— Lecture 12 —

Quantum Optics

— Optomechanical squeezing —

Review

Equations of Motion for $\delta\hat{a}$ and $\delta\hat{b}$: in a rotating frame $a_{in} = \bar{a}_{in} e^{-i\omega_L t}$, $\omega_L = \omega_c$

$$\left|\begin{array}{l} \partial_t \hat{a} = -\kappa/2\, \hat{a} + \sqrt{\kappa}\, \hat{a}_{in} + i\frac{\omega}{L} x_{zpf} (\hat{b} + \hat{b}^+) \bar{\alpha} \\ \partial_t \hat{b} = -\Gamma_m/2\, \hat{b} + \sqrt{\Gamma_m}\, \hat{b}_{in} + \frac{i\omega}{L} (\delta\hat{a} + \delta\hat{a}^+) \bar{\alpha} + i\Omega_m \end{array}\right|$$

on resonance $\Delta = 0$; ~~in [illegible]~~

(ie on resonance equations) needs to be added (w.r.t rotating frame)

Define quadratures:

$$\left|\begin{array}{ll} \hat{X} = (\hat{a} + \hat{a}^+) & \text{Amplitude} \\ \hat{Y} = (\hat{a} - \hat{a}^+)\frac{1}{i} & \text{Phase quadrature} \end{array}\right\} \hat{X}_\theta = \hat{a} e^{-i\theta} + \hat{a}^+ e^{i\theta}$$

$$\partial_t (\hat{a} - \hat{a}^+) = \partial_t \hat{Y} = -\frac{\kappa}{2}\hat{Y} + \sqrt{\kappa}\underbrace{(\hat{a}_{in} - \hat{a}^+_{in})}_{\hat{Y}_{in}\ \text{input phase quadrature}} - 2\frac{\omega}{L} x_{zpf} (\hat{b} + \hat{b}^+) \bar{\alpha}$$

$$\left|\begin{array}{l} \partial_t \hat{Y} = -\frac{\kappa}{2}\hat{Y} + \sqrt{\kappa}\, \hat{Y}_{in} - 2i\frac{\omega}{L} \bar{\alpha}\, \hat{q} \quad \wedge \quad \hat{q} = x_{zpf}(\hat{b} + \hat{b}^+) \\ \partial_t \hat{X} = -\frac{\kappa}{2}\hat{X} + \sqrt{\kappa}\, \hat{X}_{in} \\ (x)\ \partial_t \hat{q} = -\Gamma_m/2\, \hat{q} + \sqrt{\Gamma_m}\, \hat{q}_{in} + 2\frac{i\omega}{L} \cdot \underbrace{x_{zpf}(\bar{\alpha})\overbrace{(\delta\hat{a} + \delta\hat{a}^+)}^{\hat{X}_{in}}}_{QBA\text{-term}} + i\Omega_m \hat{q} \end{array}\right.$$

Lorentzian approximation.

→ 3 coupled equations.

Introduce susceptibility by transforming to Fourier domain

$$\hat{q}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t}\, dt\, \hat{q}(t) \qquad \text{Fourier operator.}$$

Note: $[\hat{a}, \hat{a}^+] = [\hat{a}(t), \hat{a}^+(t')] = \delta(t-t') \Rightarrow [\hat{a}(\omega), \hat{a}^+(\omega')] = \delta(\omega-\omega')$

Fourier domain: $\langle \hat{a}_{in}(t) a^+_{in}(t') \rangle = \delta(t-t') \to \langle \hat{a}(\omega)$

$$(a^+(t)) \hat{=} \int_{-\infty}^{\infty} e^{i\omega t} [a^+(\omega)]^+ d\omega \stackrel{!}{=} \Big(\underbrace{\int_{-\infty}^{\infty} e^{-i\omega t} a(\omega) d\omega}_{a(t)}\Big)^+ = \int_{-\infty}^{\infty} e^{+i\omega t} a^+(\omega) \neq \int e^{-i\omega t} a^+(-\omega) d\omega$$

$$\Rightarrow [a(\omega)]^+ = a^+(-\omega)$$

$$\underbrace{\boxed{[\hat{X}_{in}, \hat{Y}_{in}] = i\delta(t-t')}}_{\text{quadrature commutator}} \quad \text{but} \quad [\hat{X}_{in}(t), \hat{X}_{in}(t')] = 0$$

