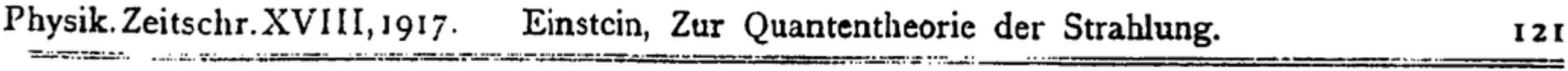
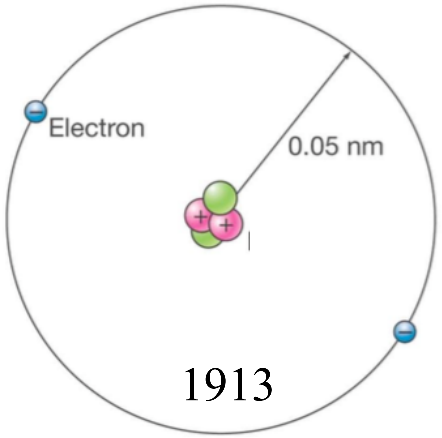

Physik. Zeitschr. XVIII, 1917. Einstein, Zur Quantentheorie der Strahlung. 121



1913

Bohr

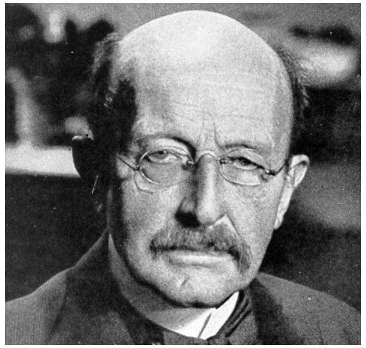
Corps noir

Niveaux discontinus

kT 1899

$h\nu, 2h\nu, 3h\nu \ \ldots$

Planck



interactions

entre matière et rayonnement

1877 **Boltzmann**

Physique statistique

Propriété intrinsèque de la lumière

quanta $h\nu$

photon: Lewis, Nature 1926

Planck’s law (1900) was established prior to the formal explanation of quantization, in an effort to reconcile the **Rayleigh–Jeans law** and **Wien’s law**.

The **Rayleigh–Jeans law** predicts the spectral energy density of blackbody radiation as: $\frac{2k_B}{c^2}\nu^2 T$

This expression agrees with experiment at low frequencies but diverges toward infinity as frequency increases $\nu \to \infty$

— a problem known as the “ultraviolet catastrophe.”

In contrast, **Wien’s law** describes the high-frequency (short-wavelength), high energy, regime as: $\alpha\nu^3 \exp(-\beta\nu/T)$

To resolve this inconsistency, **Max Planck** introduced a new universal constant, *h* (Planck’s constant), and proposed that the energy of electromagnetic oscillators is **quantized** according to: $E = nh\nu \quad (n = 1, 2, 3, \dots)$

This led to **Planck’s radiation law**, unifying the two regimes: $\frac{2\pi h\nu^3}{c^2}\frac{1}{exp\left(\frac{h\nu}{k_B T}\right)-1}$ $(h = 6{,}62.10^{-34}\,\text{J.s})$

The introduction of the constant *h* (a new universal constant) and the postulate of **energy quantization** paved the way for a new branch of physics — leading to the development of **quantum mechanics** around 1920 by Heisenberg and Schrödinger.

The profound physical significance of Planck’s formula was later elucidated by **Albert Einstein** in one of his 1905 papers on the **photoelectric effect**, describing the ejection of an electron under irradiation.

A **blackbody** may thus be regarded as an **ideal gas of photons,** each photon carrying energy $E = h\nu$.

$$E = h\nu.$$

In his first paper of March 1905, *“On a Heuristic Point of View Concerning the Emission and Transformation of Light,”* **Einstein** suggested that this discontinuity might be **intrinsic to light itself**, rather than merely a property of atomic energy levels. This was the hypothesis of the **corpuscular (particle-like) nature of light**, in direct contrast with the prevailing **wave theory of light** established since **Young’s double-slit experiment (1803)**.

Between **1905 and 1916**, Einstein focused exclusively on **special** and **general relativity**. After completing his work on general relativity and its successful application to the **perihelion advance of Mercury** (September 1916), Einstein returned to problems of **quantum theory**.

Meanwhile, **Niels Bohr** had developed his important analysis of the **quantization of matter**, later confirmed experimentally by **Franck and Hertz**. Bohr showed that electrons in an atom can occupy only specific **energy levels** or **orbits**, and cannot fall below a certain minimal orbit — the basis of the **Bohr atomic model**.

Einstein then revisited the problem of light. In his **1917 paper**, he formulated the **equations governing blackbody radiation**, synthesizing:

1. his own early work and that of Planck (1900–1905);
2. the **statistical mechanics** framework of **Boltzmann**; and
3. Bohr’s theory of the **quantization of matter**.

The only way to solve the blackbody radiation equations consistently was to introduce the revolutionary concept of **stimulated emission**, which would later become the theoretical foundation for the **laser**.

We are therefore dealing with **three distinct phenomena** within the blackbody:

1. **Stimulated absorption (by an incident photon):**
   The electron of an atom is excited from a lower to a higher energy level.
2. **Spontaneous emission:**
   After spending some time in the excited state, the atom eventually de-excites; the electron returns to a lower energy level, emitting a photon.
   In spontaneous emission, the emitted photon can be released in **any direction**.
3. **Stimulated emission (by an incident photon):**
   The atom is induced to de-excite — the electron falls back to a lower energy level — emitting a photon **in the same direction and phase** as the incident photon.

The **complete theoretical resolution** of the blackbody radiation problem thus made it possible to:

- Discover the phenomenon of **stimulated emission**;
- Confirm Einstein's 1905 intuition of the **quantum of light**, which carries a **momentum** $p = \frac{h\nu}{c}$

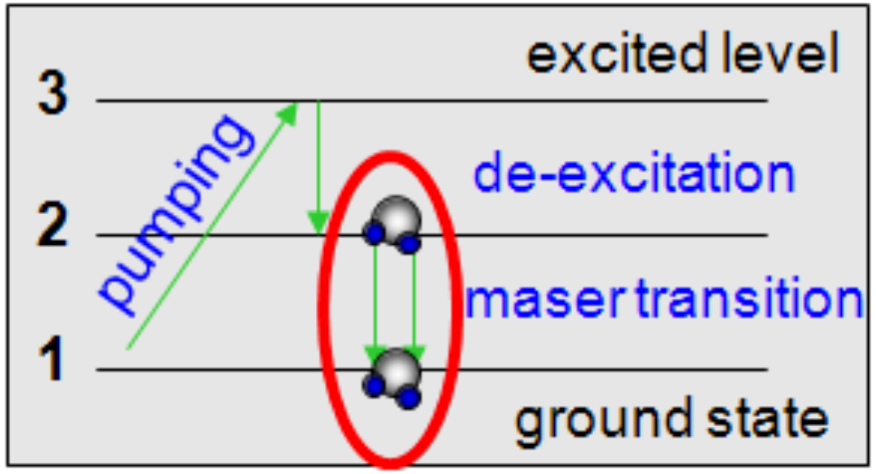
Where hν corresponds to the **transition energy** between the two atomic levels.

