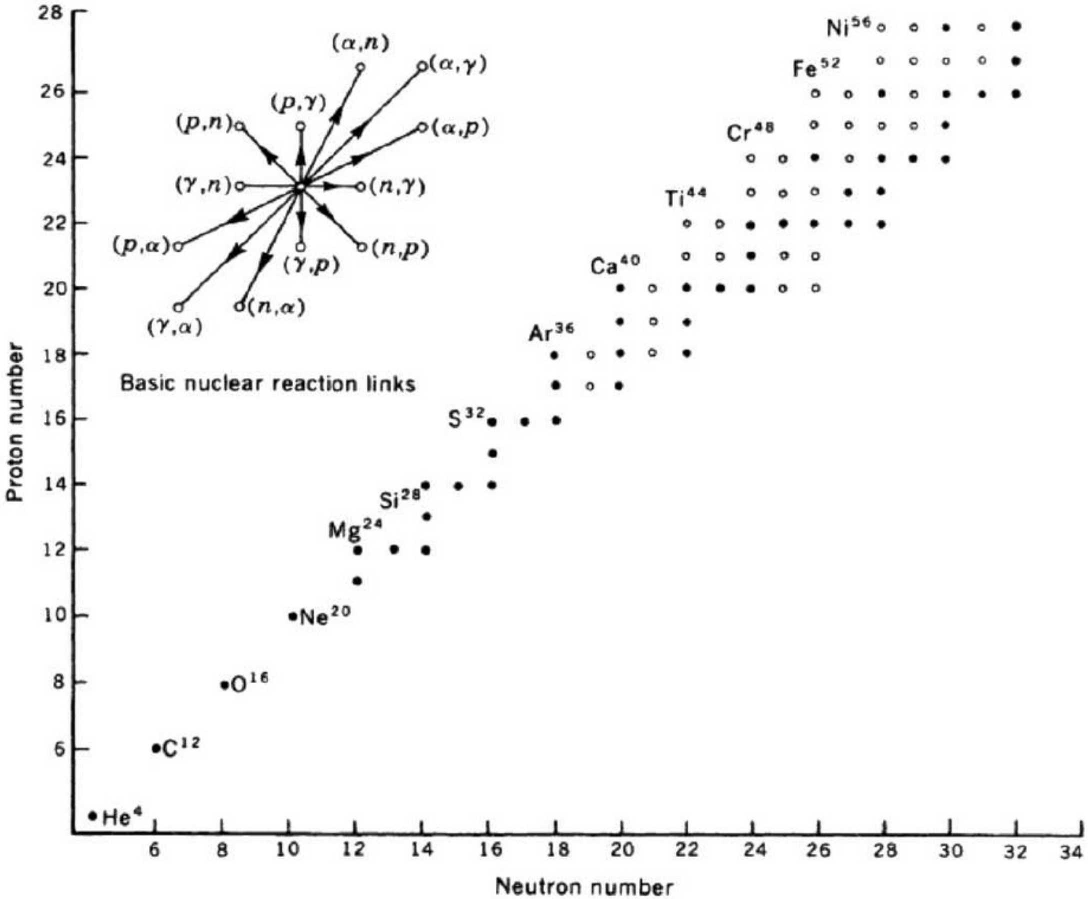
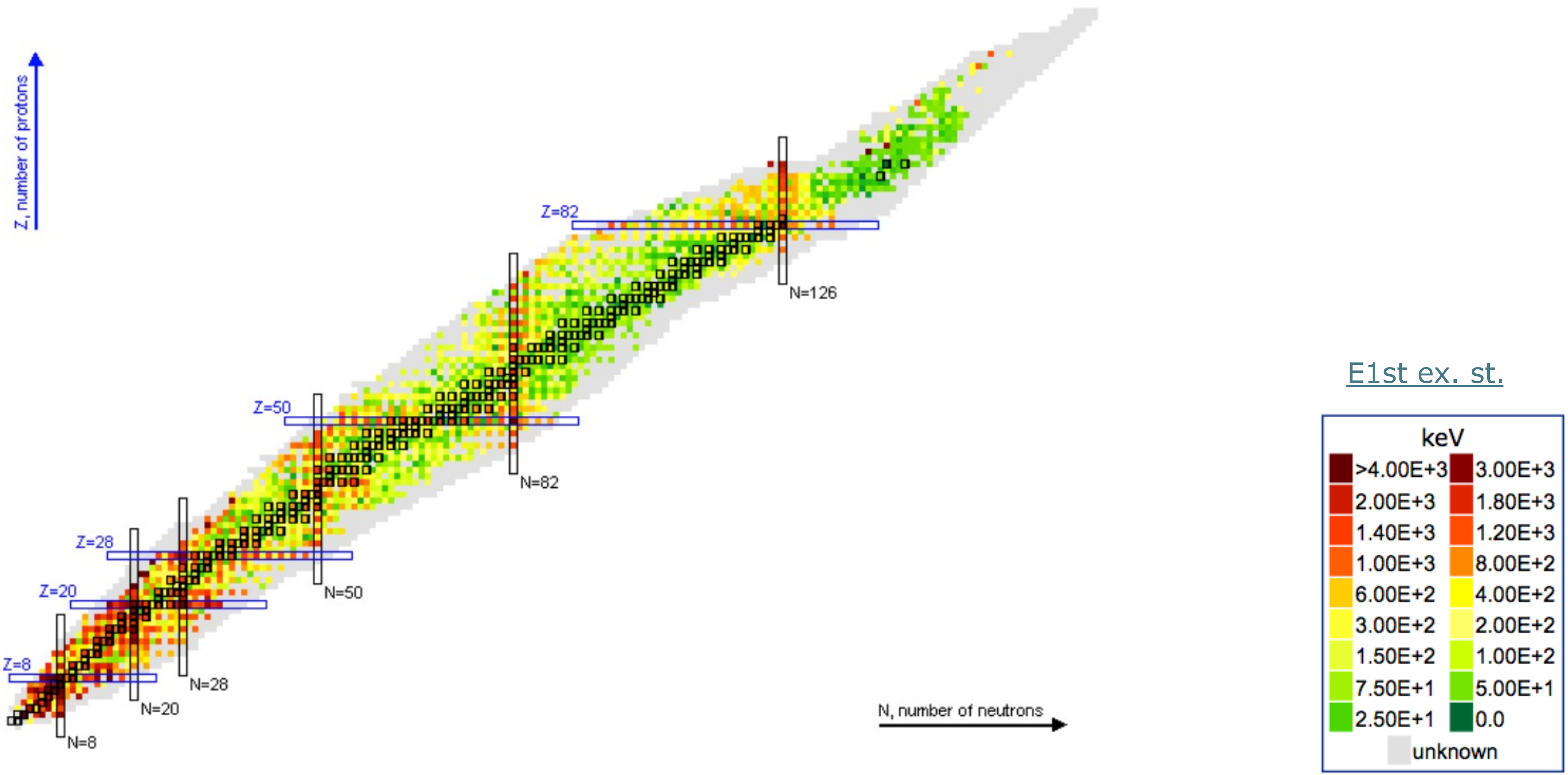