→ ie one can measure quadratures at equal time, if identical quadrature.

in Fourier domain $\hat{X}_\omega \equiv [a^+(-\omega) + a(\omega)]$ $\quad Y_\omega \equiv [a^+(-\omega) - \hat{a}(\omega)]$

$[\hat{X}_{in}(\omega), \hat{Y}_{in}(\omega)]$

$$\Rightarrow \left|\begin{array}{l} i\omega \hat{Y}(\omega) = -\kappa/2\, Y(\omega) + \sqrt{\kappa}\, Y_{in}(\omega) - 2i\frac{\omega}{L}\bar{\alpha}\, \hat{q}(\omega) \\ i\omega X(\omega) = -\kappa/2\, X(\omega) + \sqrt{\kappa}\, X_{in}(\omega) \\ i\omega\, \hat{q} = -\Gamma_m/2\, \hat{q} + \sqrt{\Gamma_m}\, \hat{q}_{in}(\omega) + 2\frac{i\omega}{L}\bar{\alpha}\, x_{zpf} \hat{X}_{in} + i\Omega_m \hat{q} \end{array}\right.$$

optical quadratures; mechanical susceptibility.

(x) Note: $m_{eff}(\Omega_m^2 - \Omega^2 - i\Gamma_m\Omega)\, \delta\hat{q}(\omega) = -\hbar\, i\omega/L\, x_{zpf} \cdot \hat{X}_{in}$ 2nd order equation.

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intracavity fields:

$$\hat{Y}(\omega) = \frac{1}{i\omega + \kappa/2}\left[Y_{in}\sqrt{\kappa} - \frac{2i\omega_c}{L}\bar{\alpha}\hat{q}\right]$$

$$\hat{X}(\omega) = \frac{1}{i\omega + \kappa/2}\left[\cancel{\hat{X}} + \sqrt{\kappa}\hat{X}_{in}\right]$$

$$\hat{q}(\omega) = \frac{1}{(i\omega - \Gamma_m/2)}\left[\sqrt{\Gamma_m}\,\hat{q}_{in} + \underbrace{\frac{2i\omega_c}{L}\bar{\alpha}\, x_{ZPF}\hat{X}_{in}}_{G\cdot\bar{\alpha} \text{ or } \hat{F}_{qba}}\right]$$

$(i\omega - \Gamma_m/2)$ → Ω_m)

$\hat{F}_{qba} = \frac{i\omega_c}{L}\bar{\alpha}\cdot\hat{X}_{in}$ → yields immediately $S_{\hat{q}\hat{q}}(\omega)$ → 1 Pole $\omega = \Omega_m$

susceptibility:

$$\chi_m = \frac{1}{i(\omega - \Omega_m) - \Gamma_m/2} \quad \text{(more precisely 2nd order)}$$

$$\chi_{cav} = \frac{1}{i\omega + \kappa/2} \quad (\text{on resonance } \Delta = (\omega_L - \omega_c) = 0)$$

Next: solve equation for $\hat{q}(\omega)$: and output fields:

$\cancel{\hat{q}(\omega)} =$

$$\hat{Y}_{out} = \hat{Y}_{in} - \sqrt{\kappa}\,Y(\omega) \quad \text{(Phase quadr.)}$$

$\Rightarrow \cancel{\hat{X}_{out} = \hat{X}_{in}}$

$\cancel{\hat{Y}_{out}} =$

Quantum Backaction term: using the spectrum:

$$2\pi\delta(\omega+\omega')\,S_{\hat{q}\hat{q}} = \langle \hat{q}^{+}(\omega)\,\hat{q}^{+}(\omega')\rangle \quad \text{i.e.} \quad S_{\hat{q}\hat{q}} = \int \langle q^{+}(\omega)\,q(\omega')\rangle\, d\omega'$$

symm:

$$S_{\hat{q}\hat{q}} = \int_{-\infty}^{\infty} e^{-i\omega T}\langle \hat{q}^{+}(t+T)\,\hat{q}(t)\rangle$$

classically: $S_{qq} = (q(\omega)\cdot q^{*}(\omega))$ spectrum.