By applying **Bohr's quantization results** to the phenomenon of stimulated emission, **Einstein** demonstrated that the quantum of light possesses a **momentum**, providing further evidence of the **particle-like nature of light**.
This "quantum of light" would later be **experimentally confirmed** and formally named the **photon** in **1926**.

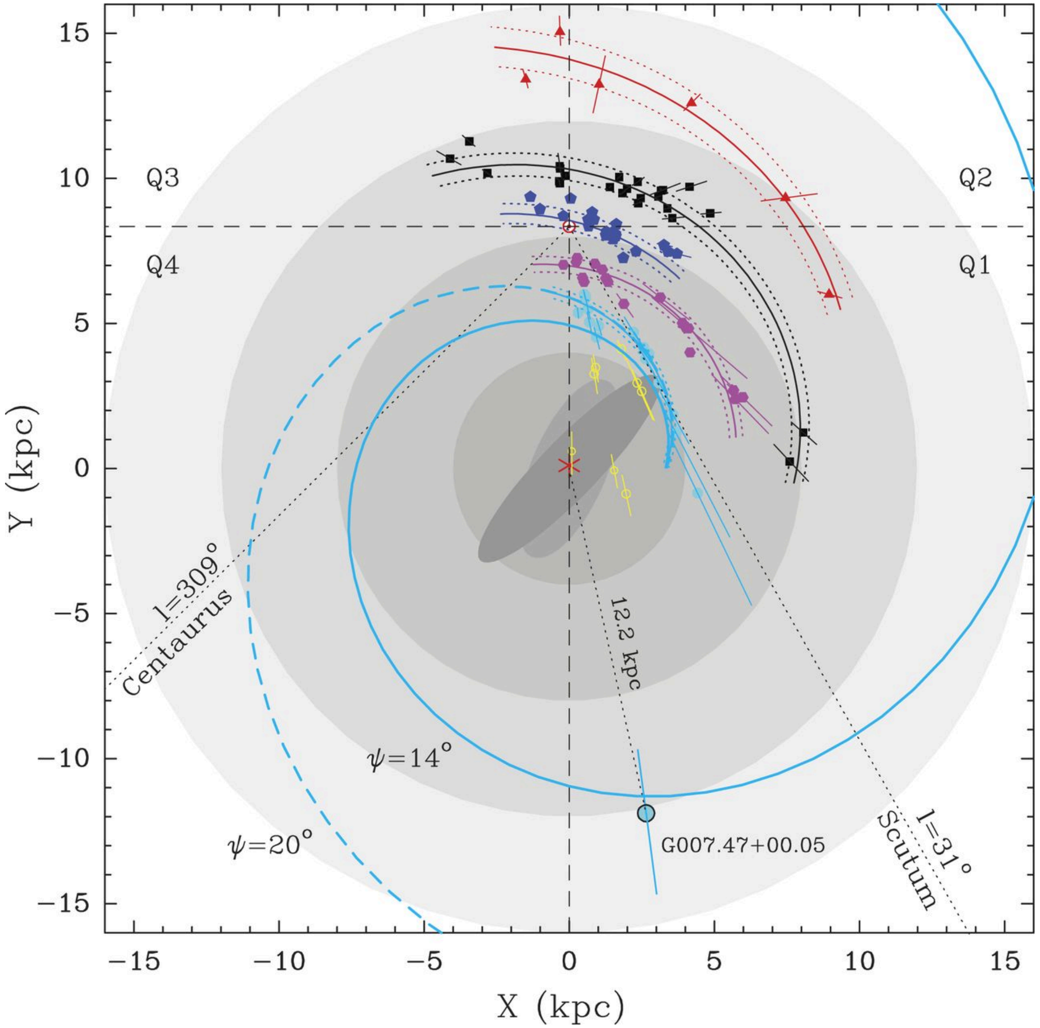
Maser

(Microwave Amplification by Stimulated Emission of Radiation)

The first papers about the MASER were published in 1954 as a result of investigations carried out simultaneously and independently by Charles Townes and co-workers at Columbia University in New York and by Dr. Basov and Dr. Prochorov at the Lebedev Institute in Moscow. All three of these gentlemen received the Nobel Prize in 1964 for their contributions to science.



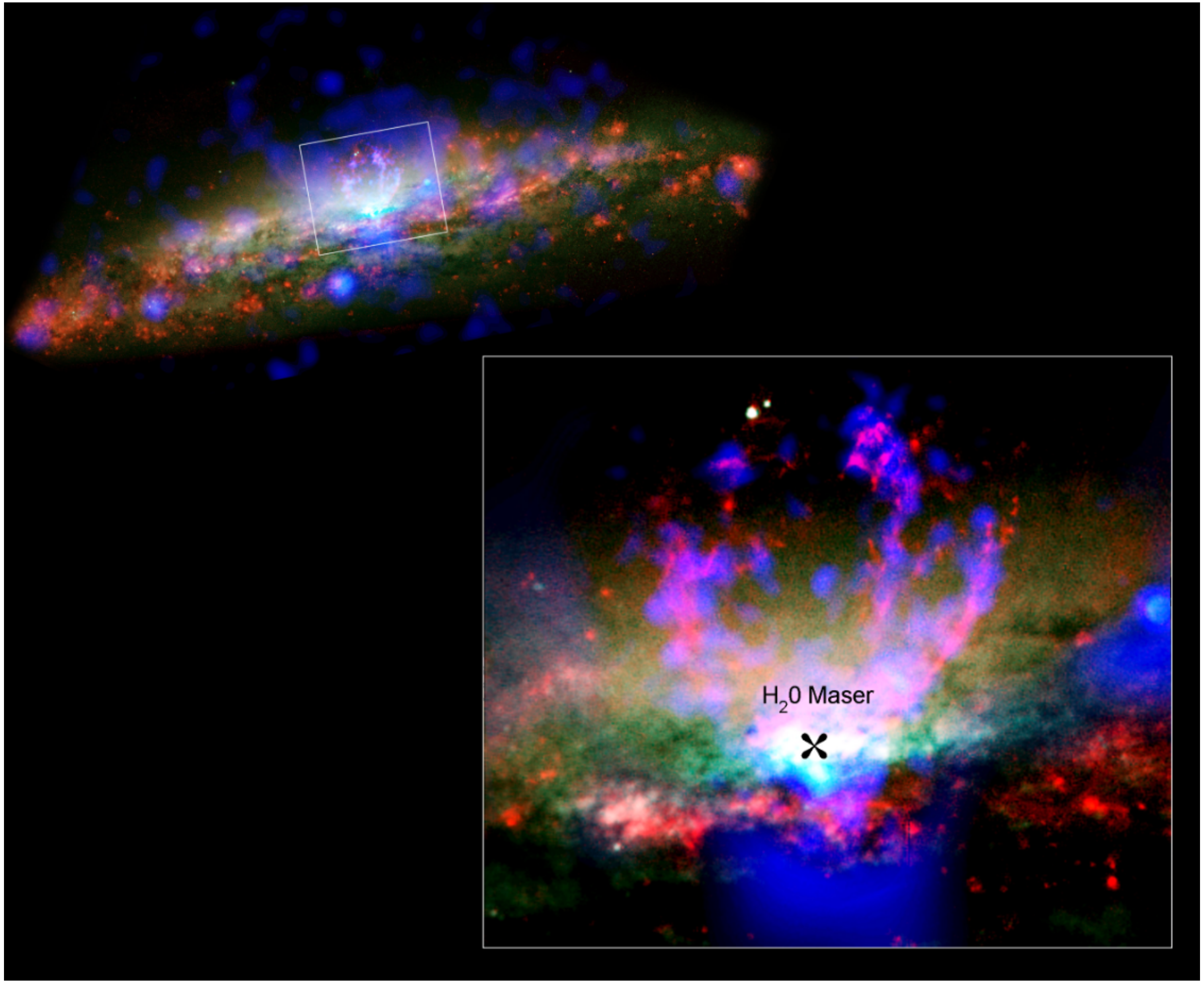
Schematic of the stimulated emission process in a maser. The molecule is pumped to an excited state and decays non-radiatively to a metastable state where a population inversion is created. An incident photon of the correct frequency stimulates the emission of another photon of the same frequency, phase and direction and both are emitted simultaneously, thus amplifying the incident radiation field.



