

Astrophysics II - Exercise Sheet 8

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1 Boltzmann and Saha equations

1. Show that for the hydrogen atom, the statistical weight of the electron is $g_n = 2n^2$, where n is the energy level of the electron.
2. Compute the partition function of hydrogen. What problem do you encounter and how can you resolve it ?
3. In the interval 6000 – 25000 K, investigate the intensity of hydrogen Balmer lines, that is, the ratio N_2/N , where N_2 is the number of atoms in level 2 and N the total number of hydrogen atoms (including ions). Some indications:
 - Ionization potential of H: 13.6 eV
 - Excitation potential of the 2nd level of H: 10.2 eV
 - Statistical weight of level n of hydrogen: $2n^2$
 - Partition function of neutral H: 2
 - Partition function of ionized H: 1
 - We will assume the electronic pressure to be constant, $P_e = 100 \text{ dyne cm}^{-2}$.

What does the plot $N_2/N(T)$ look like ? Comment. Express your conclusions in terms of spectral types, considering the following table:

Type sp.	T [K]	Type sp.	T [K]
B0	21000	F0	7200
B5	13500	F5	6500
A0	9700	G0	6000
A5	8100		

4. We will compare two stars of type G5V and G5III characterized by

$$\begin{aligned} T_{\text{eff, G5V}} &= 5400 \text{ K} & T_{\text{eff, G5III}} &= 4700 \text{ K} \\ P_{e, \text{G5V}} &= 20 \text{ dyne cm}^{-2} & P_{e, \text{G5III}} &= 1 \text{ dyne cm}^{-2} \end{aligned}$$

Compute the ionization of iron and strontium, i.e. the ratios $(N_{\text{FeII}}/N_{\text{FeI}})$ and $(N_{\text{SrII}}/N_{\text{SrI}})$. We give the following numerical values:

$$\begin{aligned} \chi_{\text{FeI}} &= 7.87 \text{ eV} & \log\left(\frac{2U_{\text{FeII}}}{U_{\text{FeI}}}\right) &= 0.40 \\ \chi_{\text{SrI}} &= 5.70 \text{ eV} & \log\left(\frac{2U_{\text{SrII}}}{U_{\text{SrI}}}\right) &= 0.32 \end{aligned}$$

Can the obtained results have an application in spectral classification ?