

Broad topic	Lecture title
Basic principles of NPP	Introduction / Review of nuclear physics
	Interaction of neutrons with matter
	Nuclear fission
	Fundamentals of nuclear reactors
	LWR plants
Modeling the beast	The diffusion of neutrons - Part 1
	The diffusion of neutrons - Part 2
	Neutron moderation without absorption
	Neutron moderation with absorption
	Multigroup theory
	Element of lattice physics
	<b>Neutron kinetics</b>
	Depletion
Reactor Concepts Zoo	Advanced LWR technology
	Breeding and LFR
	AGR, HTGR
	Channels, MSR and thorium fuel
Review session	

- Reactor Kinetics
- Kinetics without Delayed Neutrons
- Kinetics with Delayed Neutrons
- Point Kinetics Equations
- Inhour Equation
- Decay Heat
- Reactivity Variations
  - Reactivity Feedbacks
  - Reactivity Coefficients and Safety



- In general, one seeks to determine  $\phi(\vec{r}, E, t)$ 
  - Time-dependent diffusion equation needs to be solved numerically

$$P(t) = \int_E dE \int_V n(\vec{r}, t) d\vec{r}$$

- For the global behaviour, a simplification can be made
  - “Point kinetics” equations for the total neutron population
  - Does not describe spatial effects in large complex systems, but very useful...
- Two cases may be considered for the time-dependent behaviour
  - Without delayed neutrons (hypothetical)
  - Real situation (with delayed neutrons)
- One particular case, can be solved analytically
 

Step change in  $k_{\text{eff}}$

→ Leads to Reactivity Equation (Inhour Equation)

- For the neutron population:  $\frac{dP}{dt} = \bar{\nu}F(t) - (A(t) + L(t))$

- Using:  $k_{\text{eff}}(t) = \frac{\bar{\nu}F(t)}{A(t) + L(t)} \implies \frac{dP}{dt} = (k_{\text{eff}}(t) - 1) \cdot (A(t) + L(t))$

Depends on cross sections, reactor size, ...

- $[A, L] \propto P \implies A(t) + L(t) = cst \cdot P(t) \simeq \frac{P(t)}{l}$  Dimension of time

- Prompt Kinetics Equation  $\frac{dP}{dt} = \frac{k_{\text{eff}}(t) - 1}{l} P(t)$

- For a constant  $k_{\text{eff}}$  :

$$P(t) = P(0) \exp\left(\frac{k_{\text{eff}} - 1}{l} t\right) \quad \begin{array}{l} \rightarrow \text{if } k_{\text{eff}} > 1, P \nearrow \text{ (supercritical system)} \\ \rightarrow \text{if } k_{\text{eff}} < 1, P \searrow \text{ (subcritical system)} \end{array}$$

- For an hypothetical passive medium with same cross-sections but  $k_{\text{eff}} = 0$  (e.g.  $\nu = 0...$ ),  $l$  is same and

$$\frac{dP}{dt} = \frac{k_{\text{eff}}(t) - 1}{l} P(t) = -\frac{1}{l} P(t) \implies P(t) = P(0) \exp\left(-\frac{t}{l}\right)$$

- Result is analogous to the law of radioactive decay :  $1/l$  is like  $\lambda$  , i.e.  $l$  is like  $T...$

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- Result is analogous to the law of radioactive decay :  $1/l$  is like  $\lambda$  , i.e.  $l$  is like  $T \dots$
- Thus,  $l$  is *neutron lifetime*  
Measure of time taken for “disappearance” of the n’s ( $P \searrow$ ), in face of absorption, leakage...
- Like  $k_{\text{eff}}$ ,  $l$  may be calculated on the basis of different theories (diffusion, 1-group, multigroup, transport,...)  
→ Consider particular case: bare homogeneous reactor, analysed via 1-group diffusion theory

- One has:  $l = \frac{P(t)}{A(t) + L(t)}$  with  $A(t) = \int_V \Sigma_a \phi(\vec{r}, t) dV = \Sigma_a v \int_V n(\mathbf{r}, t) dV = \Sigma_a v P(t)$
- Leakage  $\sim$  supplementary absorptions corresponding to:  $\Sigma'_a \simeq DB^2$
- Thus,  $L(t) = DB^2 v P(t) \implies A(t) + L(t) = (\Sigma_a + DB^2) v P(t)$
- i.e.  $l = \frac{P(t)}{A(t) + L(t)} = \frac{1}{v(\Sigma_a + DB^2)} = \frac{1}{v\Sigma_a(1 + B^2 L^2)}$  (independent of P)
- For an infinite system:  $l = \frac{1}{v\Sigma_a} = \frac{\lambda_a}{v} = t_d$  (thermal diff. time; slowing-down time negligible...)
- One may write:  $l = \frac{1}{v\Sigma_{am}} \frac{\Sigma_{am}}{(\Sigma_a)_{tot}} = \frac{1}{v\Sigma_{am}} (1 - f) = (t_d)_m (1 - f)$

- For the reactor without delayed neutrons,  $P(t) = P(0) \exp\left(\frac{k_{eff} - 1}{l} t\right)$

$$P(t) = P(0) \exp\left(\frac{t}{T}\right) \quad \text{with} \quad T = \frac{l}{k_{eff} - 1}$$

- $t_d$  for different moderators:
 

$\text{H}_2\text{O}$	$2,1 \cdot 10^{-4}$ sec.	thus typically:	$l = (t_d)_m (1 - f)$
$\text{D}_2\text{O}$	$1,4 \cdot 10^{-1}$ sec.		$\simeq 10^{-2}$ to $10^{-4}$ s
$\text{Be}$	$3,9 \cdot 10^{-3}$ sec.		
Graphite	$1,7 \cdot 10^{-2}$ sec.		