Plan view of the Milky Way showing the location of G007.47+00.05 and other maser sources determined via trigonometric parallaxes

10.1126/science.aan5452

In our own galaxy, water molecules near hot, newly formed stars can absorb energy and then emit radio waves with centimeter wavelengths, creating the brightest spectral lines in the radio universe.



NGC 3079

In active galaxies, it is processes related to black holes rather than stars that heat the molecules.

https://www. cfa.harvard.edu/news/2006-07

NGC 4258 gri

gri + radio

gri + X-ray

Distance scale

Supermassive black holes (SMBHs) heat the surrounding gas in their accretion disks to extremely high temperatures, thereby producing strong X-ray emission. This powerful radiation field can excite various molecules within the disk—particularly water molecules—leading to **MASER** emission.

These MASERs are highly intense and compact, which enables the measurement of line-of-sight (LOS) velocities within the accretion disk at distances of thousands of Schwarzschild radii from the SMBH. Furthermore, the orbital velocities of the MASER spots around the central nucleus are sufficiently large that their LOS accelerations can be measured over timescales of only a few years.

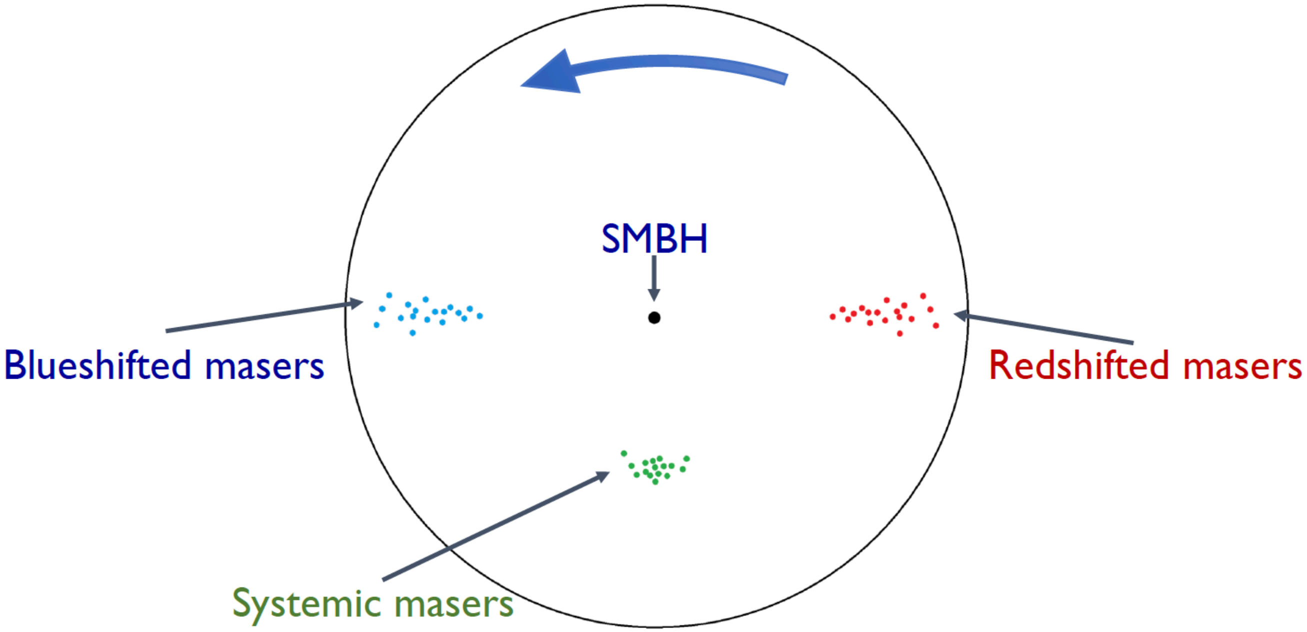
Once the acceleration, velocity, and velocity gradient have been determined, the distance can be derived according to the following relation:

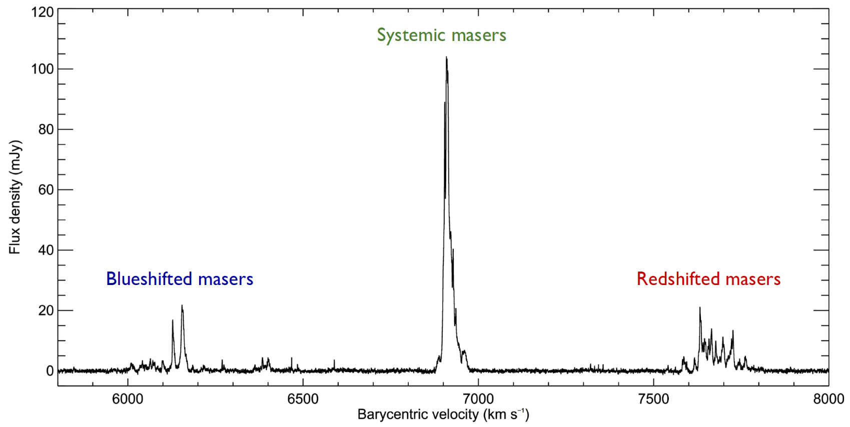
Velocity gradient vs angular position

$$d = \frac{\partial_\theta v_{\mathrm{LOS}}}{a_{\mathrm{LOS}}} v_{\mathrm{LOS}}$$