$$S_{\hat{q}\hat{q}} = \int_{-\omega}^{\infty} d\omega \,\langle a^{+}(\omega)\, a(\omega)\rangle$$

neglect $k \gg \omega$

output spectrum:

$$\hat{Y}_{out} = \hat{Y}_{in} - \sqrt{\kappa}\left(\frac{1}{i\omega + \kappa/2}\right)\left(\hat{Y}_{in}\sqrt{\kappa} - \frac{2i\omega_c}{L}\bar{\alpha}\,\hat{q}(\omega)\right)$$

Note: $\left(\sqrt{\kappa}\left(\frac{2i\omega_c}{L}\bar{\alpha}\right)\frac{1}{i\omega + \kappa/2} \equiv \sqrt{\Gamma_m}\right)$ measurement rate

$$\Rightarrow \hat{Y}_{out}(\omega) = \hat{Y}_{in}\underbrace{\left(\frac{i\omega - \kappa/2}{i\omega + \kappa/2}\right)}_{(\text{transmission coefficient})} - \frac{2i\omega}{L}\bar{\alpha}\,\hat{q}(\omega)\sqrt{\kappa} \times \left(\frac{1}{i\omega + \kappa/2}\right)$$

with $\langle \hat{Y}_{in}(\omega)\,\hat{Y}^{+}_{in}(\omega')\rangle = \delta(\omega + \omega')$

$$\Leftrightarrow S_{\hat{Y}_{out}\hat{Y}_{out}}(\omega) = \int_{-\infty}^{\infty}\underbrace{\langle \hat{Y}^{+}_{in}(\omega)\,\hat{Y}_{in}(-\omega)\rangle}_{\delta(\omega-\omega)}\left|\frac{i\omega - \kappa/2}{i\omega + \kappa/2}\right|^{2} + \left|\frac{2i\omega}{L}\bar{\alpha}\right|^{2}\cdot\langle \hat{q}^{+}_{in}(\omega)\,q_{in}(\omega)\rangle\sqrt{\kappa}^{2} \times \left|\frac{1}{i\omega + \kappa/2}\right|^{2} d\omega$$

use the fact that

$$\langle Y^{+}_{in}(\omega)\,Y_{in}(-\omega)\rangle = \delta(\omega - \omega)\cdot 1 \quad \text{quantum noise}$$

$$\langle q^{+}_{in}(\omega)\,q_{in}(-\omega)\rangle = \delta(\omega - \omega)\,\bar{n}_{TH}\cdot(x_{ZPF})^{2}$$

$$\Rightarrow \boxed{S_{\hat{Y}_{out}\hat{Y}_{out}}(\omega) = 1\cdot\left|\frac{i\omega - \kappa/2}{i\omega + \kappa/2}\right|^{2} + \left|\frac{2i\omega_c}{L}\bar{\alpha}\right|^{2}\cdot S_{\hat{q}\hat{q}}(\omega)\sqrt{\kappa}^{-2}}$$

Phase quadrature $\times\left(\frac{1}{i\omega + \kappa/2}\right)^{2}$

For case where $\cancel{\omega_m} = 0$, $\omega \ll \kappa/2$:

$$S_{\hat{Y}_{out}\hat{Y}_{out}} \cong \underbrace{1}_{S_{\hat{x}_{in}\hat{x}_{in}}(\omega)\text{: shot noise}} + \left|\frac{\omega_c}{L}\bar{\alpha}\,2\right|^{2} S_{\hat{q}\hat{q}}(\omega)\cdot\underbrace{\left|\frac{i\omega - \kappa/2}{i\omega + \kappa/2}\right|^{-2}}_{=1}\cdot\,\kappa^{-1}\cdot\left(\frac{1}{i\omega + \kappa/2}\right)^{+2}$$

note that imprecision is given by SNR of unity.