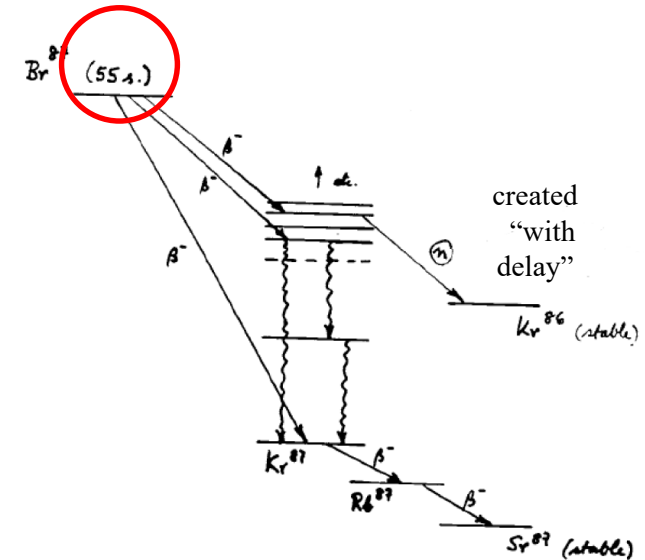
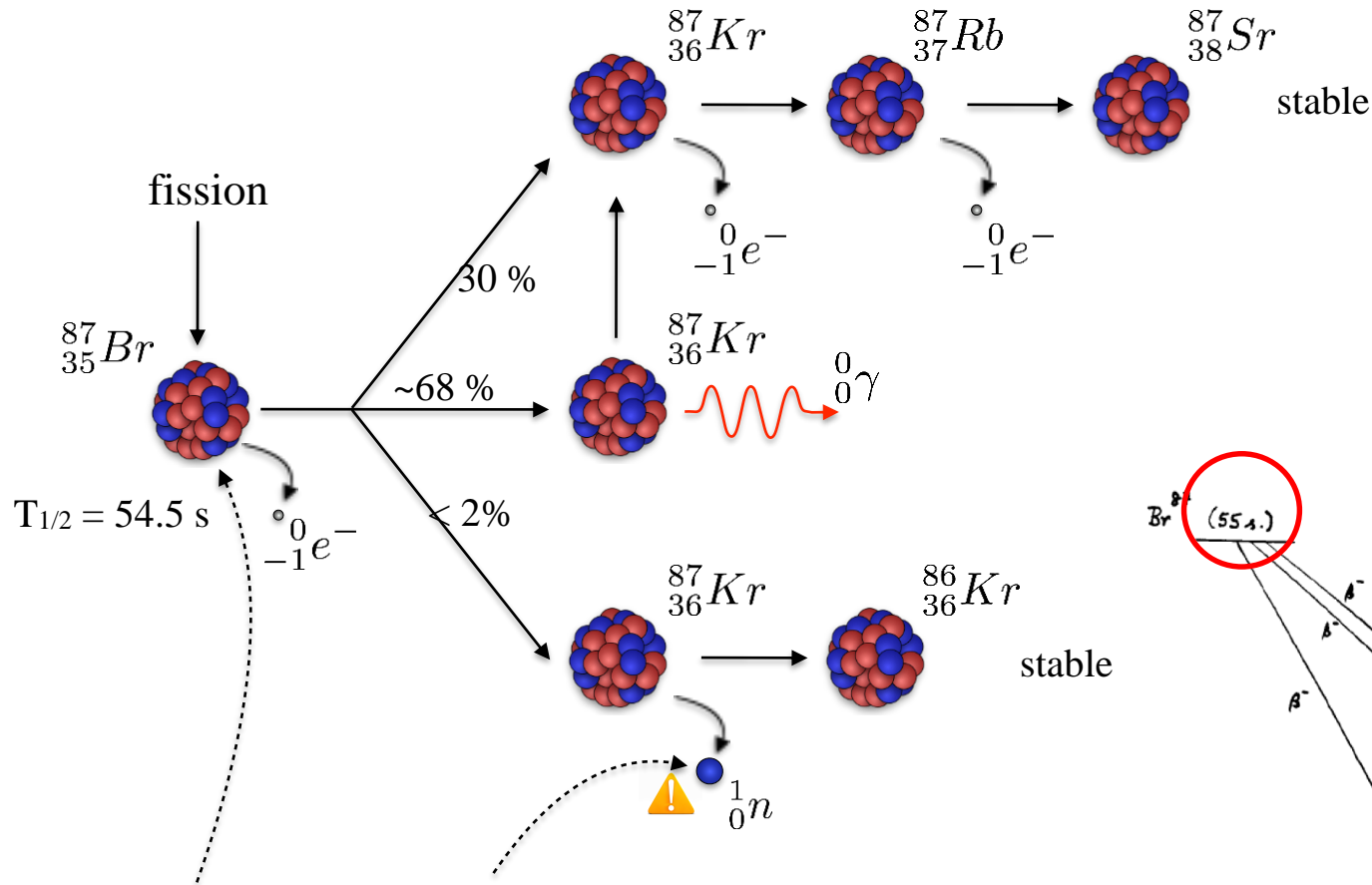
- If  $k_{eff}$ : 1.00000  $\rightarrow$  1.00500,  $P(t) = P(0) \exp\left(\frac{k_{eff} - 1}{l} t\right) \simeq P(0) \exp\left(\frac{0.005}{10^{-3}} t\right)$

- Period  $T=1/5$  s  $\rightarrow$   $P \nearrow$  by  $e^5 = 148$  in 1 s !!

- For a fast reactor,  $l=10^{-6}$  to  $10^{-7}$  s  $\rightarrow$  factor of 148 in  $< 1$  ms !

Reactors would be practically impossible to control...

- Small fraction of the neutrons, not prompt ( $\sim 0.6\%$  for  $^{235}\text{U}$ )  
Produced by decay of FP's, e.g.



- « Precursor » of « delayed neutron »  
 $\sim 6$  groups of precursors  
 $y_i, T_i \Rightarrow \beta_i, \lambda_i \quad (i = 1, 6)$

- $E_{\text{avg}}$  of delayed n's  $\sim 0.4\text{MeV}$
- $\lambda_i$ 's relatively constant
- $\beta_i$ 's depend on nuclide, e.g.  
 $\beta = \text{Sum}(\beta_i) = 0.21\%$  for  $^{239}\text{Pu}$   
 $= 0.26\%$  for  $^{233}\text{U}$
- $\beta$  small, but very important for control of the chain reaction  $\rightarrow$  *kinetic behaviour*
- Response of a reactor which becomes slightly supercritical, much slower

for  $^{235}\text{U}$ :

Group	Precursor	$T_{1/2}$ (s)	$\lambda_i$ (1/s)	$\beta_i$ (%)
1	$^{87}\text{Br}, ^{142}\text{Cs}$	55.7	0.012	0.022
2	$^{137}\text{I}, ^{88}\text{Br}$	22.7	0.031	0.142
3	$^{138}\text{I}, ^{89}\text{Br}, \dots$	6.2	0.11	0.127
4	$^{139}\text{I}, \text{Cs}, \dots$	2.3	0.30	0.257
5	$^{140}\text{I}, \text{Kr}, \dots$	0.61	1.14	0.075
6	Br, Rb, ...	0.23	3.01	0.027

$$\beta = \text{Sum}(\beta_i)$$

$$= 0.65 \%$$

- Fraction  $\beta$  of n's in reactor are delayed, so that the neutron production rate  $\neq \bar{\nu}F$

- It is, in fact: 
$$\underbrace{\bar{\nu} \cdot F \cdot (1 - \beta)}_{\text{Prompt Sources}} + \underbrace{\sum_{i=1}^6 \left[ \underbrace{\lambda_i}_{\text{Decay constant}} \times \underbrace{C_i(t)}_{\text{Precursor density at time t}} \right]}_{\text{Delayed Sources}}$$

- Thus 
$$\frac{dP}{dt} = \bar{\nu} F(t)(1 - \beta) - [A(t) + L(t)] + \sum_{i=1}^6 \lambda_i C_i(t)$$

- As before, substituting

$$k_{\text{eff}}(t) = \frac{\bar{\nu}F(t)}{A(t) + L(t)} \quad \ell = \frac{P(t)}{A(t) + L(t)} \quad \Rightarrow \quad \frac{dP}{dt} = \frac{(1 - \beta)k_{\text{eff}}(t) - 1}{\ell} \times P(t) + \sum_{i=1}^6 \lambda_i C_i(t)$$

- $k_{\text{eff}}, \ell$ : reactor characteristics indep. of P, may be calculated (e.g. 1-gp. diff. theory...)