Fig. 7-3 The nuclides which participate in the reaction network established during silicon burning. The solid dots designate stable nuclei, whose relative abundances during silicon burning must be scrutinized for correlations with natural abundances. The open dots designate unstable nuclei. Their relatively low abundances may be very important for determination of the overall rate of beta decays. [*After J. W. Truran, A. G. W. Cameron, and A. A. Gilbert, Can. J. Phys.*, **44**:576 (1966).]

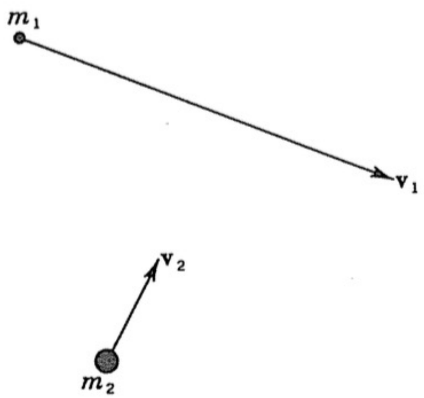


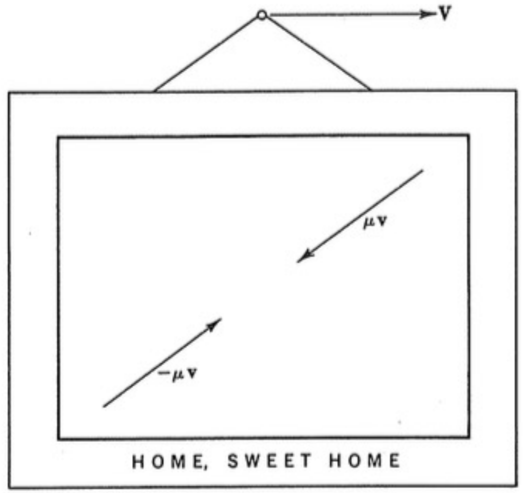
E1st ex. st.: energy of the first excited state, in keV. Only unambiguous cases are plotted.