$$\hat{S}_{\hat{x}_{in}\hat{x}_{in}} \overset{!}{=} S^{imp}_{\hat{Y}_{out}\hat{Y}_{out}} = \left|\frac{\omega_c}{L}\bar{\alpha}^{2}\right|^{2} S_{qq}(\omega)\cdot\kappa^{+1} \Rightarrow \boxed{S^{imp}_{\hat{q}\hat{q}} = \left|\frac{L}{\omega\,\bar{\alpha}}\right|^{2}\cdot S_{\hat{x}_{in}\hat{x}_{in}}\left[\kappa^{+1}/|i\omega + \kappa/2|^{2}\right]^{-1}}$$

should be $\left|\frac{\kappa}{i\omega + \kappa/2}\right|^{2}$; problem (→ Schliesser)

Quantum backaction term:

$$S^{imp}_{\hat{q}\hat{q}} = \left|\frac{L}{\omega_c \bar{\alpha}}\right|^2 \kappa^{-1} \cdot S_{\hat{x}_{in}\hat{x}_{in}}$$ (imprecision) $\times |i\omega + \kappa/2|^2$ → decreases outside of cavity bandwidth

and output $\underbrace{S_{\hat{x}_{in}\hat{x}_{in}}(\omega)}$: shot noise

$$S_{\hat{y}_{out}\hat{y}_{out}}(\omega) = 1 + \left|\frac{\omega_c}{L}\bar{\alpha}\right|^2 \kappa \cdot S_{\hat{q}\hat{q}} \cdot \underbrace{\left|\frac{1}{i\omega + \kappa/2}\right|^2}$$

implies transduction becomes inefficient

Measurement rate: $\sqrt{\Gamma_m} = \left|\frac{i\omega_c}{L}\frac{1}{\sqrt{\kappa}}\bar{\alpha}\right|$

Consider $S_{\hat{q}\hat{q}}(\omega)$ next the spectrum of H.O:

$$\boxed{\hat{q}(\omega) = \chi_m(\omega)\left(\sqrt{\Gamma_m}\,\hat{q}_{in} + \frac{2i\omega_c}{L}\bar{\alpha}\,\hat{x}\right)} \times x_{ZPF}$$

Thus the spectrum is

$$S_{\hat{q}\hat{q}} = \int_{-\infty}^{+\infty} \langle \hat{q}^{+}(\omega)\hat{q}(\omega)\rangle d\omega = \underbrace{|\chi_m(\omega)|^2\left|\frac{\omega_c 2}{L}\right|^2 \bar{\alpha}^2}_{\text{QBA - term}} \cdot \underbrace{S_{\hat{x}\hat{x}}}_{\text{intracavity field quadrature}} + |\chi_m(\omega)|^2 \cdot \underbrace{S_{\hat{q}_{in}\hat{q}_{in}}|\sqrt{\Gamma_m}|^2}_{(\bar{n}_{th}+1)\,x_{ZPF}^2}$$

Thermal noise

note that $\hat{x}_{out} = \hat{x}_{in} - \sqrt{\kappa}\,\hat{x}$
$i\omega\,\hat{x} = -\kappa/2\,\hat{x} + \sqrt{\kappa}\,\hat{x}_{in}$ → $S_{\hat{x}\hat{x}} = S_{\hat{x}_{in}\hat{x}_{in}}\left|\frac{\sqrt{\kappa}}{i\omega - \kappa/2}\right|^2 \simeq S_{\hat{x}_{in}\hat{x}_{in}}\left|\frac{1}{\sqrt{\kappa}/2}\right|^2$

hence:

$$S_{\hat{q}\hat{q}}(\omega) = \underbrace{|\chi_m(\omega)|^2}_{\text{susceptibility of oscillator.}} \cdot \left(\underbrace{\left|\frac{\omega_c 2}{L}\right|^2 \cdot \bar{\alpha}^2 \cdot S_{\hat{x}_{in}\hat{x}_{in}}(\omega)\left(\frac{2}{\sqrt{\kappa}}\right)^2}_{\text{quantum backaction force}} + \underbrace{|\chi_m(\omega)|^2(\sqrt{\Gamma_m})^2 S_{\hat{q}_{in}\hat{q}_{in}}}_{\text{thermal noise}}\right)$$

→ AM quadrature cavity.

Thus: $\hat{F}_{BA} = \hbar\left(\frac{\omega_c}{L}2.\bar{\alpha}\,.\underbrace{\hat{x}}_{\text{intracavity field}}\right)$ is Radiation pressure quantum noise (as in last lecture) ie $(F_{\text{Rad Press}} = \hbar G = \hbar \frac{\omega}{L})$

Next: squeezing of the cavity field.