- Supplementary eqns. needed for  $C_i$  's (precursor equations)

$$\frac{dC_i(t)}{dt} = \underbrace{\beta_i \bar{\nu} F(t)}_{\text{Precursor Production}} - \underbrace{\lambda_i C_i(t)}_{\text{Precursor Decay}} \Rightarrow \frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \beta_i k_{eff} (A(t) + L(t))$$

$$\frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \beta_i \frac{k_{eff}}{l} P(t)$$

- With the definitions:  $\frac{k_{eff} - 1}{k_{eff}} = \underbrace{\rho}_{\text{Reactivity}}$ ,  $\frac{l}{k_{eff}} = \underbrace{\Lambda}_{\text{Prompt Neutron Generation Time}}$

$$\Rightarrow \frac{dP}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{i=1}^6 \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda} P(t), \quad i = 1, \dots, 6$$

- complete system of 7 linear differential eqns. (**Point Kinetics Equations**)  
 → Very important basis for studies of kinetics, reactor stability, nuclear safety, etc.

*N.B.:  $\rho$ ... deviation of  $k_{eff}$  from 1 (normally very small, but very wide range:  $-\infty$  to 1)*

- Stationary Case: 
$$0 = \frac{\rho - \beta}{\Lambda} P(0) + \sum_{i=1}^6 \lambda_i C_i(0) \quad (1)$$
$$\lambda_i C_i(0) = \frac{\beta_i}{\Lambda} P(0)$$

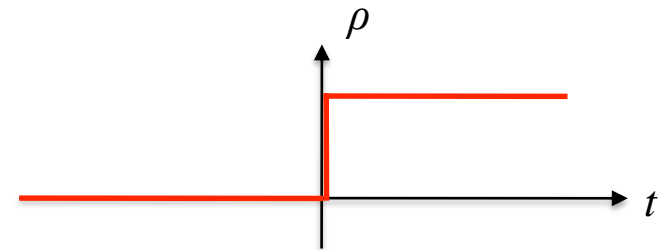
Substituting for  $C_i(0)$  into eq. 1,

$$\Rightarrow 0 = \frac{\rho - \beta}{\Lambda} P(0) + \sum_{i=1}^6 \frac{\beta_i}{\Lambda} P(0) \Rightarrow \rho = 0$$

$\Rightarrow$  Stationary states correspond to  $\rho = 0$  ( $k_{\text{eff}} = 1$ ), and the delayed neutrons have no effect...

- Important specific application of point kinetics equations  
→ Illustrative, analytical solution possible

- Constant  $\rho$  ( $\pm$ ) introduced at  $t = 0$   
→ E.g. quick movement of a control rod



- Point kinetics eqns.: 
$$\frac{dP}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_i \lambda_i C_i(t) \quad (1)$$

$$\frac{dC_i}{dt} + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda} P(t) \quad (2)$$

- For solution, one may apply method of Laplace transforms  
→ Differential equations replaced by algebraic eqns.

- Laplace Transform:  $L[F(t)] = \int_0^{+\infty} F(t)e^{-st} dt = \tilde{F}(s)$
- After certain manipulations, one “reconverts” to obtain solution of original eqn. (in time)
- Often needed information come directly from algebraic eqn. itself (e.g. via graphical soln.)
- N.B.: not necessary to compute transformations analytically:  
 → One can use “tables” of Laplace transforms

<u><math>F(t)</math></u>	<u><math>\tilde{F}(s)</math></u>	<u><math>F(t)</math></u>	<u><math>\tilde{F}(s)</math></u>
$F'(t)$	$s\tilde{F}(s) - F(0)$	$1$	$1/s$
$F''(t)$	$s^2\tilde{F}(s) - sF(0) - F'(0)$	$t$	$1/s^2$
$\vdots$	$\vdots$	$t^{n-1}/(n-1)!$	$1/s^n$
$\int_0^t F(\tau) d\tau$	$\frac{1}{s}\tilde{F}(s)$	$\vdots$	$\vdots$
$-t f(t)$	$\tilde{F}'(s)$	$e^{-at}$	$1/s+a$
$\vdots$	$\vdots$	$t e^{-at}$	$1/(s+a)^2$
		$\vdots$	$\vdots$

- From (1) & (2), taking the Laplace transform, it comes

$$s.\tilde{P}(s) - P(0) = \frac{\rho - \beta}{\Lambda} \tilde{P}(s) + \sum_i \lambda_i \tilde{C}_i(s)$$

$$s.\tilde{C}_i(s) - C_i(0) + \lambda_i \tilde{C}_i(s) = \frac{\beta_i}{\Lambda} \tilde{P}(s)$$

where  $\tilde{P}(s), \tilde{C}_i(s)$  are the Laplace transforms of  $P(t), C_i(t)$

- Eliminating  $\tilde{C}_i(s)$ ,
- $$\tilde{P}(s) = \Lambda \frac{P(0) + \sum_i \frac{\beta_i C_i(0)}{\lambda_i + s}}{\beta - \sum_i \frac{\beta_i \lambda_i}{\lambda_i + s} + \Lambda \cdot s - \rho}$$

If the reactor was critical at  $t = 0$ , using the precursor balance:

$$\lambda_i C_i(0) = \frac{\beta_i}{\Lambda} P(0) \quad \Rightarrow \quad C_i(0) = \frac{\beta_i}{\lambda_i \Lambda} P(0)$$

Partial fraction decomposition

• Thus, 
$$\frac{\tilde{P}(s)}{P(0)} = \frac{\Lambda + \sum_i \frac{\beta_i}{\lambda_i + s}}{\beta - \sum_i \frac{\beta_i \lambda_i}{\lambda_i + s} + \Lambda \cdot s - \rho} \xrightarrow{\text{Partial fraction decomposition}} \sum_{j=1}^7 \frac{B_j}{s - \omega_j}$$

where  $\omega_j$  are the roots of the denominator  $\beta - \sum_i \frac{\beta_i \lambda_i}{\lambda_i + s} + \Lambda \cdot s - \rho$

i.e. of  $\rho = \beta - \sum_i \frac{\beta_i \lambda_i}{\lambda_i + w} + \Lambda \cdot w$