Classical description

If the reaction takes place, it involves nuclear interactions and can be described as an inelastic collision in which, in the center-of-mass frame, i) momentum, ii) kinetic energy, and iii) angular momentum are conserved.

The center-of-mass velocity for two particles, 1 and 2, is determined by the conservation of momentum.





$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{V} \rightarrow \vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

We define the reduced mass of the system $\mu = \frac{m_1 m_2}{m_1 + m2}$

The relative velocity of particle 1 with respect to particle 2 is $\vec{v} = \vec{v}_1 - \vec{v}_2$

The momentum of particle 1 with respect to the center-of-mass frame is $m_1(\vec{v}_1 - \vec{V}) = \frac{m_1 m_2}{m_1 + m_2}(\vec{v}_1 - \vec{v}_2) = \mu\vec{v}.$

The momentum of particle 2 with respect to the center-of-mass frame is $m_2(\vec{v}_2 - \vec{V}) = -\mu\vec{v}.$

In the center-of-mass frame, the total momentum is zero.

Momentum is conserved in this frame, meaning that the velocity before and after the collision remains the same—i.e., the velocity V is constant and the total momentum remains zero.

For fusion to occur, the collision between the particles (nuclei) requires an energy input to overcome the repulsive forces.
Prior to the collision, the kinetic energy is

$$1/2m_1\vec{v}_1^2 + 1/2m_2\vec{v}_2^2 = \underbrace{1/2(m_1+m_2)\vec{V}^2}_{\text{Kinetic energy of center of mass}} + \underbrace{1/2\mu\vec{v}^2}_{\text{Necessary additional kinetic energy}}$$

This non-relativistic equation is valid only if the total mass of the system remains constant during the reaction. However, as we have seen, up to iron, nucleosynthesis involves exothermic reactions, in which an energy ΔMc^2 is released.
As a result, the total mass of the system changes.
At low energy, $\Delta M/M \sim 10^{-2} - 10^{-4}$
The assumption of constant mass holds to better than 1%. The kinetic energy of the center of mass is considered unchanged by the reaction, whereas the kinetic energy in the center-of-mass frame decreases or increases depending on whether the final mass is smaller or larger than the initial mass.

$$E_{aX} + (M_X + M_a)c^2 = E_{bY} + (M_Y + M_b)c^2$$

E_{aX}: kinetic energy in center of mass of system (a,X)

E_{bY}: kinetic energy in center of mass of system (b,Y)

Terms reflecting that the total rest mass of the nuclei may change during the reaction

Exemple: H → He
4x1.0081 Mu → 4.0039 Mu
$\Delta M = 2.85\times10^{-2}$ Mu = 0.7% of initial mass
$E = \Delta Mc^2 = 26.5$ MeV

In practice, atomic masses are used instead of nuclear masses. Since electric charge is conserved, the rest mass of the electrons can be included on both sides of the equation. However, the binding energy must be considered, as it may differ before and after the reaction.