Note in conclusion that

$$\hat{S}_{FF}(\omega) = \left|\left(\frac{\omega_c}{L}2\right)\bar{\alpha}\right|^2 \cdot \frac{\overbrace{S_{\hat{x}_{in}\hat{x}_{in}}(\omega)}^{\text{AM - quadrature}}}{\kappa/4}$$ is the backaction Force sp. density

$$S^{imp}_{xx}(\omega) = \left|\frac{L}{\omega\bar{\alpha}}\right|^2 \underbrace{S_{\hat{y}_{in}\hat{y}_{in}}}_{\text{Phase quadrature}} \cdot \kappa/4$$

and

$$\boxed{S_{\hat{F}\hat{F}}\, S^{imp}_{\hat{x}\hat{x}} = \left(\frac{\hbar^2}{4}\right)\underbrace{S_{\hat{y}_{in}\hat{y}_{in}} \cdot S_{\hat{x}_{in}\hat{x}_{in}}}_{\text{vacuum noise} \to 1}}$$ Heisenberg uncertainty.

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— Squeezing —

(later Squeezing effect: consider generalized quadrature for this)

$$X_{out}(\theta) = (\hat{a}e^{-i\theta} + \hat{a}^+ e^{+i\theta})$$

First note that QBA couples phase and AM quadrature.

$$\hat{Y}_{out} = -\left(\frac{\kappa/2+i\omega}{\kappa/2-i\omega}\right)\hat{Y}_{in} - \frac{2i\omega\bar{\alpha}}{L}\underbrace{\left(\frac{\sqrt{\kappa}}{i\omega+\kappa/2}\right)}_{\approx \frac{2}{\sqrt{\kappa}}}\hat{q}(\omega)$$

and $\hat{q}(\omega) = \chi_m(\omega)\left[\sqrt{\Gamma_m}\,\hat{q}_{in}(\omega) + \underbrace{\frac{2i\omega}{L}\bar{\alpha}\,\hat{X}}_{\text{qba driven motion}}\right] x_{ZPF}^2$

or $\delta q_{qba}[\omega] = \chi_m[\omega]\overbrace{\left(\frac{2i\omega}{L}x_{ZPF}\right)}^{\sqrt{C}}\sqrt{\kappa\Gamma_m} \times \sqrt{\kappa}\hat{X}\cdot\sqrt{\Gamma_m}$

$$\Leftrightarrow \hat{Y}_{out} = -\underbrace{\left(\frac{\kappa/2+i\omega}{\kappa/2-i\omega}\right)}_{-1}\hat{Y}_{in} - \frac{2i\omega\bar{\alpha}}{L}\left(\frac{2}{\sqrt{\kappa}}\right)\cdot\left[\chi_m(\omega)\sqrt{\Gamma_m}\,q_{in}(\omega) + \chi(\omega)\frac{2i\omega}{L}\bar{\alpha}\,\hat{X}\right] x_{ZPF}^2$$

$\kappa/2 \gg \omega$ Bad cavity limit

intracavity amp. quadrature $\rightarrow \hat{X} \cong X_{in}\cdot\frac{\sqrt{\kappa}}{i\omega-\kappa/2} \simeq X_{in}(\sqrt{\kappa})^{-1}2$ ← input quadr. noise [$\omega \ll \kappa$] bad cavity limit

$$\hat{Y}_{out} = \hat{Y}_{in} - \frac{2i\omega\bar{\alpha}}{L}\frac{2}{\sqrt{\kappa}}\left[\chi_m\sqrt{\Gamma_m}\,\hat{q}_{in}(\omega) + \frac{2i\omega}{L}\bar{\alpha}\,\hat{X}_{in}\left(\frac{2}{\sqrt{\kappa}}\right)(\chi_m)\right] x_{ZPF}$$

(intracavity)

$$\Rightarrow \hat{Y}_{out} = \hat{Y}_{in} - \frac{2i\omega\bar{\alpha}}{L}\frac{2}{\sqrt{\kappa}}\chi_m(\omega)\sqrt{\Gamma_m}\,q_{in}(\omega)\bar{\alpha} + \left(\frac{2i\omega}{L}\right)^2\frac{4}{\kappa}\bar{\alpha}^2\hat{X}_{in}\cdot\chi(\omega)\,x_{ZPF}^2$$