$$\rho = \Lambda \cdot w + \sum_i \frac{\beta_i w}{\lambda_i + w}$$

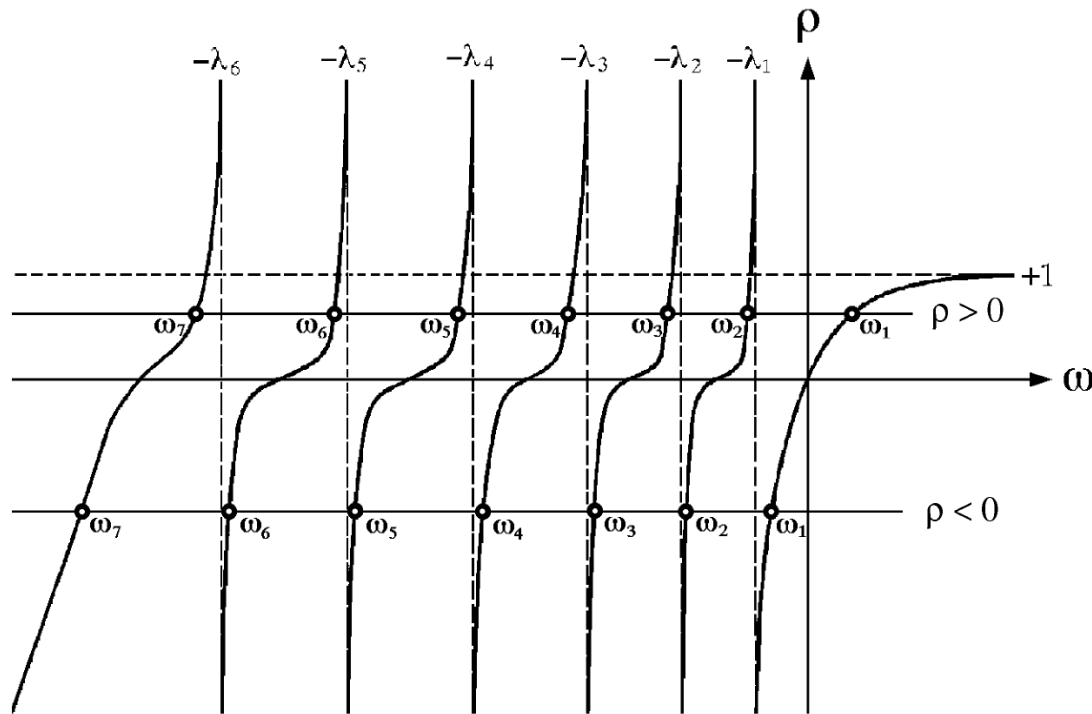
*Reactivity Equation (Inhour Equation)*

• The solution has the following form:

$$\frac{P(t)}{P(0)} = \sum_{i=1}^7 B_i \exp(\omega_i t)$$

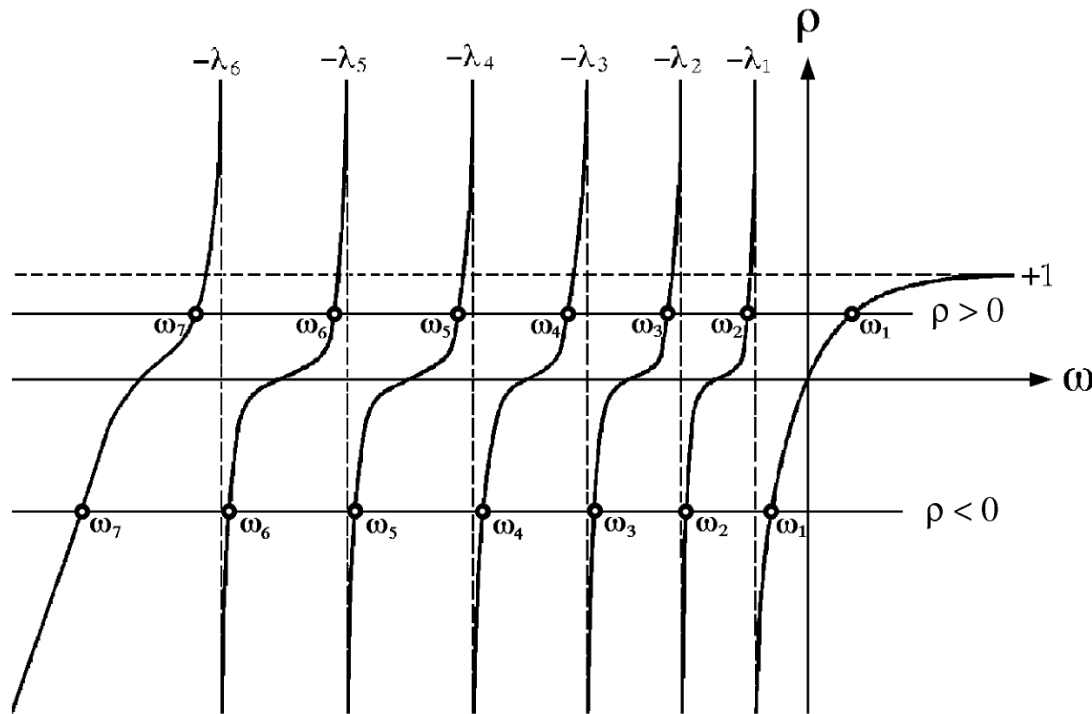
(from inversion of the Laplace-transform equation (1) above)

- The solution is therefore:  $P(t) = \sum_{i=1}^7 B_j \exp(\omega_j t)$   $\omega_j$  solutions of  $\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{(\omega + \lambda_i)}$
- One may solve the equation (Reactivity Eqn.) graphically:



- R.H.S.... function of  $\omega$ ,  
L.H.S....  $\rho (\pm)$ , constant

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- One may solve the equation (Reactivity Eqn.) graphically:



- R.H.S.... function of  $\omega$ ,  
L.H.S....  $\rho (\pm)$ , constant
- For positive  $\rho$  (supercritical reactor), one value of  $\omega$  is positive, the others negative
- For negative  $\rho$  (subcritical reactor), all 7 roots  $\omega$  are negative

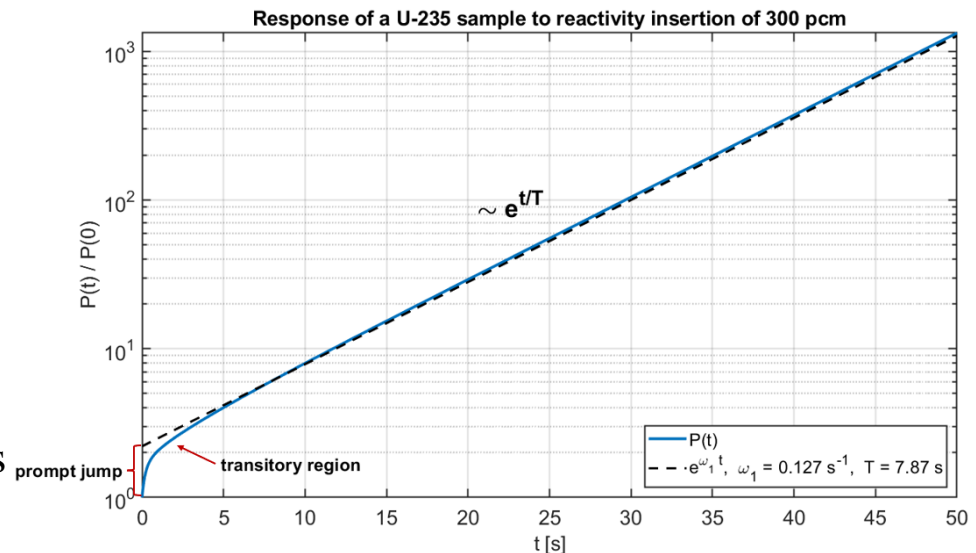
- For  $\rho > 0$ , after a certain time:  $P(t) = B_1 \exp(\omega_1 t)$   
 → All the other  $\exp(\omega_j t)$  terms disappear (negative  $\omega_i$ 's)