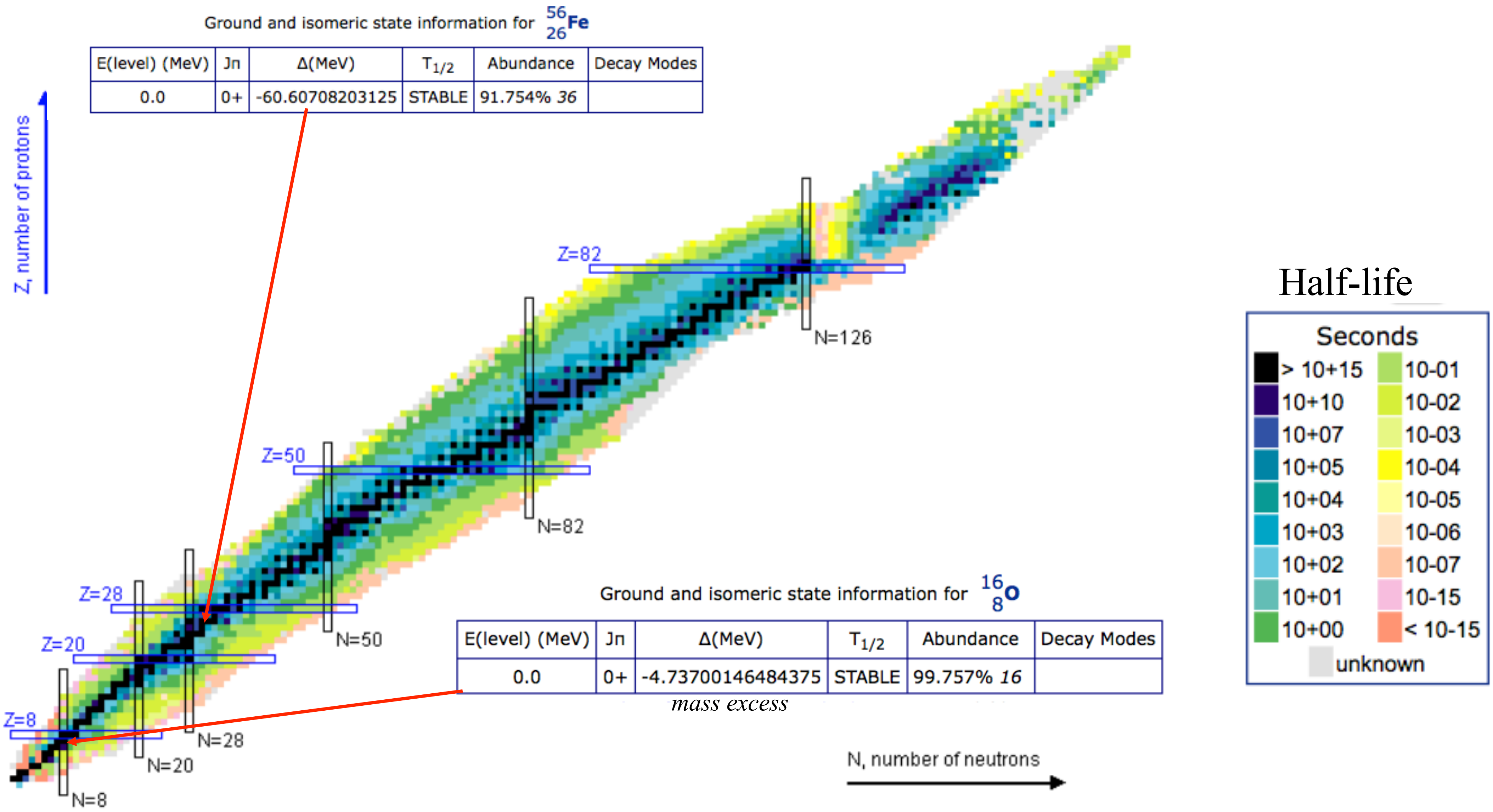
Atomic mass excess resulting from this approximation:

$$\Delta M_{AZ} = (M_{AZ} - AM_u)c^2 = 931.478(M_{AZ} - A)\text{Mev}$$

1 Mu = 931.4943 MeV/c^2

We can write: $$E_{aX} + (\Delta M_X + \Delta M_a) = E_{bY} + (\Delta M_Y + \Delta M_b)$$

The released energy is $$Q = (\Delta M_a + \Delta M_X) - (\Delta M_b + \Delta M_Y) = E_{bY} - E_{aX}$$



Ground and isomeric state information for $^{56}_{26}Fe$

E(level) (MeV)	Jπ	Δ(MeV)	$T_{1/2}$	Abundance	Decay Modes
0.0	0+	-60.60708203125	STABLE	91.754% *36*	

Ground and isomeric state information for $^{16}_{8}O$

E(level) (MeV)	Jπ	Δ(MeV)	$T_{1/2}$	Abundance	Decay Modes
0.0	0+	-4.73700146484375	STABLE	99.757% *16*	

Mass excess for different elements and isotopes

(source: D. D. Clayton, « principles of stellar evolution and nucleosynthesis », McGraw-Hill, 1968)