$C \equiv \underbrace{\left(\frac{\omega}{L}\right)^2}_{G}\bar{\alpha}^2\,4/\kappa\Gamma_m \cdot x_{ZPF}^2$ cooperativity ~~(intracavity, but dimensional)~~ $\hat{=} \left(\frac{4\omega_c^2}{L^2}x_{ZPF}^2\right)\frac{|\alpha|^2}{\kappa} \hat{=} C\cdot\Gamma_m$

$$\hat{Y}_{out} = \hat{Y}_{in} - 2\sqrt{C}\,(\chi_m)\sqrt{\Gamma_m}\,q_{in}^{(\omega)} + 4C\cdot\hat{X}_{in}\cdot\chi_m(\omega)\,\Gamma_m$$

$$\Leftrightarrow \boxed{\hat{Y}_{out} = \hat{Y}_{in} - \sqrt{\Gamma_m}\cdot\sqrt{C}\,2\,\chi_m(\omega)\left(\frac{\hat{q}_{in}(\omega)}{x_{ZPF}}\right) + \underbrace{4C\,\chi_m(\omega)\,\hat{X}_{in}(\omega)\,\Gamma_m}_{\text{QBA driven motion}}}$$

$\Rightarrow$ The phase and amplitude quadrature are coupled!

$$\boxed{C \equiv 4\frac{\left(\frac{\omega}{L}x_{ZPF}\right)^2}{\kappa\cdot\Gamma_m}\bar{\alpha}^2 = \frac{4g_0^2}{\kappa\Gamma_m}}$$ COOPERATIVITY Definition

spectrum

$\langle\hat{Y}_{out}\hat{Y}_{out}(\omega)\rangle + \langle\hat{Y}_{out}(\omega)\hat{Y}_{out}(\omega)\rangle = S_{Y_{out}Y_{out}}(\omega) = S_{Y_{in}Y_{in}}(\omega) + \Gamma_m 4C|\chi_m|^2 S_{q_{in}q_{in}}(\omega) + 16C^2|\chi_m(\omega)|^2\Gamma_m\cdot S_{X_{in}X_{in}}$

$\hat{Y}_{out}(-\omega)$ hence output phase quadrature

RMP Clerk symmetrized sideband $\langle\hat{Y}_{out}\hat{Y}_{out}(T) + Y_{out}(T)Y_{out}(0)\rangle$

$S_{q_{in}q_{in}}(\omega)/x_{ZPF}^2$ spectra noise

$$\Rightarrow S_{Y_{out}Y_{out}}(\omega) = \underbrace{1}_{\text{shot noise}} + \underbrace{8\Gamma_m C|\chi_m|^2 (\bar{n}_{th}+1)}_{\text{Thermal noise symmetrized}} + \underbrace{16C^2|\chi_m(\omega)|^2\Gamma_m}_{\text{QBA noise}}\,\underbrace{S_{X_{in}X_{in}}(\omega)}_{1}$$

output phase quadrature

$\rightarrow$ shot noise limits measurement & QBA

(check)

$$S_{Y_{out}Y_{out}} = 1 + 16\Gamma_m C|\chi_m|^2\left(n_{th} + \frac{1}{2} + n_{QBA}\right)$$

$\hat{=}$? (check!)

$\Rightarrow$ No correlations.

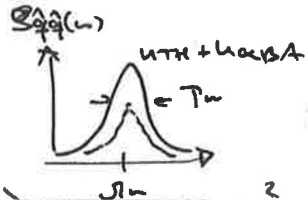
Optimum C.

$\boxed{n_{QBA} = C}$ Cooperativity. Intuitive!

redone ↓

⇒ The Standard Quantum Limit (SQL)

$$C_{eff}^{opt} = \frac{1}{\Gamma_m \chi(\omega)}$$ optimum choice of C



evident when considering: "output referred back to INPUT!"

