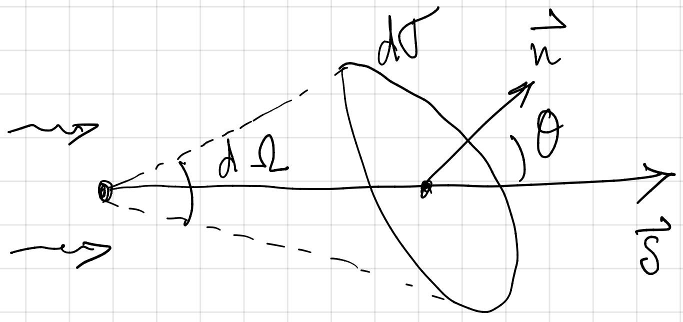
Basic definitions - General principles

Specific Intensity



Consider a radiation field propagating along a direction $\vec{s}$ forming an angle θ with the normal $\vec{n}$ to the surface $d\sigma$

One defines dU as the amount of energy in the frequency interval $[\nu, \nu + d\nu]$ crossing $d\sigma$ in the solid angle $d\Omega$ in dt.

$$dU = I_\nu \, d\nu \, dt \, d\sigma \cos\theta \, d\Omega$$

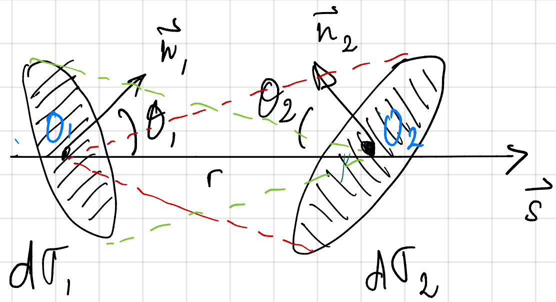
I_ν is therefore the "energy flux" per unit frequency
surface
time
solid angle.

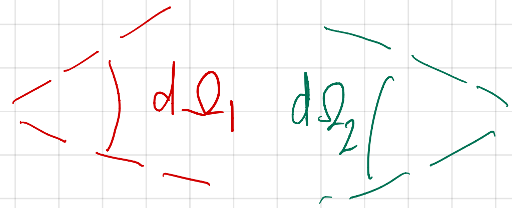
$I_\nu d\nu$ is in units Watts / m^2 / stéradians

on can define I_ν or I_λ

$$I = \int_0^\infty I_\nu \, d\nu$$

What happens in the vaccum?





$d\Omega_1$ = solid angle subtended by $d\sigma_2$ seen from the center of $d\sigma_1$

$d\Omega_2$ = ——————— $d\sigma_1$ ——————— $d\sigma_2$

By definition
$$d\Omega_1 = \frac{d\sigma_2 \cos\theta_2}{r^2}$$
$$d\Omega_2 = \frac{d\sigma_1 \cos\theta_1}{r^2}$$

$$dU_1 = I_{\nu_1} \cos\theta_1 \, d\sigma_1 \, d\Omega_1 \, d\nu \, dt$$
$$= I_{\nu_1} \cos\theta_1 \, d\sigma_1 \frac{d\sigma_2 \cos\theta_2}{r^2} d\nu \, dt.$$

Similarly

$$dU_2 = I_{\nu_2} \cos\theta_2 \, d\sigma_2 \frac{d\sigma_1 \cos\theta_1}{r^2} d\nu \, dt.$$

Radiation energy is conserved (no absorption)

$\Rightarrow$ The specific intensity is conserved in the vaccum.

Remarque: This is independent of distance, provided the source is <u>resolved</u>

What does "resolution" mean?

at distance D from the observer

Now doubling the distance to the observer ⟶ 2D

Star : distance $\times 2$ $\Longrightarrow$ flux $\div 4$ (r^2)

Galaxy : Each new surface unit will contain $\times 4$ more previous surface elements

But flux is $\div 4$

Factors cancel out

Mean specific intensity

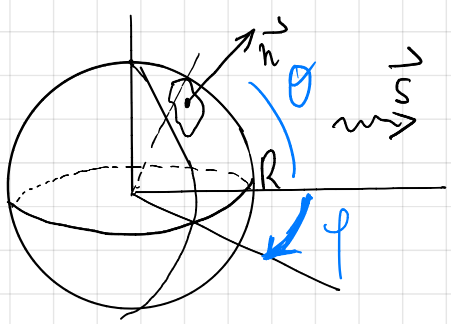
written $< I_\nu >$, also named surface brightness

WHY ? A star does not radiate the same intensity in all points of its surface. We can think of the spots on the Sun. However we do not "resolve" all stars in the same detail due to their much larger distances.