Table 4-1 Atomic mass excesses†

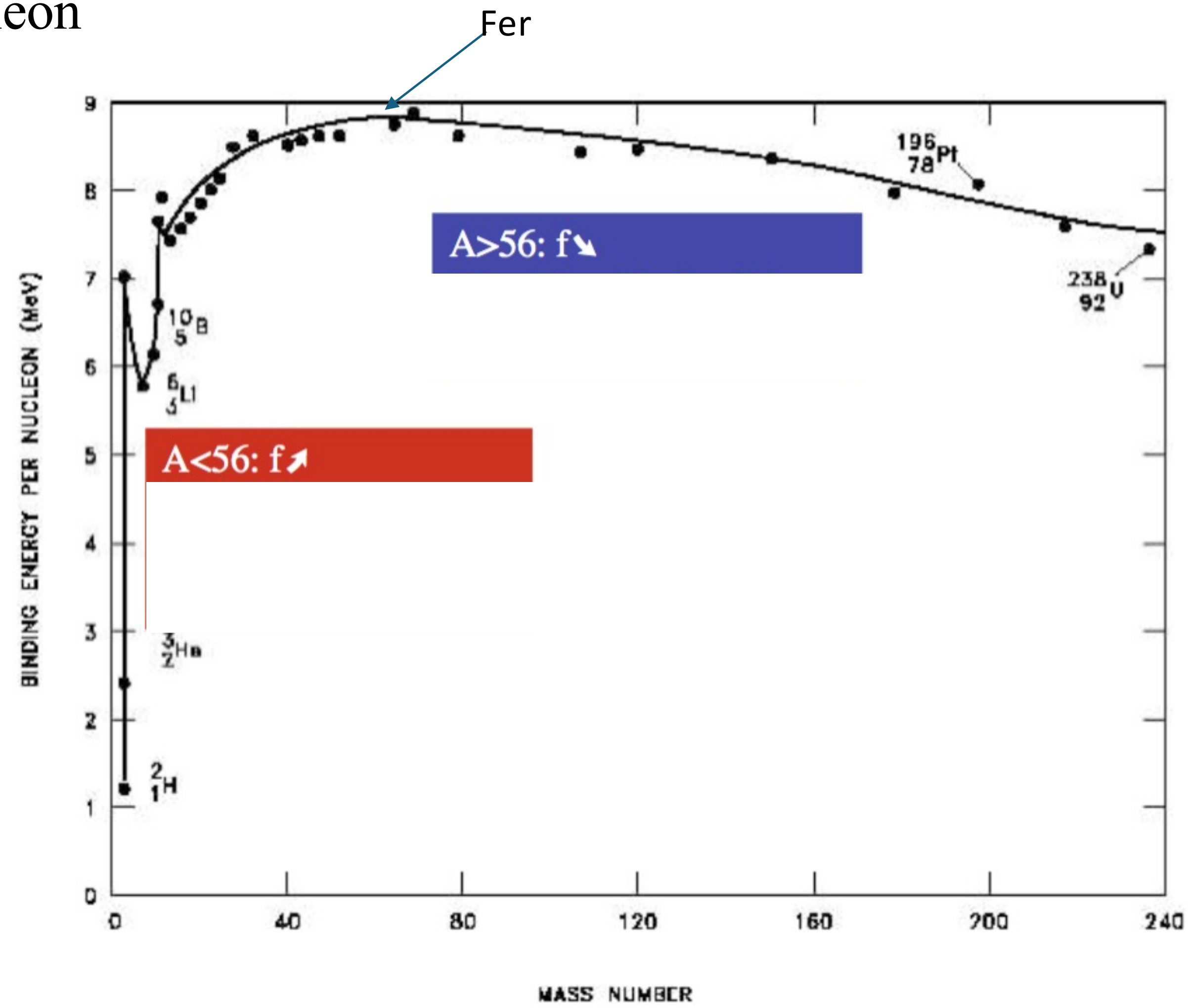
Z	Element	A	M − A, Mev
0	n	1	8.07144
1	H	1	7.28899
	D	2	13.13591
	T	3	14.94995
	H	4	28.22000
		5	31.09000
2	He	3	14.93134
		4	2.42475
		5	11.45400
		6	17.59820
		7	26.03000
		8	32.00000
3	Li	5	11.67900
		6	14.08840
		7	14.90730
		8	20.94620
		9	24.96500
4	Be	6	18.37560
		7	15.76890
		8	4.94420
		9	11.35050
		10	12.60700
		11	20.18100
5	B	7	27.99000
		8	22.92310
		9	12.41860
		10	12.05220
		11	8.66768
		12	13.37020
		13	16.56160
6	C	9	28.99000
		10	15.65800
		11	10.64840
		12	0
		13	3.12460
		14	3.01982
		15	9.87320
7	N	12	17.36400
		13	5.34520
		14	2.86373
		15	0.10040
		16	5.68510
		17	7.87100