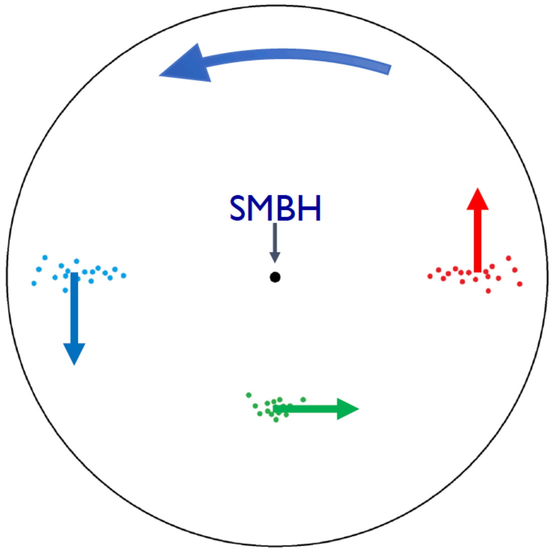
Acceleration

Velocity

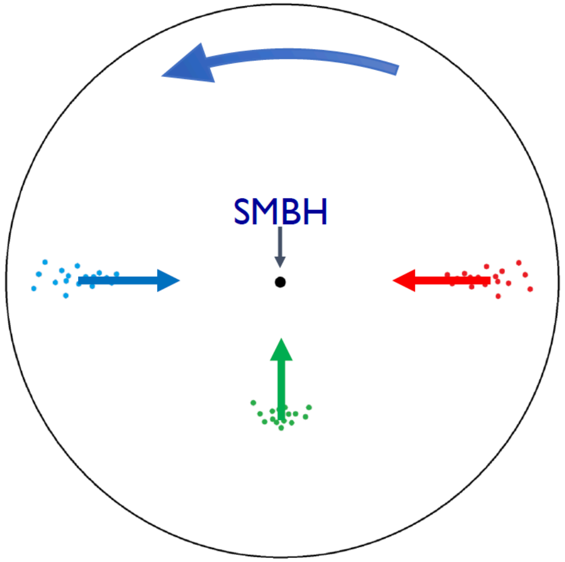




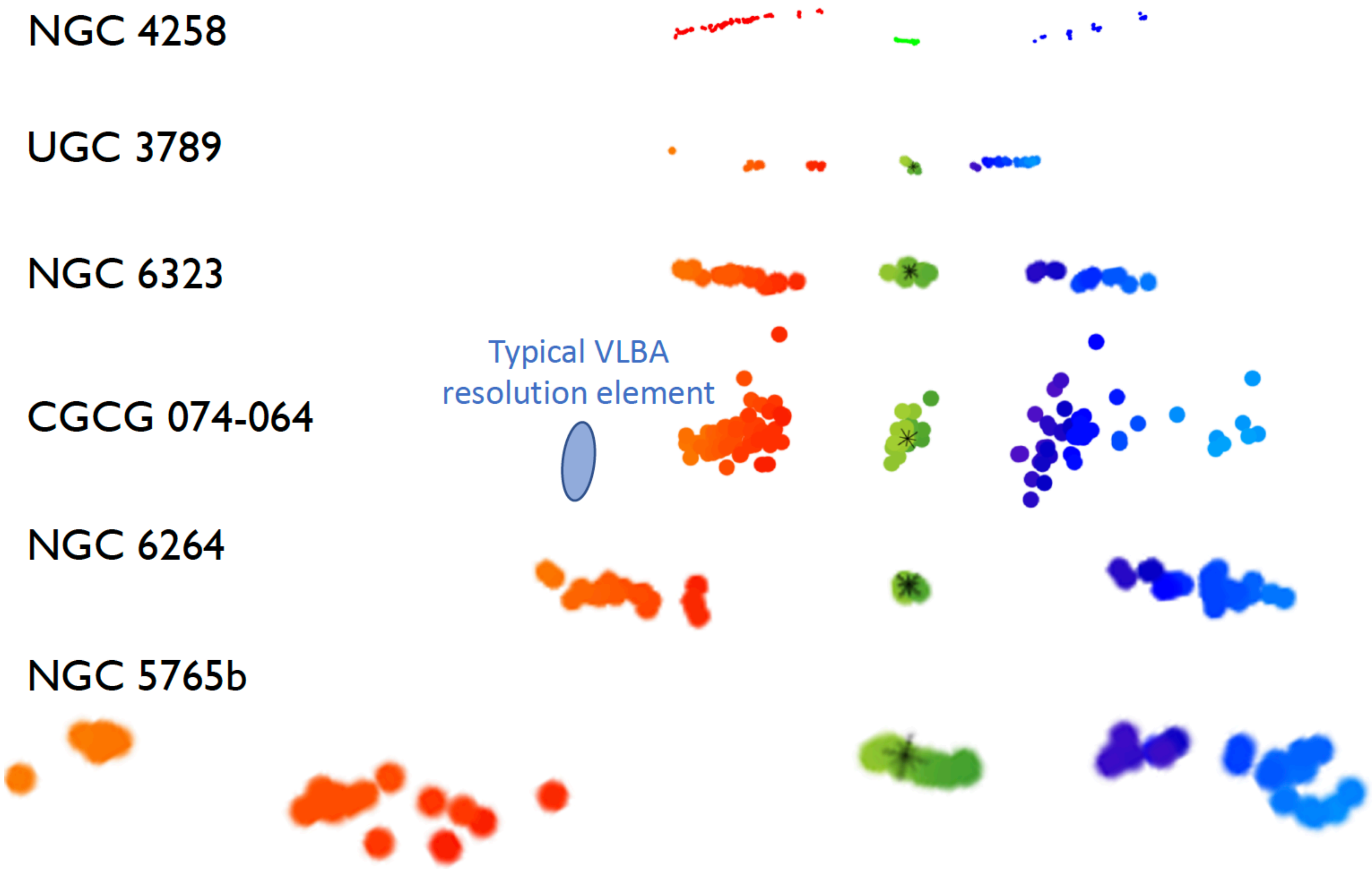
velocity vectors



acceleration vectors



The Megamaser Cosmology Project



NGC 4258

UGC 3789

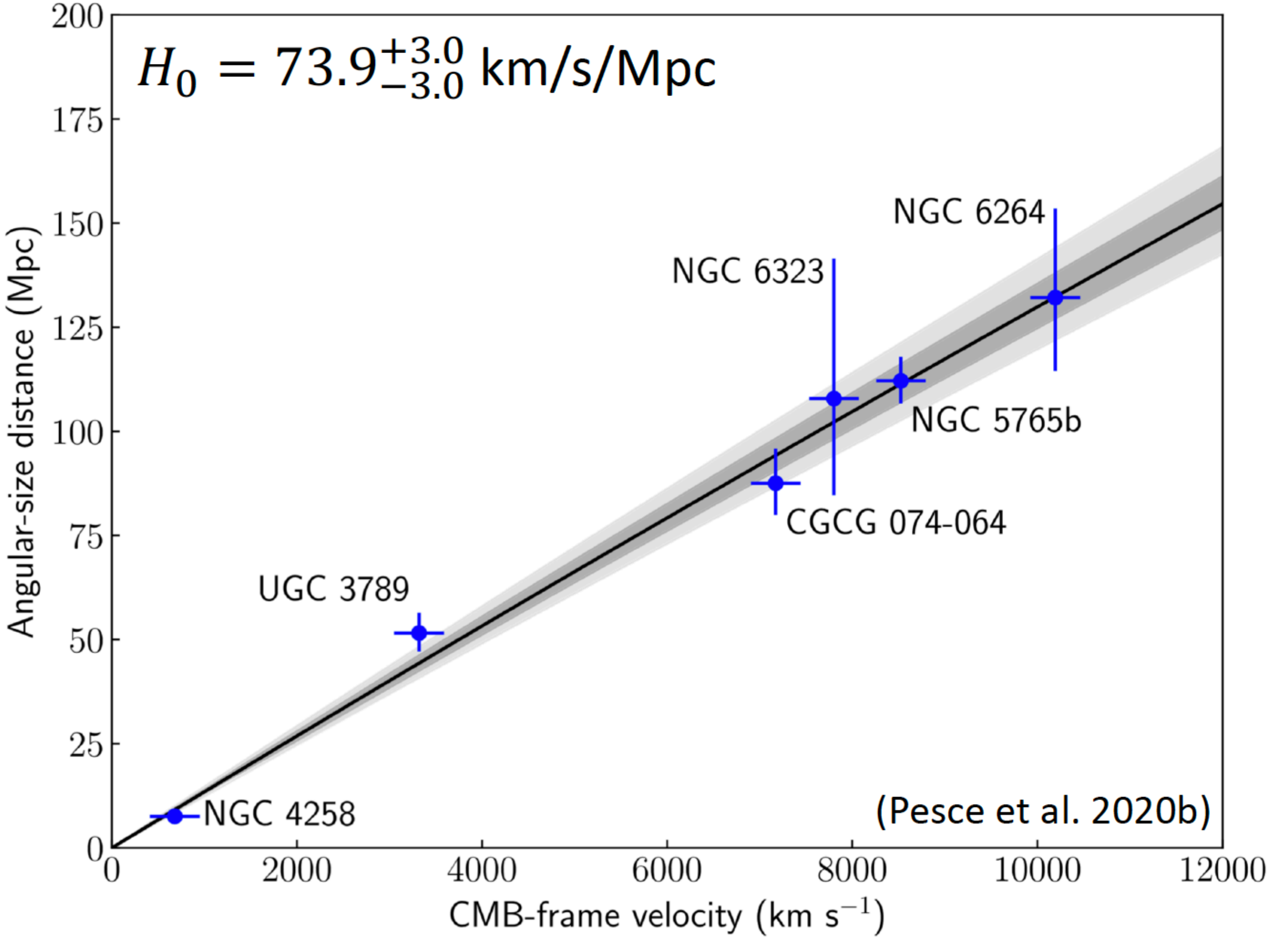
NGC 6323

CGCG 074-064

NGC 6264

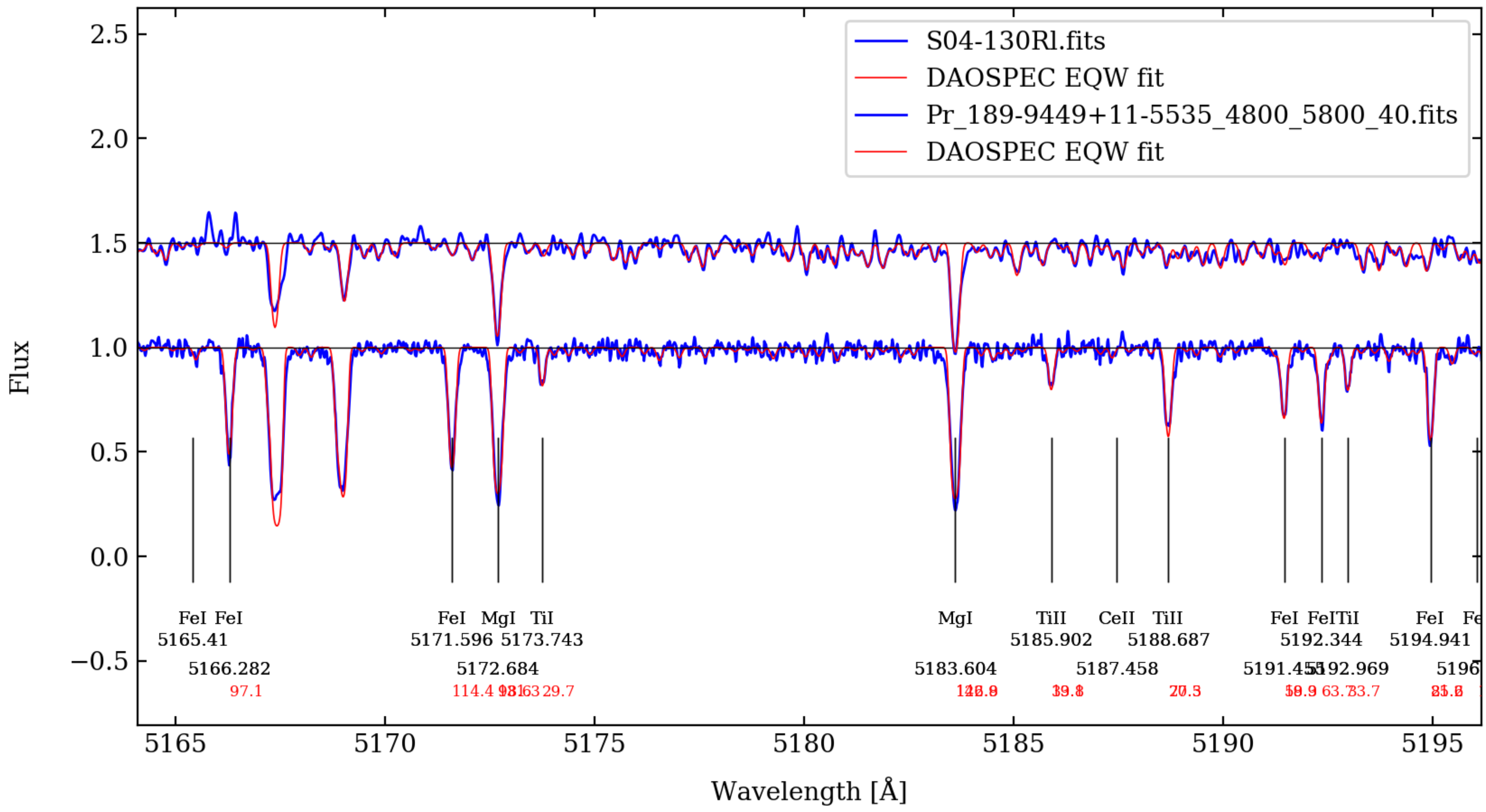
NGC 5765b

$$D_i = \frac{c}{H_0(1+z_i)} \int_0^{z_i} \frac{dz}{\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}}$$



$H_0 = 73.9^{+3.0}_{-3.0}$ km/s/Mpc

(Pesce et al. 2020b)



S04-130Rl.fits
DAOSPEC EQW fit
Pr_189-9449+11-5535_4800_5800_40.fits
DAOSPEC EQW fit
Flux
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
Wavelength [Å]
5165
5170
5175
5180
5185
5190
5195
FeI
5165.41
FeI
5166.282
97.1
FeI
5171.596
114.4
MgI
5172.684
TiI
5173.743
29.7
MgI
5183.604
TiII
5185.902
CeII
5187.458
TiII
5188.687
FeI
5191.455
FeI
5192.344
63.7
TiI
5192.969
33.7
FeI
5194.941
Fe
5196

$\Psi(\nu) = \sum_k \phi_i(\nu_k)\,\phi_j(\nu - \nu_k)\delta\nu_k$ en passant à la limite continue, $\Psi(\nu) = \int \phi_i(\nu')\phi_j(\nu - \nu')\mathrm{d}\nu'$