Alpha Centauri : 4.22 astronomical unit (UA)

$1\ UA =$ Distance Earth - Sun $\simeq 150.10^6$ km

Integral of flux over one hemisphere :



$\theta \in \left[0, \frac{\pi}{2}\right]$ $\varphi \in [0, 2\pi]$

Summing flux on the surface, then dividing by the surface (= mean)

Surface in spherical coordinates
$= R^2 \sin\theta \, d\theta \, d\varphi$

$$I_\nu \, d\sigma \cos\theta = \frac{dU_\nu}{d\nu \, dt \, d\Omega}$$

Integral of the Specific intensity

$$\int I_\nu \, d\sigma \cos\theta = \int \frac{dU_\nu}{d\nu \, dt \, d\Omega}$$

$$\int_0^{\frac{\pi}{2}} \int_0^{2\pi} I_\nu \cos\theta \, R^2 \sin\theta \, d\theta \, d\varphi$$

$\theta \in [0, \frac{\pi}{2}]$
$\varphi \in [0, 2\pi]$

$$= 2\pi R^2 \int_0^{\frac{\pi}{2}} I_\nu \cos\theta \sin\theta \, d\theta .$$

Surface

$$: 2\pi R^2 \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta = \int_0^{\pi/2} \underbrace{d\sigma}_{d\sigma'} \cos\theta$$

$$\mu = \cos\theta \qquad d\mu = -\sin\theta \, d\theta .$$

$$2\pi R^2 \int_1^0 -\mu \, d\mu = 2\pi R^2 \int_0^1 \mu \, d\mu .$$

$$2\pi R^2 \left[\frac{1}{2} \mu^2 \right]_0^1 = \pi R^2$$

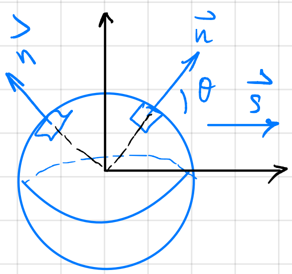
$$\langle I_\nu \rangle = 2 \int_0^{\pi/2} I_\nu \cos\theta \sin\theta \, d\theta .$$

One can choose to calcule the mean over the solid angle

$$\frac{\int I_\nu \, d\Omega}{\int d\Omega} = \frac{1}{4\pi} \int I_\nu \, d\Omega$$

Flux

Monochromatic flux



Amount of energy emitted or received, in all directions
per unit time
surface
in $[\nu, \nu + d\nu]$

$$dF_\nu = \frac{dU_\nu}{d\sigma \, d\nu \, dt} = I_\nu \cos\theta \, d\Omega$$

One can also write $dF_\nu = I_\nu \cos\theta \sin\theta \, d\theta \, d\varphi$

unit : $W / m^2 / Hz$

Astrophysical fluxes are very small, it is convenient to change units

$$1 \text{ Jansky} = 1 \text{ Jy} = 10^{-26} \, W \, m^{-2} \, Hz^{-1} = 10^{-23} \, erg . s^{-1} \, cm^{-2} \, Hz^{-1}$$

$$F_\nu = \int_\Omega dF_\nu$$ Total monochromatic flux

Bolometric flux = integrated over all frequencies $F = \int_0^\infty F_\nu \, d\nu$

QUESTION: If intensity is isotropic, how much is the total flux?

$$F_\nu = \int_\Omega I_\nu \cos\theta \, d\Omega = I_\nu \int_0^{2\pi} \int_0^{\pi} \cos\theta \sin\theta \, d\theta \, d\varphi .$$

$$= 2\pi I_\nu \int_0^{\pi} \cos\theta \sin\theta \, d\theta$$

$\mu = \cos\theta \qquad d\mu = -\sin\theta \, d\theta .$

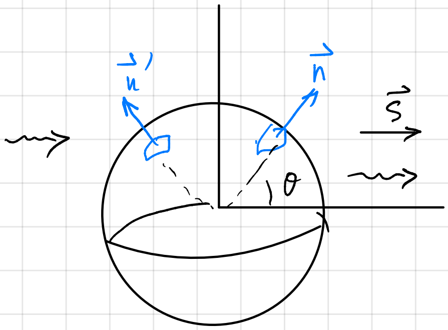
$$F_\nu = 2\pi I_\nu \int_1^{-1} -\mu \, d\mu = 2\pi I_\nu \left[\frac{1}{2} \mu^2 \right]_{-1}^{1} = 0$$

The flux of an isotropic radiation is null

Flux requires deviation from isotropy

Incident and emergent flux

$$F_\nu = \underbrace{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_\nu \cos\theta \sin\theta \, d\theta \, d\varphi}_{\text{emergent flux}} + \underbrace{\int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi} I_\nu \cos\theta \sin\theta \, d\theta \, d\varphi}_{\text{incident flux}}$$