Z	Element	A	M − A, Mev
		19	3.33270
		20	3.79900
9	F	16	10.90400
		17	1.95190
		18	0.87240
		19	−1.48600
		20	−0.01190
		21	−0.04600
10	Ne	18	5.31930
		19	1.75200
		20	−7.04150
		21	−5.72990
		22	−8.02490
		23	−5.14830
		24	−5.94900
11	Na	20	8.28000
		21	−2.18500
		22	−5.18220
		23	−9.52830
		24	−8.41840
		25	−9.35600
		26	−7.69000
12	Mg	22	−0.14000
		23	−5.47240
		24	−13.93330
		25	−13.19070
		26	−16.21420
		27	−14.58260
		28	−15.02000
13	Al	24	0.1000
		25	−8.9310
		26	−12.2108
		27	−17.1961
		28	−16.8554
		29	−18.2180
		30	−17.1500
14	Si	26	−7.1320
		27	−12.3860
		28	−21.4899
		29	−21.8936
		30	−24.4394
		31	−22.9620
		32	−24.2000

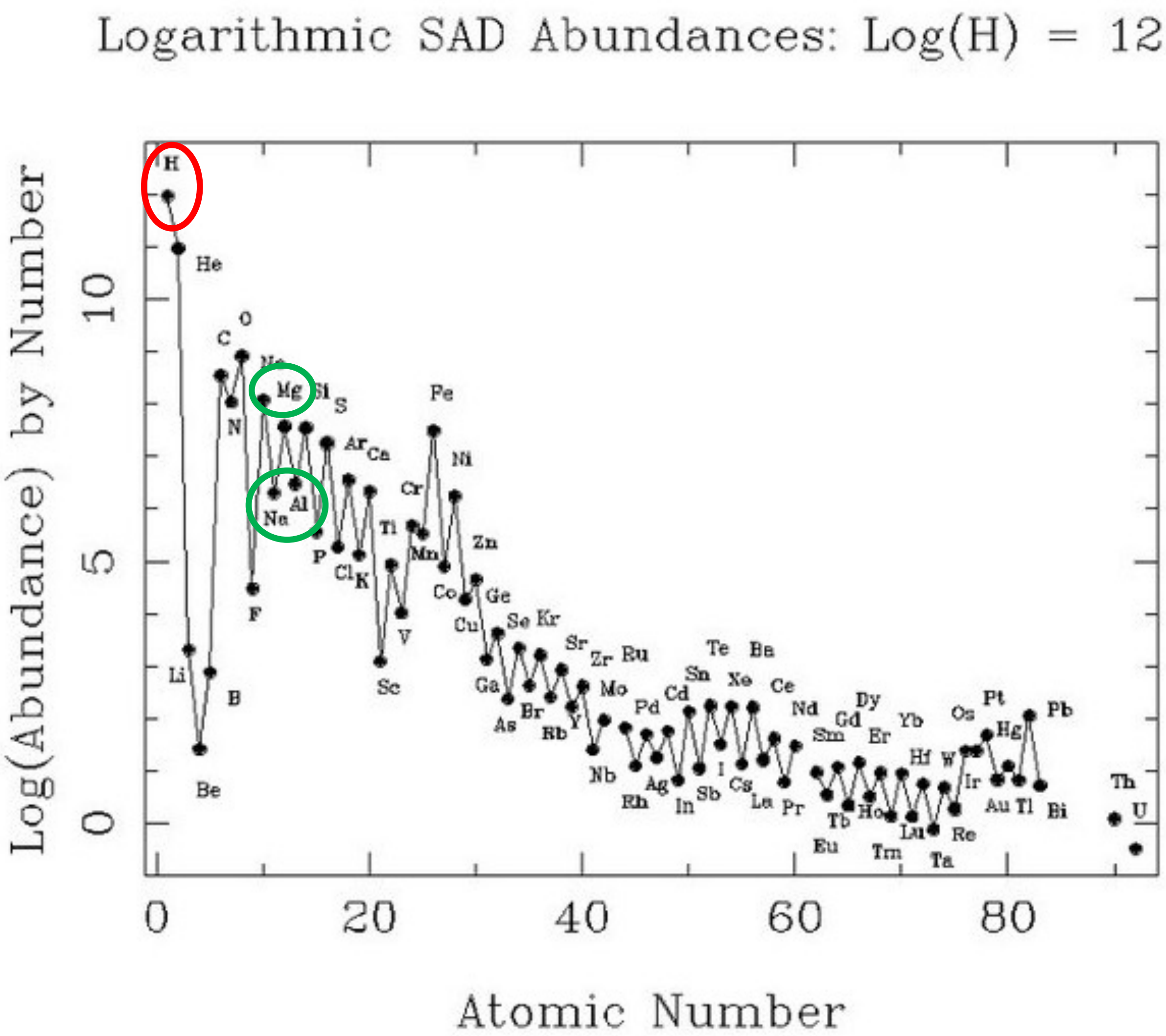
The energy released is a function of the binding energies. $Q = E_l(Y) + E_l(b) - E_l(X) - E_l(a)$

$f = E_l / A$ mean binding energy per nucleon

All fusion reactions release energy, but the amount decreases as heavier elements are fused, starting from hydrogen.
When iron is reached, the net energy gain becomes zero.
Beyond iron, fusion reactions require an input of energy instead of releasing it.