The displacement sensitivity:

$$\boxed{S_{\hat{q}\hat{q}} = \frac{S_{\text{y output}}}{8|C|^2 \Gamma_m / x_{ZPF}^2} - \underbrace{(n_{TH}+\tfrac{1}{2})|\chi_m|^2}_{\text{mech. signal}} + \underbrace{\frac{x_{ZPF}^2}{8C\Gamma_m}}_{\substack{\text{shot noise floor}\\ S_{xx}^{imp}}} + \underbrace{(2 x_{ZPF}^2 |\chi_m|^2 C)}_{\substack{\text{QBA}\\ S_{xx}^{qba}}}}$$

* output spectrum of $S_{\text{y output}}$ converted to $S_{\hat{q}\hat{q}}$

$$S_{\hat{q}\hat{q}}(\omega) = \underbrace{x_{ZPF}^2}_{\text{apparent noise}} \underbrace{(n_{TH}+\tfrac{1}{2})}_{\text{thermal noise}}|\chi_m|^2 + \underbrace{\frac{x_{ZPF}^2}{16\Gamma_m C}}_{\text{imprecision noise floor}} + \underbrace{2|\chi_m|^2 C_{eff}\, x_{ZPF}^2}_{\text{QBA}}$$

both terms are equal $n_{imp} \, \& \, n_{qba}$

→ optimum is the SQL (on resonance): $C_{eff}^{opt} = \frac{1}{\Gamma_m \chi(\omega)}$

[...] Find SQL

i.e. $\frac{x_{ZPF}^2}{8C\Gamma_m} \triangleq 2\Gamma|\chi|^2 \frac{C}{x_{ZPF}^2} \rightarrow C = \frac{1}{\Gamma_m \chi}$

cooperativity?

?

$$\boxed{S_{\hat{q}\hat{q}}^{SQL} = \frac{x_{ZPF}^2}{8C\Gamma_m} \triangleq |\chi(\omega)| \cdot x_{ZPF}^2}$$ SQL of mech oscillator detection

$\triangleq 2/\Gamma_m$ on resonance

SQL: $C=1$, $N_{added} = 1/2$

Note minimum added noise corresponds to → corresponds to zero point fluct. spectral density.

$$\left[\begin{array}{l} 1/2 \text{ quanta total noise} \end{array} \left\{ \begin{array}{l} n_{BA} = 1/4 \text{ quanta of backaction noise} \\ n_{imp} = 1/4 \text{ quanta} \end{array} \right.\right]$$

$\sqrt{n_{qba} \, n_{imp}} = \frac{1}{4}$ → Heisenberg uncertainty.

→ $n_{tot} = n_{imp} + n_{BA} = 1/2$

⇒ ZPF noise added.

SQL from measurement:

$$\boxed{S_{FF}(\omega) = \frac{(\hbar m \Omega)}{|\chi(\omega)|^2} S_{qq}(\omega)}$$ since: $q(\omega) = \chi(\omega) \cdot F(\omega)$.

$\frac{1}{i(\omega-\Omega)-\Gamma_m/2} \cdot m^{-1}$

In here: $(S_{\hat{F}\hat{F}} = 2\Gamma_m \cdot m \hbar \Omega \, S_{\hat{q}_{in}\hat{q}_{in}})$ thermal noise in Bowen def.)

nomenclature....

But note that:

$$S_{\hat{q}\hat{q}}(\omega) = x_{ZPF}^2 \left\{ \bar{n} \frac{\Gamma_m}{(\omega+\Omega_m)^2 + \Gamma_m^2/4} + (\bar{n}+1) \frac{\Gamma_m}{(\omega-\Omega_m)^2 + \Gamma_m^2/4} \right\}$$ i.e. an asymmetric noise spectrum.

$$\boxed{\text{Bowen: } \quad \chi_m \equiv \frac{\Omega}{(\Omega_m^2-\omega^2) - i\omega\Gamma_m}}$$ (No mass!!)

$$S_{SQL}^{x_p} = \hbar \Big(\frac{x_{ZPF}^2}{\hbar} \underbrace{\tilde{\chi}[\omega]}_{\chi_m[\omega]} \Big) \underset{\omega=\Omega_m}{\triangleq} x_{ZPF}^2 \left(\frac{1}{\Gamma_m/2} \right) = \frac{2 x_{ZPF}^2}{\Gamma_m}$$ on resonance.