F_ν^+ $\qquad$ $-F_\nu^-$

$$F_\nu = F_\nu^+ - F_\nu^-$$

If isotropy : $|F_\nu^+| = |F_\nu^-| \Rightarrow F_\nu = 0$

$$F_\nu^+ = \pi \langle I_\nu \rangle$$

Luminosity $\qquad$ $\boxed{\text{Emitted}}$ energy in all directions per unit time frequency

$$dL_\nu = \frac{dU_\nu}{dt\, d\nu} = I_\nu \cos\theta \, d\sigma \, d\Omega$$

$$L_\nu = \int_{\text{surface}} \int_{\Omega^+} I_\nu \cos\theta \, d\sigma \, d\Omega = \int_{\text{surface}} F_\nu^+ \, d\sigma$$

ensemble of outgoing direction

If spherical symetry $\quad L_\nu = 4\pi R^2 F_\nu{}^{+}$

Bolometric luminosity $\quad L_{Bol} = \int_\nu L_\nu \quad$ (unité : Watt)

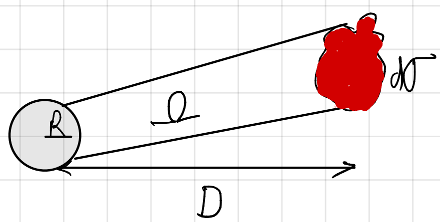
Bolometric magnitude $\quad M_{Bol} = -2.5 \log L + cte$

$$= -2.5 \log \frac{L}{L_\odot} + 4.75$$

$$L_\odot = 3.846 \; 10^{26}\,W = 3.846 \; 10^{33}\ erg/s$$

Radiance $\quad \boxed{\text{Received}} \quad$ flux per surface unit.

$$dF_\nu = \frac{dU_\nu}{d\sigma\, dt\, d\nu} \longrightarrow E_\nu = \frac{dU_\nu}{dt\, d\nu}$$



$$dU_\nu = \underbrace{I_\nu}_{} \, d\nu\, dt \, \underbrace{d\Omega \, d\sigma \cos\theta}_{}$$

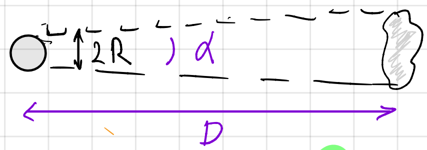
Emission : $\quad \langle I \rangle$ is averaged over πR^2

solid angle $= \dfrac{d\sigma}{D^2}$

One can write $\quad \pi R^2 \langle I_\nu \rangle \dfrac{d\sigma}{D^2} \equiv$ received flux

E_ν, per surface unit $\Rightarrow \quad E_\nu = \dfrac{\pi R^2}{D^2} \langle I_\nu \rangle = \Omega_{Source} \langle I_\nu \rangle$

It is common to use the angular size of the source, rather than its solid angle.



$$\alpha = \frac{2R}{D}$$ (for small angles, given that sources are distant)

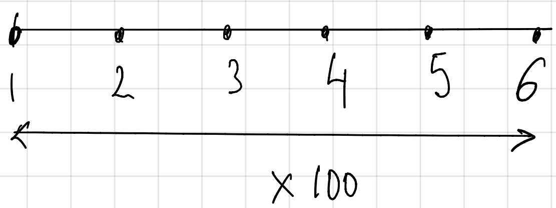
$$E_\nu = \frac{\pi}{4} \alpha^2 \langle I_\nu \rangle = \frac{\alpha^2}{4} F_\nu^{+}$$

Apparent magnitude $\qquad m_\nu = -2.5 \log E_\nu + cte$.

Why do we use magnitudes?

→ Hipparque (II^e century before JC) observed 6000 stars which could be observed with naked eyes. He defined 6 groups.

→ In 1856, Pogson calculated that group 1 was 100 times more luminous than group 6



⟿ 5 intervals

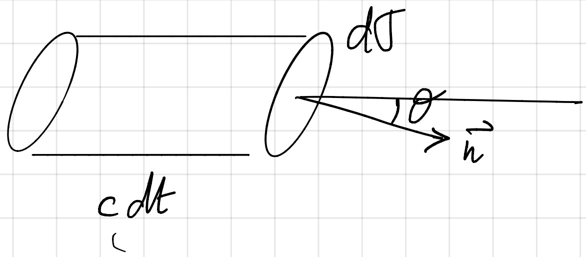
$$5 = 2.5 \log_{10}(100)$$

$$\frac{2.5}{5} \log_{10}(100) = 1 = 2.5 \log\left(100^{1/5}\right)$$

Radiative energy density

$$\frac{dU_\nu}{d\nu\, d\Omega} = I_\nu \cos\theta \, d\sigma \, dt$$

$\equiv$ Energy going through the surface $d\sigma$, of angle θ vs the direction of radiation



This energy is contained in the volume $c\,dt\, d\sigma \cos\theta$

$$\frac{dU_\nu}{d\nu\, d\Omega \underbrace{c\,dt\, d\sigma\cos\theta}} = \frac{I_\nu}{c}$$

Summing over all directions

$$u_\nu = \frac{1}{c}\int_\Omega I_\nu \, d\Omega$$

Isotropy

$$u = \frac{4\pi}{c} I \qquad \left(\text{Bolometric} \right)$$

$$I = \int_\nu I_\nu \, d\nu .$$