- Stable period:  $T = \frac{1}{\omega_1}$   
 → Time (in stable region) for P (or  $\Phi$ ) to increase by factor of e

- Solution  $\omega_1$  obtained graphically  
 → For a given set of kinetics parameters ( $A, \beta_i, \lambda_i$ ), each  $\rho \Leftrightarrow$  specific  $\omega_1$   
 → E.g. with

$$\Lambda = 10^{-3} \text{ sec.}; \beta_i, \lambda_i \Rightarrow U^{235}$$

$$\rho = 3 \cdot 10^{-3} \Rightarrow \omega_1 = 0,127 \text{ sec}^{-1} \Rightarrow T = 7,9 \text{ sec.}$$



transitory region:  
 negative terms which vanish rapidly

“prompt jump”:

$$\frac{P(t)}{P(0)} = \frac{\beta}{\beta - \rho}$$

- Without delayed n's, one had:  $P(t) = P(0) \exp\left[\frac{k_{\text{eff}} - 1}{l} t\right] \cong P(0) \exp\left[\frac{\rho}{\Lambda} t\right] = \exp\left[\frac{3 \cdot 10^{-3}}{10^{-3}} t\right]$

i.e  $T = \frac{\Lambda}{\rho} = \frac{10^{-3}}{3 \cdot 10^{-3}} = 0,33 \text{ sec} \Rightarrow$  a factor of  $\sim 24$  on period (for this example)

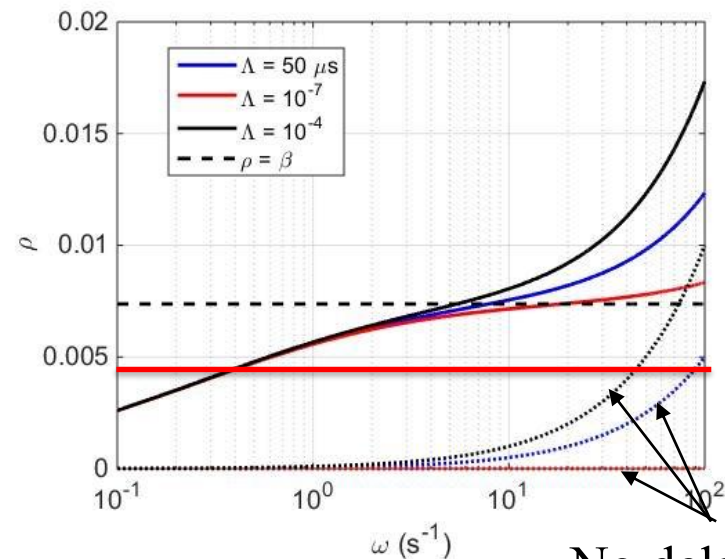
$\rightarrow$  Delayed n's render the reactor controllable

- For “ $^{235}\text{U}$  systems” ( $\beta_i, \lambda_i$ ):

- For  $\rho < \beta/2$ , no dependence on  $\Lambda$

- In absence of delayed n's ( $\beta = 0$ ),

$$\omega = \frac{\rho}{\Lambda}$$



No delayed n's


$\rightarrow$  For small  $\rho$ 's, differences with/without delayed n's, very large

$\rightarrow$  For  $\rho \sim \beta$ , differences decrease strongly (role of  $\Lambda$  becomes much more important)

- Normally, reactor is “delayed critical”:  $\text{Production} = (1 - \beta)k_{\text{eff}} + \sum_{i=1}^6 \lambda_i C_i$
- If  $\rho = \beta$ , reactor is critical only with prompt n’s ( $T$  becomes very short)
- For  $\rho \gg \beta$ , delayed neutrons no longer important  
 →  $\rho$  - vs -  $\omega$  curves become asymptotic to  $\rho = \Lambda \omega + \beta$

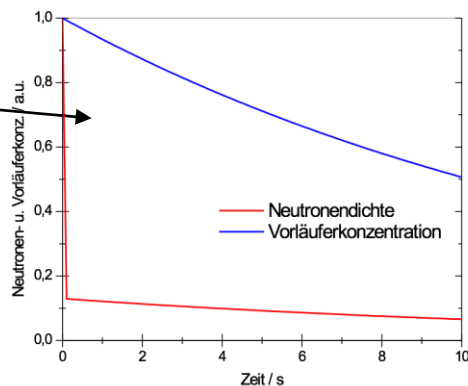
$$\rho = \Lambda \omega + \sum_i \frac{\beta_i \omega}{(\omega + \lambda_i)} \approx \Lambda \omega + \beta \quad \text{for } \lambda_i \ll \omega$$

- Extremely important that all reactivity insertions are significantly less than  $+\beta$   
 → Withdrawal of a control rod  
 → Effects of temperature, voidage (e.g. due to boiling of a liquid moderator)
- $\rho = \beta$  important enough limit to provide a special unit for reactivity, the *dollar*  
 →  $\rho = 0.65\%$  ( $^{235}\text{U}$  system)  $\Rightarrow$  1 dollar (100 cents)

- $\Delta\rho$  usually small: 0.65% or 0.2% not really convenient   
 0.65% = 650 **pcm** “per cent mille” (*one-thousandth of a percent*) is preferred for the reactivity

- For systems with  $^{239}\text{Pu}$ ,  $^{233}\text{U}$ , reactivity insertions have to be smaller  
→ It is  $\frac{\rho}{\beta}$  which matters, i.e. the value of  $\rho$  in \$ or ¢ ...
- Fast reactors (with  $^{239}\text{Pu}$  as primary fuel material), more sensitive because of their lower value of  $\beta$ , not because of their very small  $\lambda$   
→ However,  $\rho_{\text{abs}}$  values are also generally lower in fast reactors (cross sections  $\searrow$ )

- For a “prompt drop”



Flux immediately after drop:

$$\frac{P(t)}{P(0)} = \frac{\beta}{\beta - \rho}$$

- For the stable period, one needs to consider the roots of the Reactivity Equation
  - All the  $\omega$  values are negative with  $|\omega_7| > |\omega_6| > |\omega_5| > \dots > |\omega_1|$
  - The stable region is where only term  $-\omega_1$  remains and the flux is:

