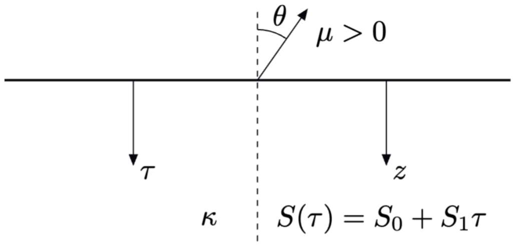
Astrophysics II - Exercise Sheet 5

EPFL Physics section, Prof. Pascale Jablonka

November 7, 2025

1 Intensity emerging from a star

We consider a semi-infinite homogeneous medium (this means that the absorption coefficient κ is uniform). The source function S is assumed to be isotropic, but not homogeneous: it increases linearly with the optical depth: $S(\tau) = S_0 + S_1\tau$, where S_0 and S_1 are constants with respect to τ (both positive, $S_1 > 0$ physically represents an increase in temperature with optical depth). We suppose that there is no diffusion. The incoming intensity $I(\tau = 0, \mu < 0)$ vanishes, and we will assume that at large optical depths, $J - S$ remains finite. (Here, J is the mean specific intensity which in the course is denoted by $\langle I \rangle$).



1. Show that the expression for the emerging intensity is $I(\tau = 0, \mu > 0) = S_0 + \mu S_1$.
2. Derive from this the value of the mean intensity $J(\tau = 0)$ and the flux $F(\tau = 0)$ as a function of S_0 and S_1. Interpret your result.
3. We now apply the Eddington approximation by adopting a semi-isotropic model to represent $I(\tau, \mu)$. Express the intensities $I^+(\tau)$ and $I^-(\tau)$ as functions of $J_a(\tau)$ and $dJ_a(\tau)/d\tau$, and translate the limiting conditions to the surface and in optical depth to obtain the approximate solution $J_a(\tau)$.

In the Eddington approximation, the radiation pressure can be written

$$p_\alpha = \frac{4\pi}{3c} J_a \tag{1}$$

and one can show that the radiative transfer equation reads

$$\frac{1}{3}\frac{d^2 J}{d\tau^2} = J - S \tag{2}$$

with solution

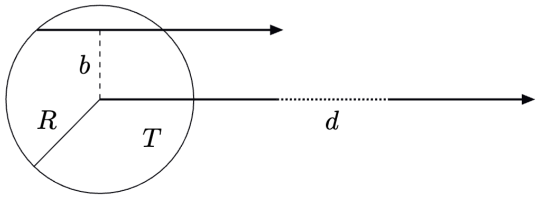
$$J_a(\tau) = S_0 + S_1\tau + C_1 e^{-\sqrt{3}\tau} + C_2 e^{+\sqrt{3}\tau} \tag{3}$$

where C_1 and C_2 are integration constants.

4. Comment on the variation of $I^+(\tau)$, $I^-(\tau)$ and $J_a(\tau)$ with τ.
5. Compare the exact value $J(\tau = 0)$ with the approached value $J_a(\tau = 0)$.
6. Same question for the flux, $F(\tau = 0)$ v.s. $F_a(\tau = 0)$.

2 Brilliance of an interstellar cloud

We consider an interstellar cloud of gas at temperature T, emitting thermal radiation with an emissivity integrated over all directions given by $P(\nu)$. The cloud is spherical, of radius R, and located at a distance $d \gg R$ of the observer.



1. Supposing that the cloud is optically thin, what is its brilliance as measured by the observer ? We will express the result as a function of the distance b to the center of the cloud, projected on the plane of the sky, as indicated on the figure, by making the further simplifying assumption that we can consider the observer as being located at infinity.

2. What is the effective temperature of the cloud ? Compare this to its actual physical temperature T.

3. Compute the spectral flux density F_ν measured by the observer.

4. Repeat the previous questions by now assuming the cloud to be optically thick.