

# Astrophysics II - Exercise Sheet 10

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## Degenerate Gases

1. Show that the velocity of a partly relativistic particle of mass  $m_0$  can be written as

$$v = \frac{p/m_0}{\left[1 + \left(\frac{p}{m_0 c}\right)^2\right]^{1/2}} \quad (1)$$

2. Show that, in the case of fully degenerate, partly relativistic gas, the pressure is given by

$$P = \frac{\pi m_0^4 c^5}{3 h^3} f(x) \quad (2)$$

with

$$f(x) = x(x^2 + 1)^{1/2} (2x^2 - 3) + 3 \ln(x + \sqrt{1 + x^2}) \quad (3)$$

and  $x = \sinh \theta_F = p_F / m_0 c$ ,  $\sinh \theta = \frac{p}{m_0 c}$ .

3. Establish a mass-radius relation in white dwarfs. We will assume a fully degenerate electron gas.

*Indication:* Start from the equation of hydrostatic equilibrium and show that, at first order, we have

$$\frac{P_c}{R} \simeq \frac{3}{4\pi} \frac{GM^2}{R^5}. \quad (4)$$

Next, identify the central pressure with the electronic pressure of the non-relativistic fully degenerate electron gas, to finally establish the relationship between mass and radius.

4. To which temperature does a bar of iron need to be heated to break the degeneracy of the electron gas? Assume the density to be  $\rho \simeq 7 \text{ g cm}^{-3}$ .