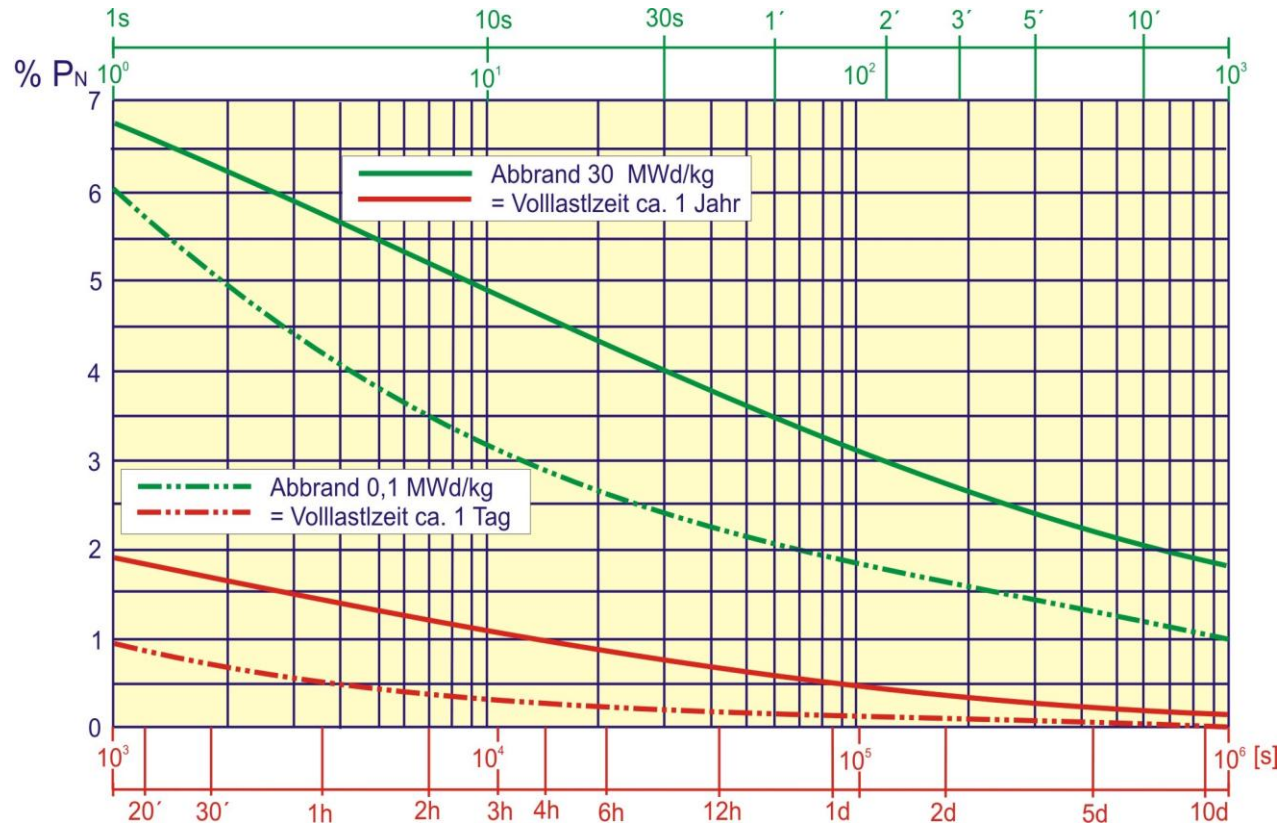
$$\Phi(t) = \Phi_1 \exp(-t/T) \quad \text{with } T = \frac{1}{|\omega_1|}$$

- For negative  $\rho$  with  $|\rho| \gg \beta$ ,  $\omega_1 \rightarrow \lambda_1$  (decay const. of 1<sup>st</sup> gp.)  $T = \frac{1}{\lambda_1} \cong 80 \text{ sec.}$

- Thus, a reactor cannot be shut down more quickly than with a negative period of  $\sim 80\text{s}$
- The “neutronic” power is determined by the prompt jump and T (80s)
- After a certain time, thermal power  $\sim$  fission-product “decay power” (residual heat)

Decay Heat (Way-Wigner):  $P_N = 6,22 \% \cdot (t^{-0,2} - (t_0 + t)^{-0,2}) \stackrel{t_0 \rightarrow \infty}{=} 6,22 \% \cdot t^{-0,2}$

- Directly after Scram 6,4 % of full power level
- after one minute half thereof (~ 3,2 %)
- after one hour half thereof (~ 1,6 %)
- after one day half thereof (~ 0,8 %)



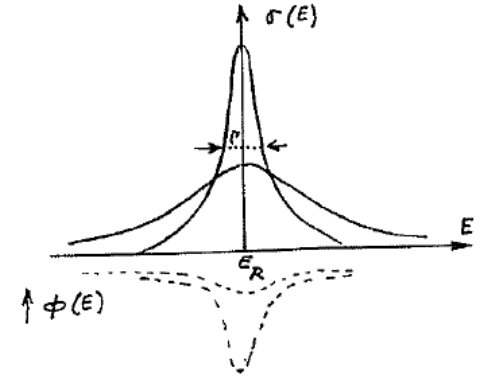
- Normally,  $\rho$  change not sudden, e.g. could be  $\sim$  linear ramp in reactivity:  $\rho(t) = K.t$ 
  - Point kinetics equations need to be solved numerically
- Change in power changes thermal balance, affects temperatures, hence  $\sigma$ 's and  $\rho$ 
  - Feedback effects: have to be negative (act as “brakes”)... safety studies involve coupling of thermal-hydraulics, neutron kinetics...
- Various “time constants” involved (e.g. time for power change to affect temperature,...)
  - Values quite different for different types of “feedbacks”
- Short-term causes for  $\rho$ -variation
  - Fuel temperature (*Doppler effect*),  $< 1$  sec (effect  $\sim$  prompt  $\rightarrow$  most important...)
  - Moderator temperature, secs - mins
  - Voiding of liquid moderator/coolant, secs (boiling, bubble formation, effect on density...)
- Medium-term causes
  - Principal effect: Fission product  $\text{Xe}^{135}$  in a thermal reactor, hours - days
- Long-term effects
  - Fuel composition changes with irradiation (burnup), days - months
  - Largest effect in power reactors (“burning” of fissile, Pu-production, accumulation of FPs,...)