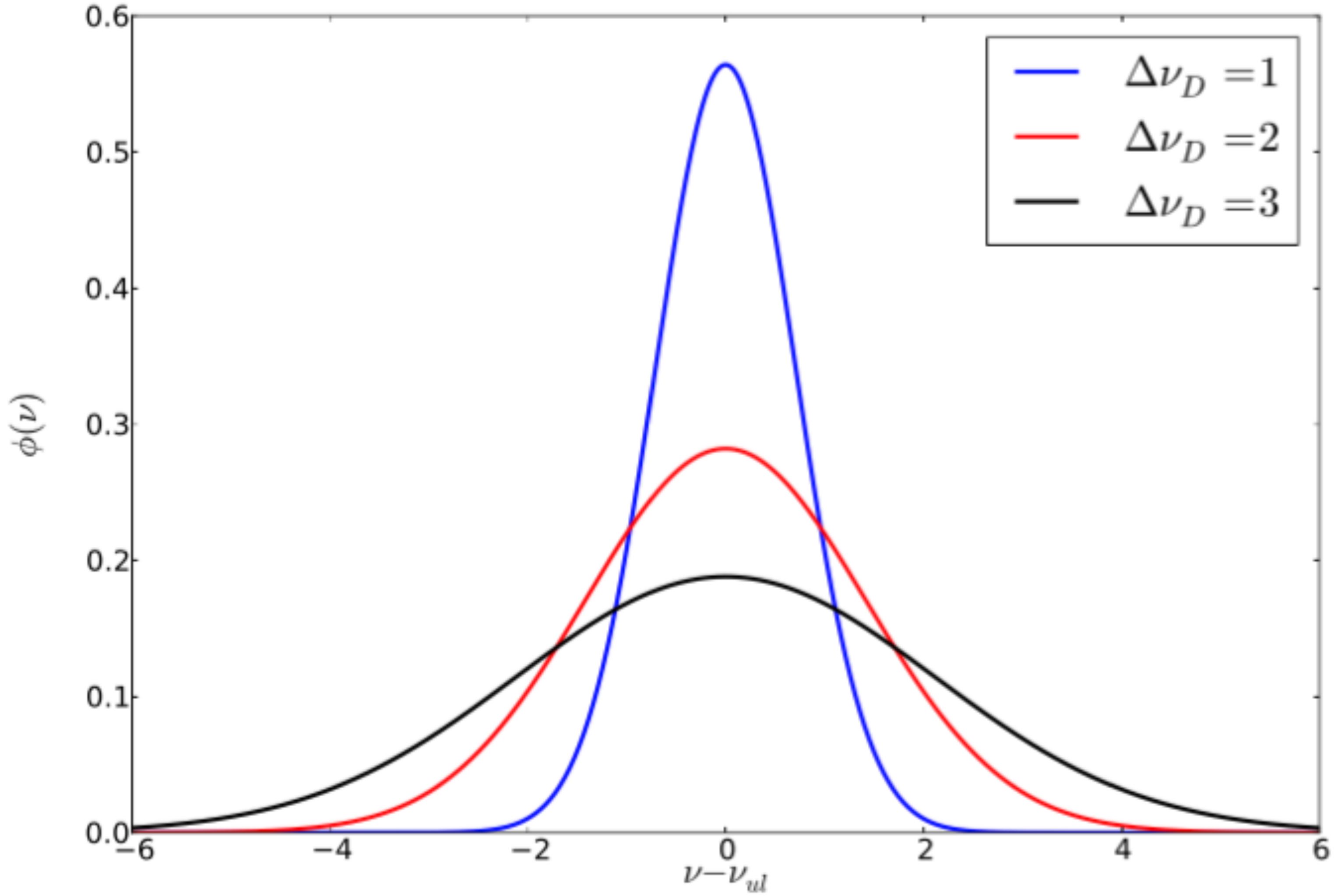
The convolution of two Gaussian functions, with standard deviations σ_1 and σ_2, results in another Gaussian function with a width given by

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}.$$

Similarly, the convolution of two Lorentzian profiles, with respective parameters γ_1 and γ_2, also yields a Lorentzian profile characterized by

$$\gamma = \gamma_1 + \gamma_2.$$

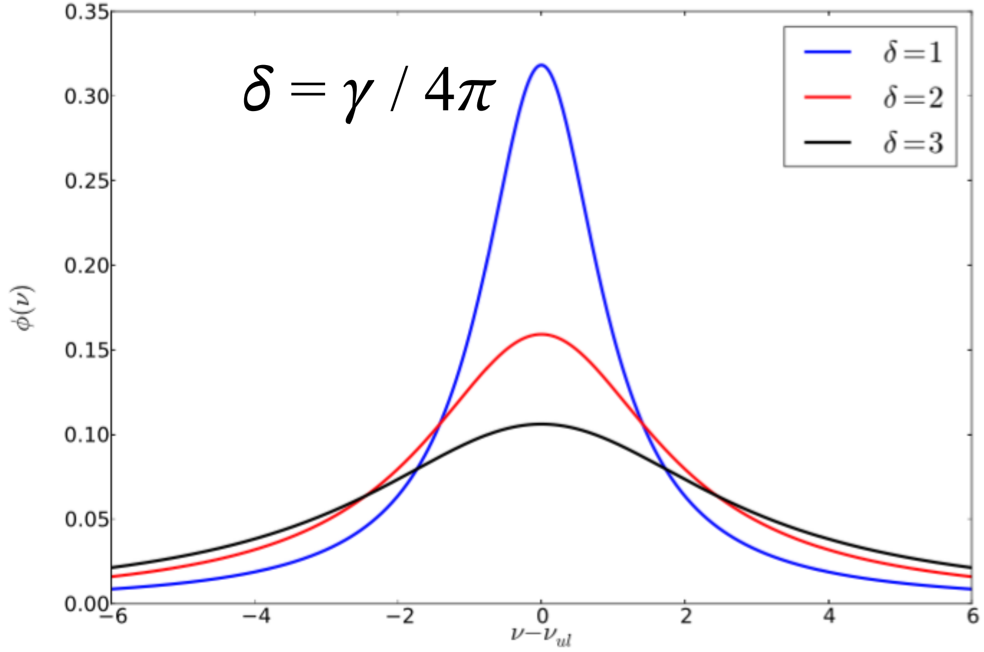
We therefore identify two types of fundamental line profiles:
(i) **Gaussian profiles**, associated with thermal and turbulent Doppler broadening; and
(ii) **Lorentzian profiles**, resulting from natural and collisional broadening.



Profils de Gauss(Doppler) pour $\Delta\nu_D = 1$ (bleu), $\Delta\nu_D = 2$ (rouge) et $\Delta\nu_D = 3$ (noir). Les unités de $\Delta\nu_D$ sont les mêmes que celles de l'axe $\nu - \nu_{ul}$.

Courtesy F. Levrier

$$\gamma_{\text{tot}} = \gamma_{\text{naturelle}} + \gamma_{\text{coll}}$$



$\delta = \gamma / 4\pi$

· Profils de Lorentz pour $\delta = 1$ (bleu), $\delta = 2$ (rouge) et $\delta = 3$ (noir). Les unités d sont les mêmes que celles de l'axe $\nu - \nu_{ul}$.