* Repeat analysis!

OM-
Squeezing

Arbitrary quadrature: $\boxed{X^{\theta}_{out} = \underbrace{\hat{X}_{out}}_{\text{Phase}} \cos\theta + \hat{Y}_{out}\sin\theta}$ arbitrary quadrature

$\theta = \pi/2 \Rightarrow$ ~~AM~~ Phase

$\theta = 0 \rightarrow$ ~~Phase~~ AM

$$\hat{Y}_{out} = \left(\frac{-\frac{\kappa}{2} + i\omega}{-\kappa/2 - i\omega}\right)\hat{Y}_{in} + 2T\chi(\omega)\cdot\left[\sqrt{C}\,\underbrace{\hat{q}_{in}}_{\substack{x_{ZPF} \\ \text{thermal}}} - \underbrace{2C\hat{X}_{in}}_{\substack{\text{QBA} \\ \text{driven mode oscillator}}}\right]$$

$$\hat{X}_{out} = \left(\frac{-\kappa/2 + i\omega}{-\kappa/2 - i\omega}\right)\hat{X}_{in} \simeq \hat{X}_{in}$$

$\omega \ll \kappa$ term is unity

Quantum correlation term between quadratures $\langle \hat{X}_{out}\hat{Y}_{out}\rangle$

Thus arbitrary quadrature

$$X^{\theta}_{out} = \left[\left(\frac{-\frac{\kappa}{2} + i\omega}{-\kappa/2 + i\omega}\right)\cos\theta + \sin\theta\,.\underbrace{2C\chi 2T_m}_{\substack{\text{QBA} \\ \text{Quantum correlation term}}}\right]\hat{X}_{in}$$

$$+ \left[\left(\frac{-\kappa/2 + i\omega}{-\kappa/2 + i\omega}\right)\sin\theta\,\hat{Y}_{in} + \underbrace{2T\chi(\omega)2\sqrt{C}}_{\text{thermal noise}}\underbrace{\hat{q}_{in}}_{x_{ZPF}}\right]$$

$$\int_{-\infty}^{\infty} \langle \hat{X}^{\dagger}_{out}(\omega)\hat{X}_{out}(\omega)\rangle\, d\omega = S_{\hat{X}_{out}\hat{X}_{out}}(\omega)$$

$$\underbrace{S_{outxout}}_{\text{symmetric}} = \tfrac{1}{2}(\cos^2\theta + \sin^2\theta)\left(\frac{-\kappa/2 + i\omega}{-\kappa/2 + i\omega}\right)^2 + 16T^2|\chi(\omega)|^2 C\left(\bar{n}_{th} + \tfrac{1}{2} + \overset{\text{QBA}}{C_{...}}\right)\sin^2\theta$$

$$+ 2T.C\,(\underbrace{\chi(\omega) + \chi^{*}(\omega)}_{2\mathrm{Re}\{\chi_m\}})\sin(2\theta)$$

quantum correlation from AM–PM coupling.

correlation
Sqp

→ squeezing depends on real part of χ_m.

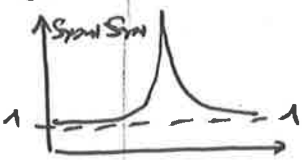
→ and cancel shot noise ($\frac{1}{2}$) term.

key point: sin(2θ) can be negative

$\sin(2\theta) = -1 \rightarrow \theta \simeq \pi/4$

→ cancellation of QBA possible

$\theta = \pi/2$



condition: $\left| \bar{n}_{th} + |C| + \tfrac{1}{2} < \dfrac{\Omega^2 - \omega^2}{2T\Omega\tan\theta} \right|$ condition

resonant
off-resonant.

For $\theta = \pi/4$ we have:

$$S_{outout}(\omega) = 1 - \frac{4}{\Omega_m}\left(1 - \bar{n}/Q_m\right)\left(\underbrace{\frac{4g_0^2\bar{n}}{\kappa}}_{\simeq T_{meas}}\right)$$

$\bar{n} < Q_m$
condition for squeezing.

$\theta = \pi/4$

$\theta = -\pi/4$

quantum noise reduction

Can be used to measure force.