- When  $T_F \uparrow$ ,  $U^{238}$  resonances broadened due to increased thermal agitation of nuclei

➤ Area under resonance constant, but flux is less depressed

⇒ Effective resonance integral,  $I_{eff} \uparrow$

➤ In  $k_{\infty} = \eta f p \varepsilon$ ,  $p \downarrow$



$$p(E, T) = \exp \left( - \underbrace{\int \frac{\bar{\sigma}_{A,\gamma}(E, T)}{N_A \bar{\sigma}_{A,\gamma}(E, T) + N_H \sigma_{S,H}} \frac{dE}{E}}_{I_{eff}} \right)$$

$$I_{eff}(T) \cong I_{eff}(300 \text{ K}) \cdot \left[ 1 + C \cdot (\sqrt{T} - \sqrt{300}) \right]$$

- Fuel Temperature Coefficient of Reactivity (*Doppler Coeff.*)

$$\Rightarrow \alpha_T = \frac{\partial \rho}{\partial T_F} = \frac{1}{k^2} \frac{\partial k}{\partial T_F} \cong \frac{1}{k} \frac{\partial k}{\partial T_F} = \frac{1}{\eta} \frac{\partial \eta}{\partial T_F} + \frac{1}{f} \frac{\partial f}{\partial T_F} + \frac{1}{p} \frac{\partial p}{\partial T_F} + \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial T_F}$$

$$\Rightarrow \alpha_T \cong \frac{1}{p} \frac{dp}{dT_F} = - \frac{C}{2\sqrt{T_F}} \ln \left[ \frac{1}{p(300 \text{ K})} \right]$$

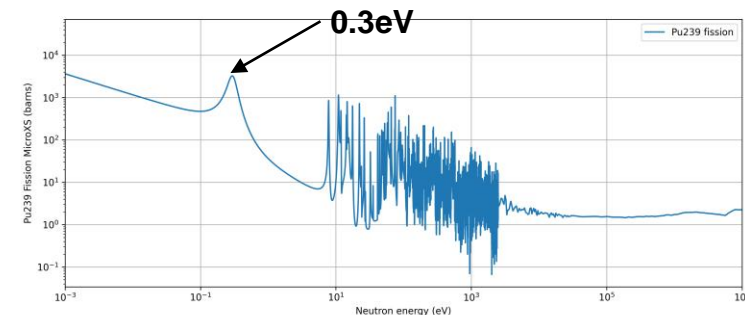
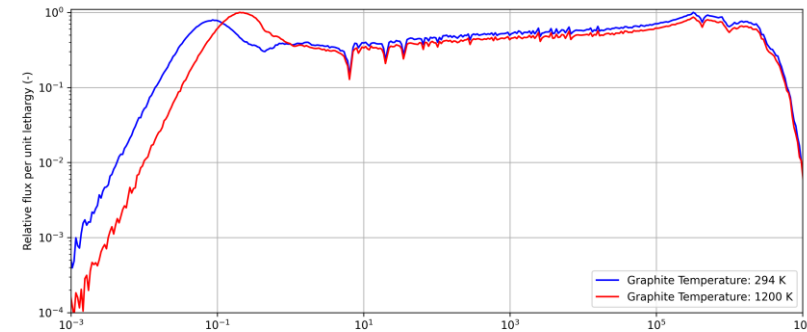
- Neutron spectrum effects

- Maxwellian part shifted to right when  $T_m \uparrow$
- $\sigma_{th}$ 's  $\sim 1/v$  (i.e.  $1/\sqrt{E}$ ), but not exactly...
- For  $\eta = \bar{\nu} \frac{\Sigma_f}{\Sigma_a}$ , individual changes of  $\Sigma_f, \Sigma_a$  important
- For  $U_{nat}$ ,  $\eta \downarrow$  when  $T_m \uparrow$
- In presence of Pu, this changes (Pu<sup>239</sup> resonance at 0.3 eV, positive effect)
  - ✓ Partly compensating effect from Pu<sup>240</sup> (large capture resonance at 1ev)

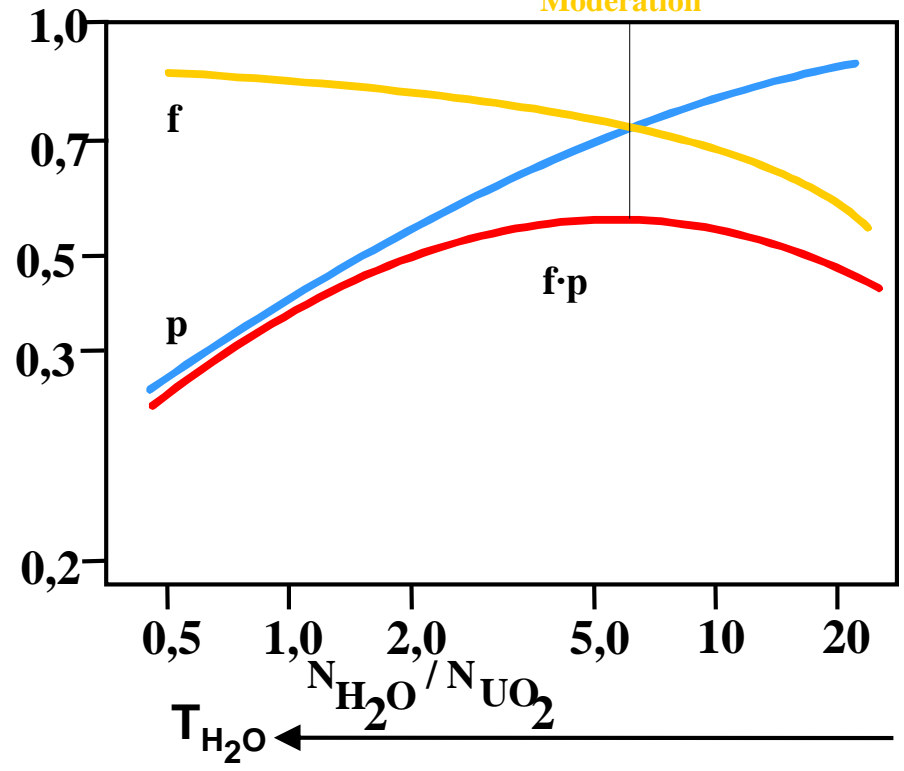
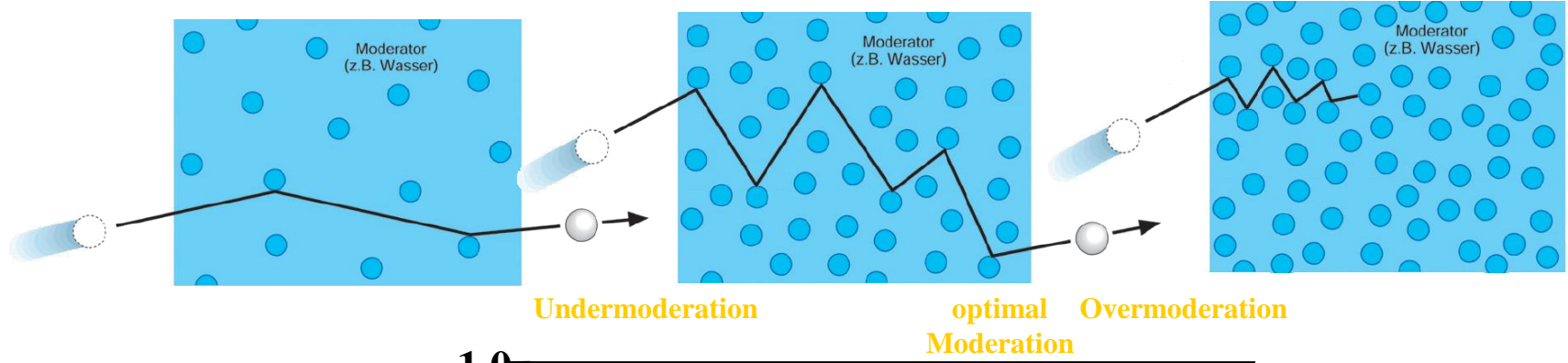
- Spectrum effect most important for solid moderator, e.g. graphite

