Astrophysics II - Exercise Sheet 3

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1 Pressure

In this exercise we will estimate the contribution of two components to the global cosmological pressure in the universe.

1. What is the cosmological pressure due to galaxies? For simplicity, we will consider that each galaxy is a point mass with $M_{\rm gal} \sim 3 \cdot 10^{44}$ g, that galaxies have a number density $n \sim 0.1$ Mpc^{-3}, and move with an average random velocity of 100 km s^{-1}. *Hint:* first find the mean kinetic energy density due to galaxies.

2. How does this pressure component compare to the CMB radiation pressure (recall that $T_{\rm CMB} = 2.725$ K) ?

2 Radiative Transfer

Show that the equation of radiative transfer can be written in the form

$$\frac{dP_{\rm rad}}{dr} + \frac{1}{r} \cdot (3P_{\rm rad} - U) + \frac{\rho\kappa}{c} F = 0, \tag{1}$$

where κ is the mean Rosseland opacity and F is the bolometric flux.

3 Lifetime of a star

We wish to make an order of magnitude estimate of a few properties of a star based on the laws of hydrostatic equilibrium and energy conservation. We will assume that the matter making up the star can be treated as a perfect gas of atomic hydrogen.

1. Give the hydrostatic equilibrium condition for a column of matter located at a distance r of the star's center. Derive from this two relations giving the order of magnitude of the pressure $P_\star$ and temperature $T_\star$ at the center of a star in terms of its mass $M_\star$ and radius $R_\star$. Give a numerical expression for the case of our sun.

2. Find the order of magnitude of the gravitational potential energy of the star, assuming that the mass density ρ is uniform across its volume. Derive the energy released during the process of the star's formation, when its parent gas cloud collapses. To this end, assume that in the initial state all the gas is located infinitely far from the final position of the star. Again, do the numerical application for the case of the sun.

3. Suppose that the sun draws all of its luminosity ($L_\odot \simeq 3.828 \cdot 10^{26}$ W) from the energy released due to gravitational contraction. Based on this, derive an order of magnitude estimate of its age (assuming its luminosity to remain constant in time). Does this seem reasonable to you ?

4. What other energy source needs to be accounted for ? Knowing that the fusion of hydrogen into helium releases an energy $\varepsilon = 0.007 m_{\rm H} c^2$ per nucleon and that a star can use all the fuel contained in a fraction $x < 1$ of its total mass, give an o.o.m. estimate of the total available energy, and re-estimate the lifetime (we'll assume $x = 0.1$).