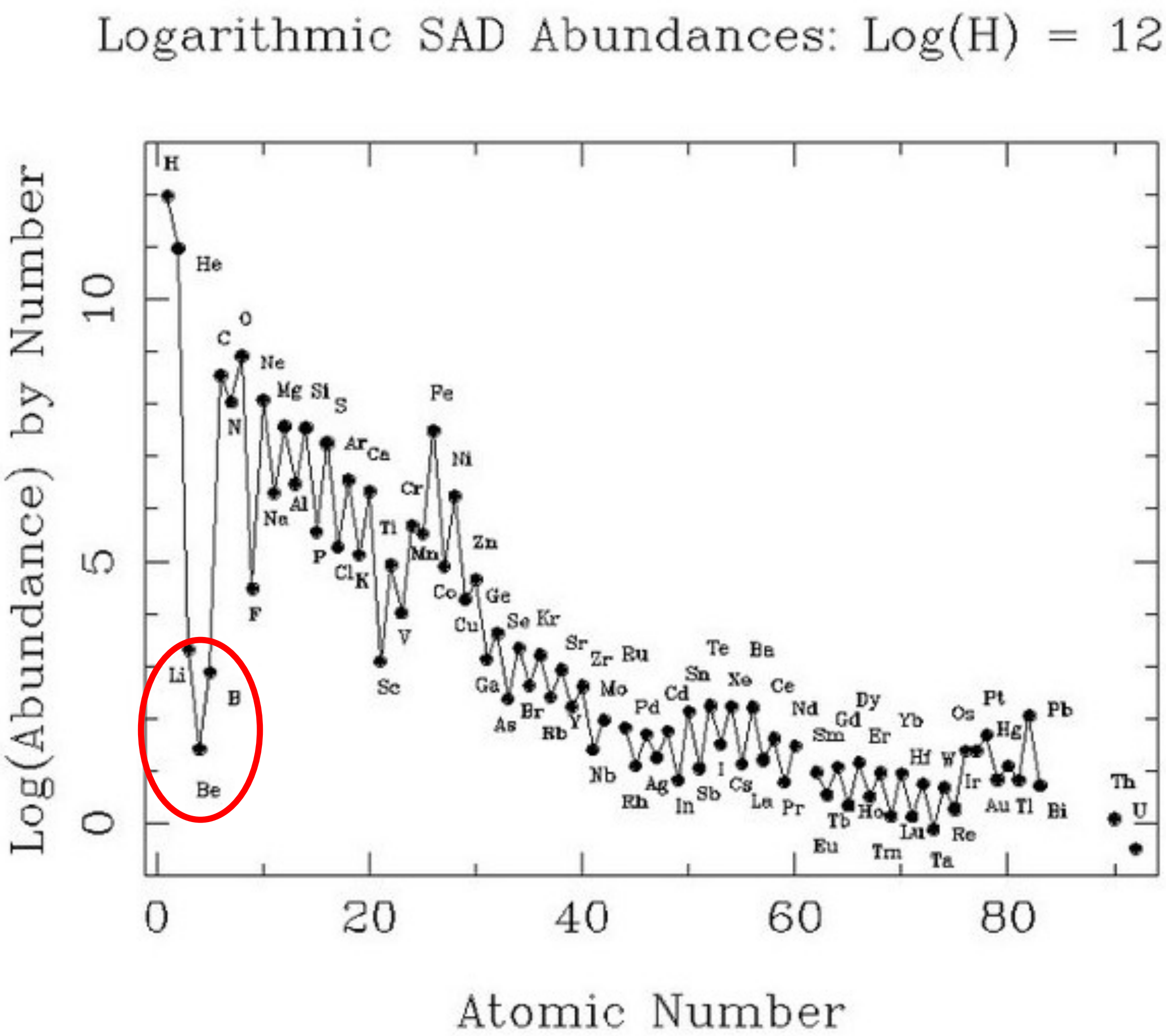
Solar abundances



- Hydrogen peak. Two elements are extremely abundant: hydrogen makes up 90% by number and 70% by mass of solar matter, while helium accounts for 10% by number and 28% by mass. All other elements together constitute only 2% by mass.
- Abundances decrease very rapidly with increasing atomic mass, though the decrease is not monotonic. Elements whose nuclei contain an integer number of α particles tend to have higher abundances.