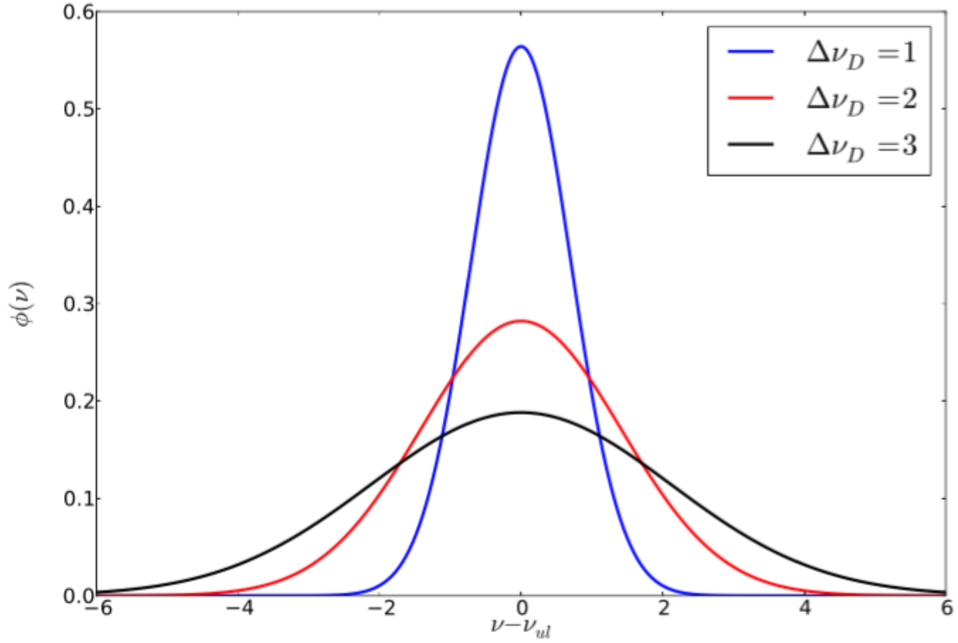
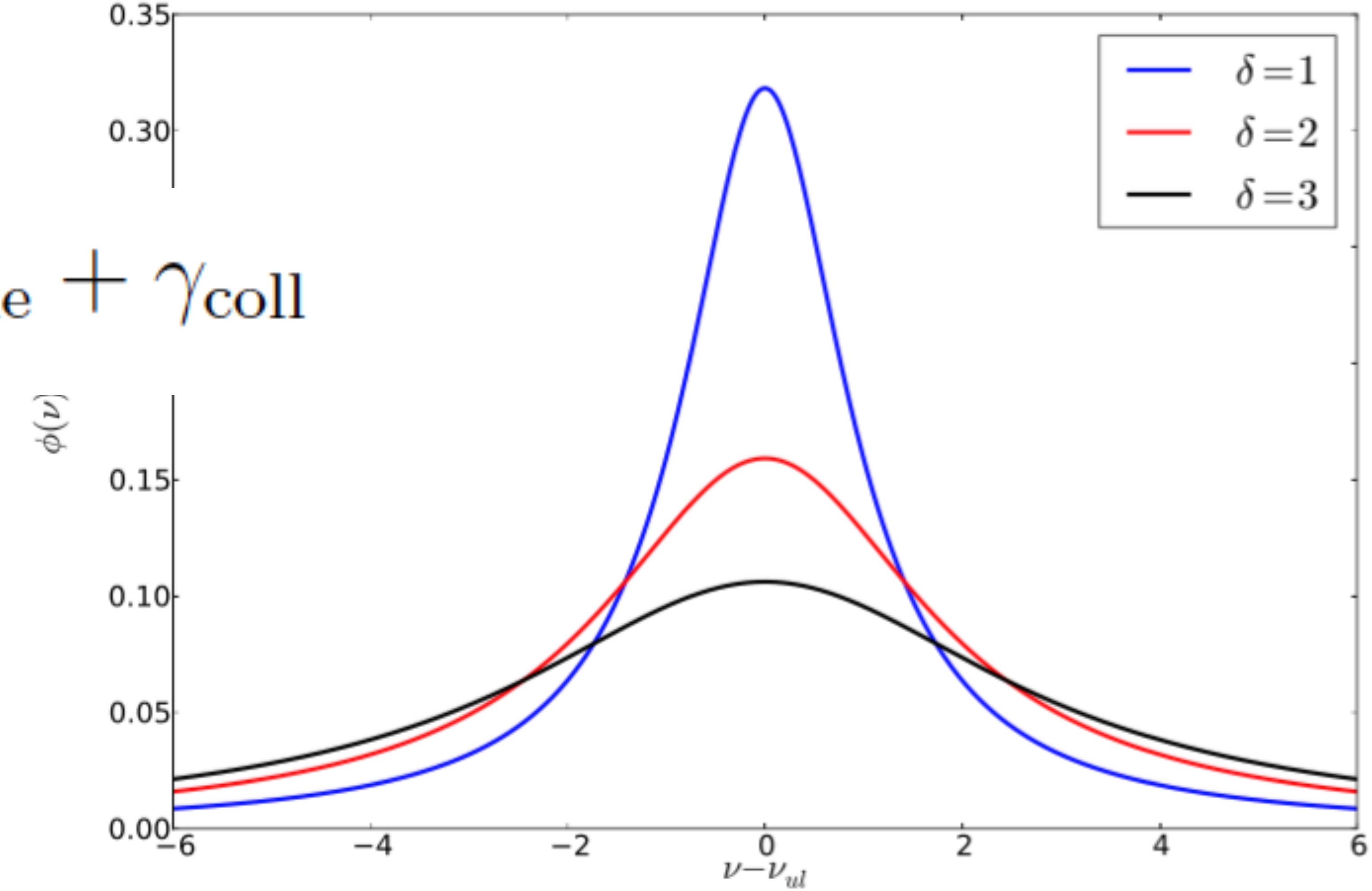


FIGURE 4.2 – Profils de Gauss (Eq. 4.2) pour $\Delta\nu_D = 1$ (bleu), $\Delta\nu_D = 2$ (rouge) et $\Delta\nu_D = 3$ (noir). Les unités de $\Delta\nu_D$ sont les mêmes que celles de l'axe $\nu - \nu_{ul}$.