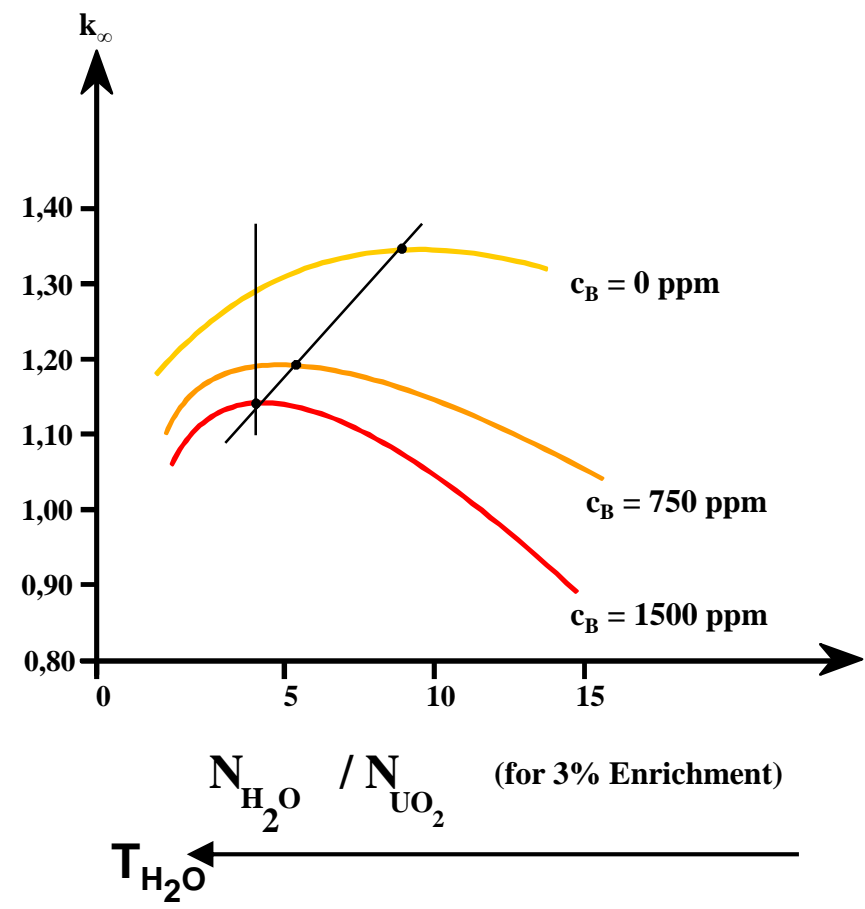
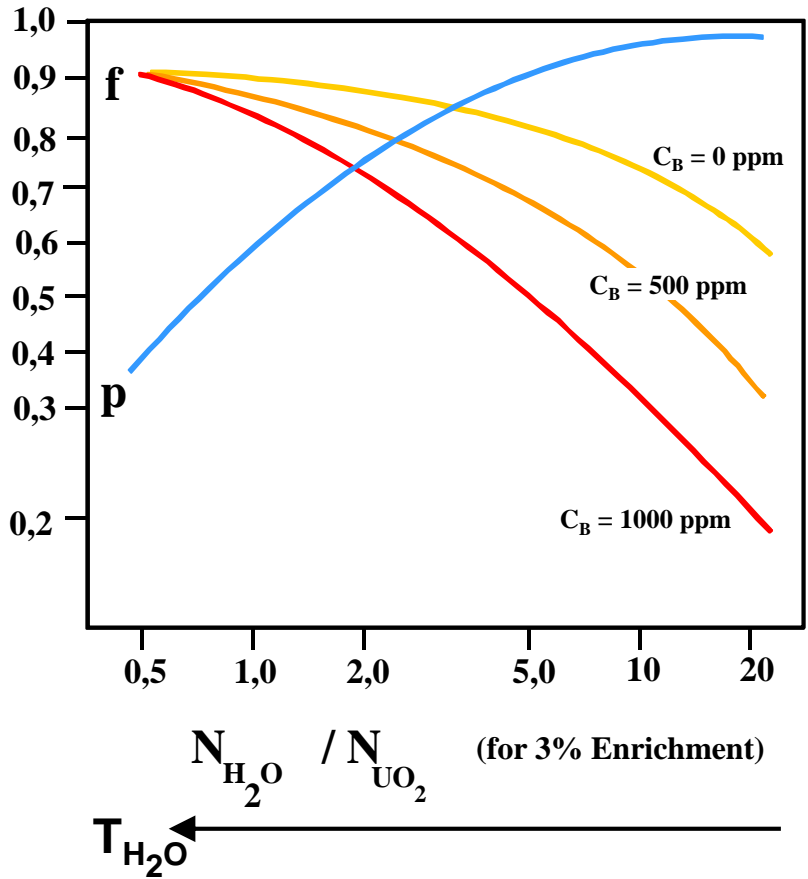
- For a liquid moderator (coolant), density variation much more important effect

$$\alpha_m = \frac{\partial \rho}{\partial T_m} \cong \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial T_m}$$



# MODERATOR (COOLANT) TEMPERATURE COEFFICIENT, $\alpha_M$





- $$\alpha_v = \frac{\partial \rho}{\partial V} \cong \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial V} \quad (\text{v : volumetric fraction of steam...})$$
- Very important to have negative  $\alpha_v$  for liquid moderator/coolant (Chernobyl...!)
- Boiling implies a strong reduction of density
  - ✓ As for  $\alpha_m$ , thermal reactor needs to be undermoderated

- For the short-term effects, one may write:  $\Delta\rho \cong \alpha_F \Delta T_F + \alpha_M \Delta T_M + \alpha_V \Delta V + \dots$ 
  - However, this does not give the true “dynamic” behaviour
    - ✓ No consideration of the time constants
- One needs proper time-dependent “modelling” of the power reactor (including the secondary cooling system), with “coupling” betn. neutronics, thermal-hydraulics
  - Safety studies:
    - ✓ Numerical simulation and analysis of hypothetical accident situations
- In general, if all the  $\alpha$ 's are negative, reactor “inherently” safe from viewpoint of uncontrolled reactivity insertion
- Calculation of  $\alpha$ 's generally very delicate
  - Compensation of individual effects, e.g. sodium  $\alpha_v$ , or  $\alpha_m$  in HTR (graphite)
  - Necessary to carry out “checks” on power reactor before start-up

- Reactivity Equation (constant reactivities)
- Roots of Reactivity Equation
  - Stable Reactor Period
- Delayed, prompt criticality
  - Dollar
- “Prompt jump” (small positive, all negative  $\rho$ 's)
- Negative reactivities
- Internal reactivity variations
  - Reactivity feedbacks (fuel temperature, moderator temperature, coolant voidage, etc.)
  - Importance for “inherent” safety