Astrophysics II - Exercise Sheet 2

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1 Black-body radiation

A black body is an idealized system which perfectly absorbs radiation at all wavelengths, and is in thermal equilibrium at a temperature T assumed to be constant. It emits isotropic radiation whose specific intensity is given by the *Planck law*,

$$I_\nu = B_\nu(T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}. \tag{1}$$

This relation describes the distribution of a photon gas in thermodynamic equilibrium, following a Bose-Einstein statistic with null chemical potential $\mu = 0$. The radiation of stars is very close to that of a black body, and the cosmic microwave background (CMB) is the most 'perfect' example known in nature, with $T_{\mathrm{CMB}} = 2.725$ K.

Questions:

1. Graphically represent $B_\nu(T_1)$ and $B_\nu(T_2)$ for two temperatures $T_2 > T_1$.
2. Compute the specific intensity $B_\lambda(T)$ of a black body as a function of wavelength rather than frequency. Recall exercise **(2)** from last week.
3. Give approximate forms of B_ν and B_λ in the limit of low and high frequencies. The former is known as the *Rayleigh-Jeans* law, and the latter as the *Wien* law.
4. Determine the frequency ν_{m} which maximizes the function $B_\nu(T)$, and do the same for the wavelength λ_{m} for $B_\lambda(T)$. Do you have $\lambda_{\mathrm{m}}\nu_{\mathrm{m}} = c$?
5. Compute λ_{m} for the CMB and the following stars:

Star Name	Mintaka	Regulus	Sirius	Polaris	Sun	Aldebaran	Antares
Spectral type	O9.5II	B7V	A1V	F7Ib	G2V	K5III	M1.5Iab-b
T [K]	31800	12460	9940	6015	5778	3910	3400

6. We consider a special realization of a black body in the form of a container at temperature T. The interior walls of this container are completely opaque, except for a small hole pierced at one point of the system. Show that the total flux F (at all frequencies) emitted through this hole is proportional to T^4. This relation is known as the *Stefan-Boltzmann law*. Explicitly compute the proportionality constant σ, often called *Stefan's constant*. We give for reference the following integral,

$$\int_0^\infty \frac{x^3}{e^x - 1} \mathrm{dx} = \frac{\pi^4}{15} \tag{2}$$

7. Compute the volumetric energy density u and the radiation pressure p of black-body radiation as a function of T, σ and c.

2 Wien's law

Recall that: $h = 6.6262 \cdot 10^{-34}$ J s, $c = 2.998 \cdot 10^{8}$ m s^{-1} and $k_B = 1.3807 \cdot 10^{-23}$ J K^{-1}.

1. Starting from the Wien approximation for $B_\lambda(T)$, show that $\lambda_\mathrm{m} T = \frac{1}{5}\frac{hc}{k_B} = 0.288$ cm K.
2. Show that the factor 1/5 becomes 0.2014 if you instead start from the full Planck law (recall question 4 of the previous exercise).
3. Assuming the following systems can be described as black bodies, at what wavelength would you need to observe:
   - A cloud of neutral hydrogen ($T \sim 100$ K) ?
   - A cloud of dust around a star ($T \sim 300$ K) ?
   - The CMB ($T \sim 3$ K) ?
   - A (healthy) human person ($T \sim 310$ K) ?
   - Frozen spinach ($T \sim 255$ K) ?