Courtesy F. Levrier

$$\gamma_{\mathrm{tot}} = \gamma_{\mathrm{naturelle}} + \gamma_{\mathrm{coll}}$$

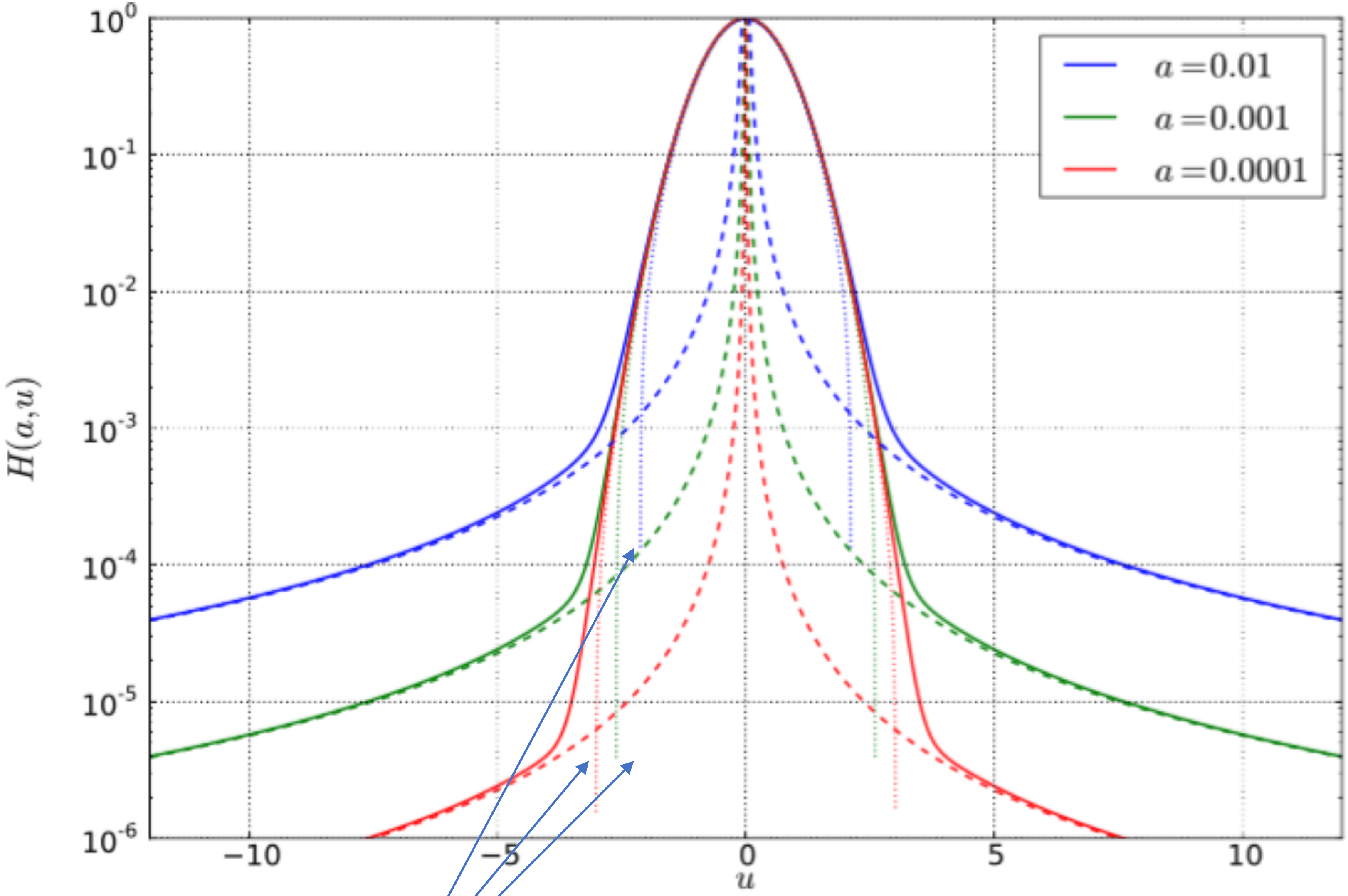


$$\delta = \gamma \,/\, 4\pi$$

Profils de Lorentz pour $\delta = 1$ (bleu), $\delta = 2$ (rouge) et $\delta = 3$ (noir). Les unités de δ sont les mêmes que celles de l'axe $\nu - \nu_{ul}$.

Courtesy F. Levrier

Combinaison: Convolution and Voigt Profile

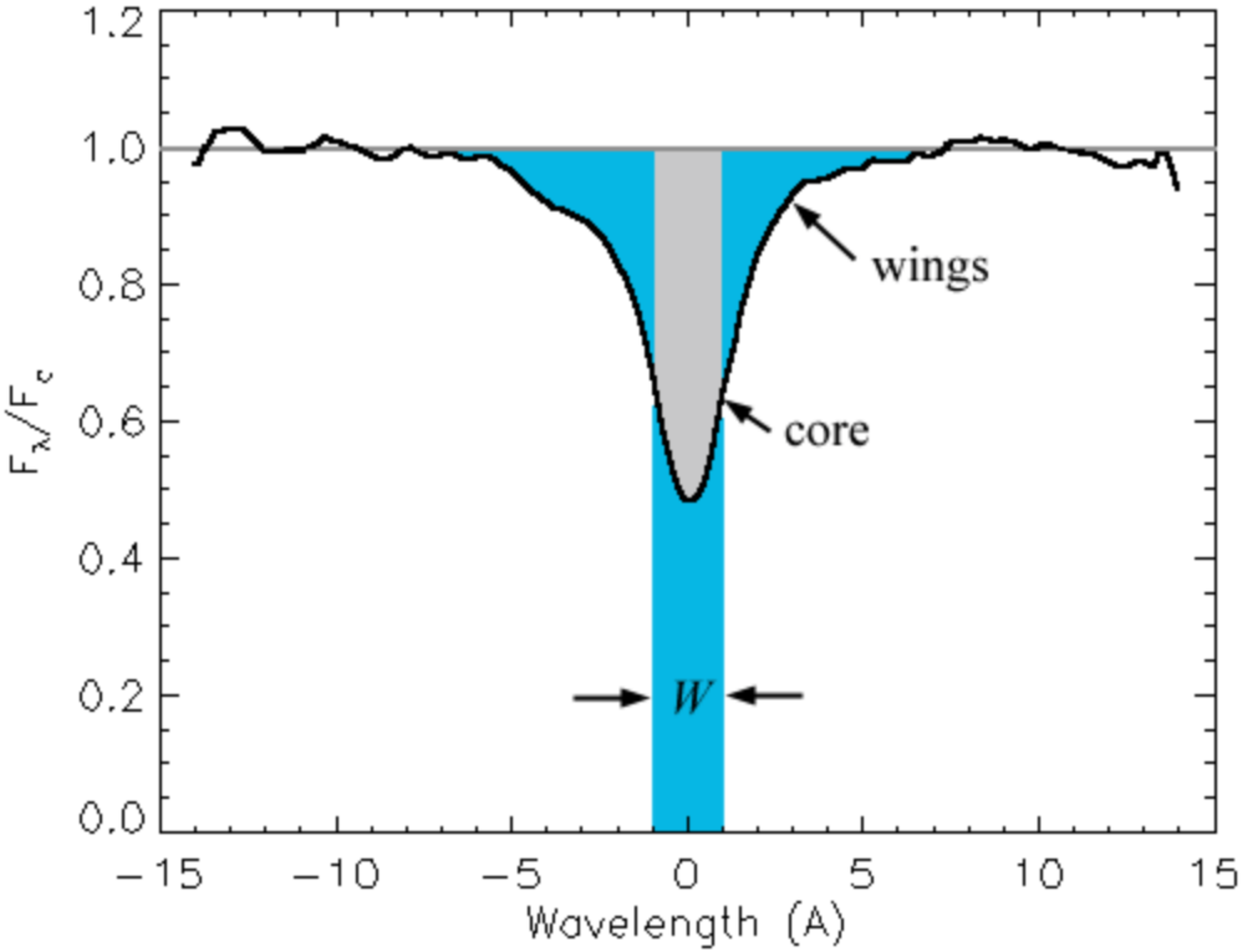


pour $u \to 0$, et en se limitant au premier ordre en a,

$$H(u) \simeq e^{-u^2} - \frac{2a}{\sqrt{\pi}}(1 - 2u^2)$$

lorsque $u \gg 1$, toujours avec $a \ll 1$

$$H(a,u) \simeq \frac{1}{u^2}\frac{a}{\pi}\int_{-\infty}^{+\infty} e^{-y^2} \mathrm{d}y = \frac{a}{\sqrt{\pi}u^2}$$



Profil de Voigt =
Doppler core
and
Lorentz wings

W = equivalent width