Moreover, elements with even-mass nuclei generally have higher abundances than those with odd-mass nuclei; for instance, $^{24}Mg/^{23}Na = 16$.

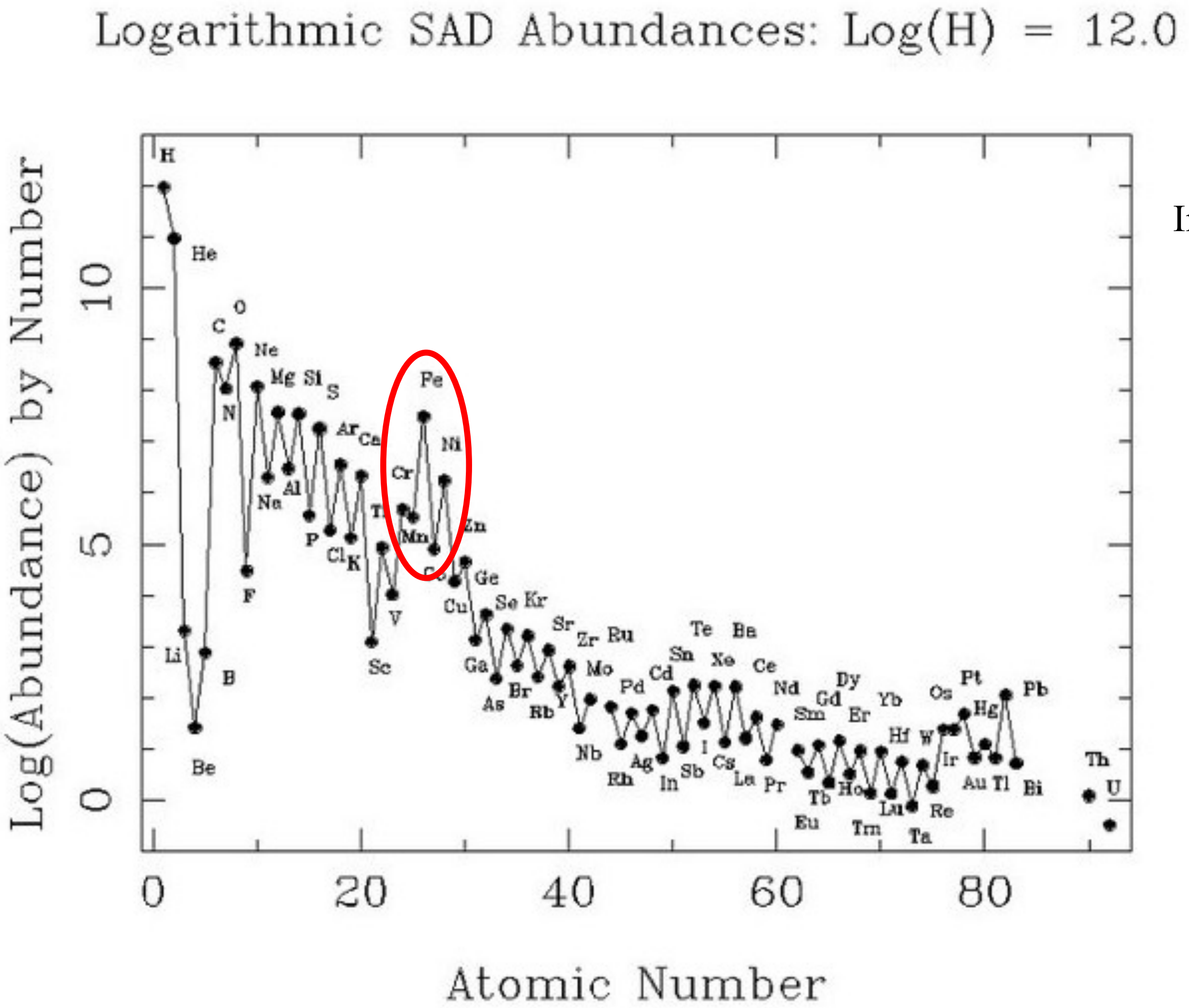
Solar abundances



A small group of very rare light elements consists of Li, Be, and B, with abundances 10^8–10^{10} times lower than those of H and He.

These elements originate from different nucleosynthetic processes. Peaks in relative abundances are seen for elements with high binding energy per nucleon.

Solar abundances



Iron Peak