Squeezed States and Sub-Poissonian Photon Statistics

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Outline

- Introduction
- Quadratures (Q1)
- Photon statistics (Q2)
- Detection with LO (Q3 and Q4)
- Classical vs Quantum squeezing
- Conclusion

Introduction

- Squeezing is reducing quadrature fluctuations below vacuum noise
- Sub-Poissonian statistics have less photon-number fluctuations compared to Poisson statistics
- These two phenomena have no direct relation
- Phase-sensitive interference detection of squeezed states automatically generates sub-Poissonian photon statistics

Quadratures and squeezing

Q1. What are the quadratures of the field $\hat{E}_1$ and $\hat{E}_2$ and how are they related to the positive- and negative-frequency parts of the field $\hat{E}^{(+)}$ and $\hat{E}^{(-)}$?

Quadratures

A real field amplitude $\hat{E}$:

$$\hat{E}_1 \propto \hat{E}^{(+)} + \hat{E}^{(-)}, \qquad \hat{E}_2 \propto -i\left(\hat{E}^{(+)} - \hat{E}^{(-)}\right)$$

Commutators

$$[\hat{E}^{(+)}, \hat{E}^{(-)}] = C > 0$$

$$[\hat{E}_1, \hat{E}_2] = 2iC$$

Variances

$$\langle(\Delta\hat{E}_1)^2\rangle = C + \langle: (\Delta\hat{E}_1)^2 :\rangle$$

$$\langle(\Delta\hat{E}_2)^2\rangle = C + \langle: (\Delta\hat{E}_2)^2 :\rangle$$

Squeezed state if one of $\langle: (\Delta\hat{E}_i)^2 :\rangle < 0$

Photon statistics

Q2. What is poissonian, super-poissonian, and sub-poissonian light? Why is this distinction important?

Variance of the photon number operator

$$\langle(\Delta\hat{n})^2\rangle = \langle n\rangle + \langle : (\Delta\hat{n})^2 : \rangle$$

Sub-Poissonian fluctuations

$$\langle : (\Delta\hat{n})^2 : \rangle < 0$$

Q parameter: measurement of the departure from Poisson statistics

$$Q \equiv \frac{[\langle(\Delta n)^2\rangle - \langle n\rangle]}{\langle\hat{n}\rangle}$$

- Poissonian: $Q = 0$ (coherent state)
- Sub-Poissonian: $Q < 0$ (non-classical)
- Super-Poissonian: $Q > 0$ (thermal light)

Squeezed state detection

Phase-sensitive interference experiment:

- Local oscillator is a pure coherent reference beam
- Interference at beam splitter with squeezed field
- Measurement of intensity fluctuations (ΔI) proportional to $\hat{E}_1$ or $\hat{E}_2$ (homodyne-like)

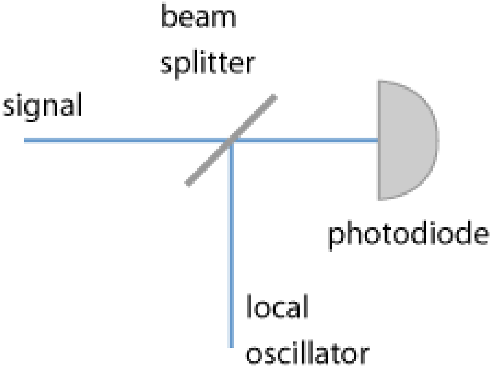

Figure: Setup for optical heterodyne detection

How interference maps squeezing

Q3. What is the relation between photon statistics and squeezing?

Q in a strong coherent field

$$Q \approx \begin{cases} \alpha T\langle : (\Delta \hat{E}_1)^2 : \rangle, & \text{if } \theta = 0, \\ \alpha T\langle : (\Delta \hat{E}_2)^2 : \rangle, & \text{if } \theta = \pi/2, \end{cases}$$

where α characterises collection and quantum efficiency; T is the time interval (short).

- Changing phase angle (θ) of LO measures ΔI in either $\hat{E}_1$ or $\hat{E}_2$
- $Q < 0$ always for one of $\hat{E}_1$ or $\hat{E}_2$
- Measurement method generates sub-Poissonian statistics

How interference maps squeezing

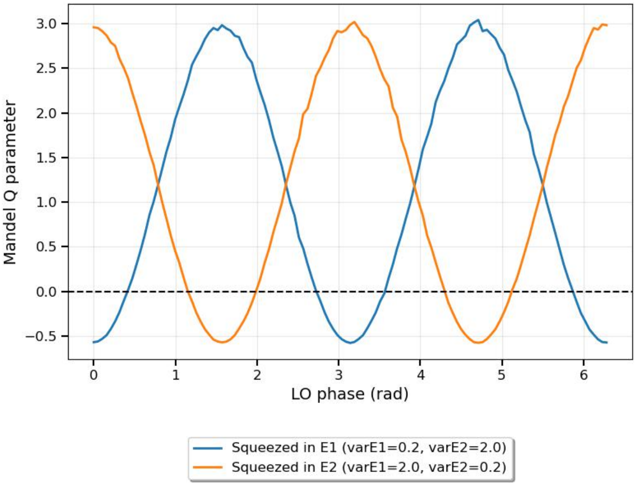

Figure: Q vs LO phase for both squeezed quadratures

Displacement operator

Q4. How does a displacement operator influence the variances of the field quadratures?

LO is a coherent state

$$\left|\{v\}\right\rangle = \hat{D}(\{v\})\left|0\right\rangle$$

where $\{v\}$ is a multimode set of complex amplitudes, $\hat{D}(\{v\})$ is the displacement operator.

$$\hat{E}^{(+)}\left|v\right\rangle = \mathcal{E}\left|v\right\rangle$$

where $\mathcal{E} = |\mathcal{E}|e^{i\theta}, |\mathcal{E}| >> 1$ (coherent).

- $D\{v\}$ shifts the mean field $\{v\} \to \{v\} + \mathcal{E}$
- $D\{v\}$ leaves the variances of quadrates unchanged: squeezing survives displacement
- Interference with strong LO ($|\mathcal{E}| >> 1$) converts small $\langle(\Delta\hat{E}_{1,2})^2\rangle$ into ΔI (measurable)

Classical vs quantum squeezing

Classical

- Either or both dispersions can be below C
- Photoelectric counting fluctuations always super-Poissonian before and after interference with LO

Quantum

- Quadrature variance reduced below vacuum limit $\langle : (\Delta \hat{E}_i)^2 : \rangle < 0$
- Detection shows sub-Poissonian photon statistics
- No classical analogue: diagonal coherent-state representation cannot be non-negative

Conclusions

- Squeezing and sub-Poissonian statistics have no direct connection
- Detection method forces them to occur together
- Phase of LO can choose between the quadratures
- Mixing with a strong coherent LO displaces the signal but quadrature variance remains
- There is no classical analogue to sub-Poissonian photon statistics
- Detection matters: how you measure changes what nonclassical features appear