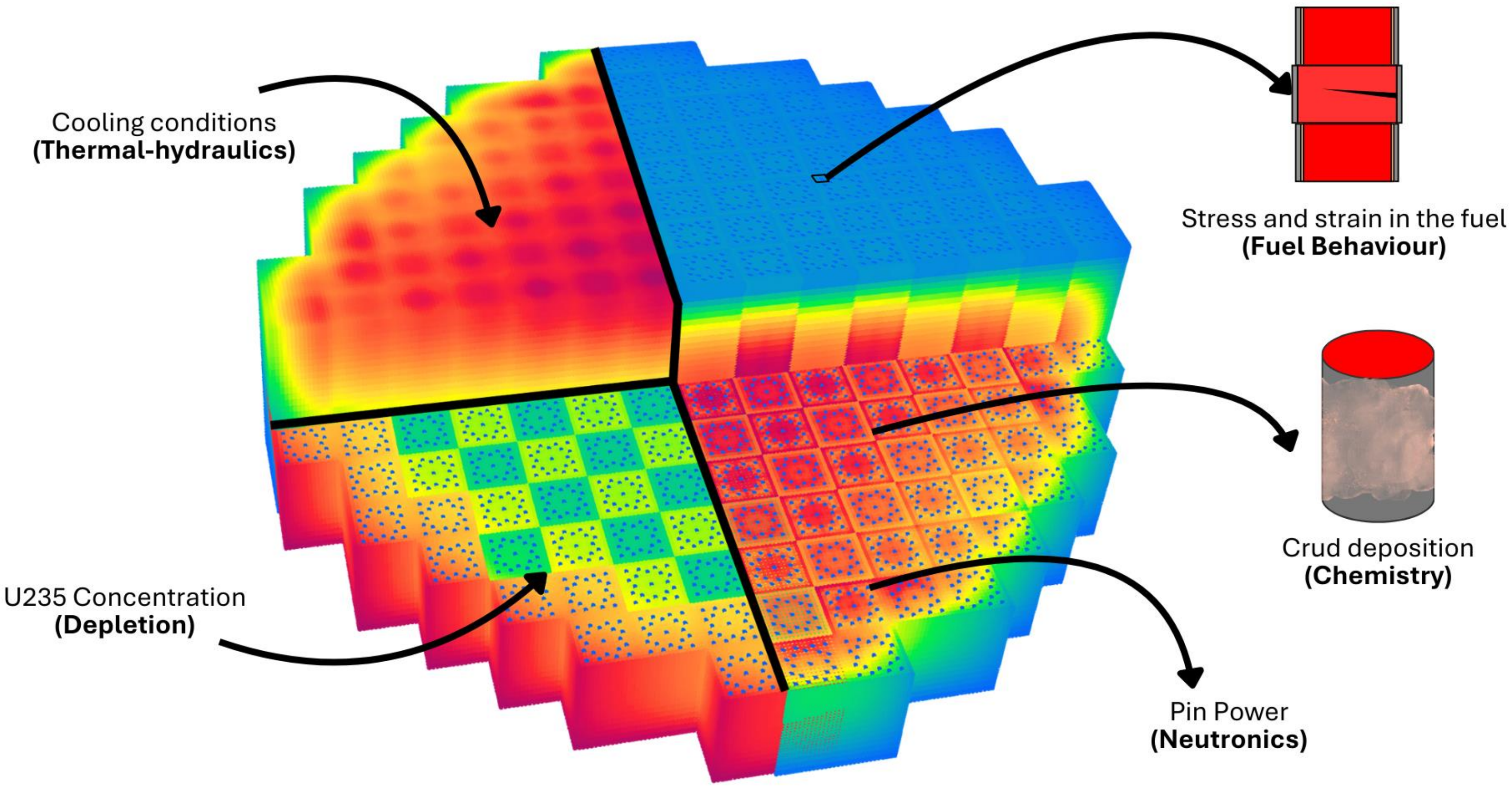
Physics of Nuclear Reactors



Diffusion Theory II

Daniele Timpano
Milica Krstovic

Exercises of today

7 DIFFUSION PART II

7.1 - Critical Pu mass
7.2 - k_∞ for parallelepiped reactor
7.3 - Critical cylinder radius
7.4 - Leakage as absorption
7.5 - Flux normalization
7.6 - Flux peaking factor

Divide in groups of 5:

- *We do Exercise 1 at the board*
- *I will leave you 8 minutes for Exercise 2 and 8 minutes for Exercise 3*
- *I will leave you 8 minutes for Exercise 5 focusing only on cylindrical reactor*
- *We will do Exercise 6 at the board*

1. Critical Pu Mass

Exercise description:

Calculate the critical mass of a sphere of metallic plutonium (95% ^{239}Pu, 5% ^{240}Pu; density: 19 g/cm^3). This is effectively related to an assembly simulating the conditions of an atomic bomb, with a very "hard" fast-neutron spectrum. You may apply 1-group diffusion theory to this situation, using the following set of spectrum-averaged microscopic cross-sections for the plutonium (both the isotopes have been combined into a single, effective nucleus): $\sigma_t = 5.87b$, $\bar{\nu}\sigma_f = 5.52b$, $\sigma_a = 1.81b$

Knowledge to be applied: $k_\infty = \frac{\bar{\nu}\Sigma_f}{\Sigma_a}$, $D = \frac{\lambda_t}{3}$, $L^2 = \frac{D}{\Sigma_a}$, $B_m^2 = \frac{k_\infty - 1}{L^2}$, $d = 0.71\lambda_t$

Expected results: R_c=5.59 cm, m_c ~ 13.9 kg

1. Critical Pu Mass – Let's take a step back!

Write one group diffusion equation in multiplying medium ($\bar{\nu}\Sigma_f$, Σ_a, D) in stationary state

Derive one-group reactor equation

What is the criticality condition?

1. Critical Pu Mass – Let's take a step back!

- One uses the **one-group** diffusion equation for the stationary case (critical reactor)

$$D\nabla^2\phi(\vec{r}) - \Sigma_a(\vec{r})\phi(\vec{r}) + S(\vec{r}) = 0$$

- The source is coming from fissions: $S(\vec{r}) = Q_f(\vec{r}) = \bar{\nu}\Sigma_f(\vec{r})\phi(\vec{r})$

- Thus $D\nabla^2\phi + (\bar{\nu}\Sigma_f - \Sigma_a)\phi = 0 \Rightarrow \nabla^2\phi + \dfrac{\frac{\bar{\nu}\Sigma_f}{\Sigma_a} - 1}{\frac{D}{\Sigma_a}}\phi = 0$

$$\nabla^2\phi + \frac{k_\infty - 1}{L^2}\phi = 0$$

$$\nabla^2\phi + B_m^2\phi = 0$$

One-group Reactor Equation

$$B_m^2 = \frac{k_\infty - 1}{L^2}$$

Material Buckling
(depends only material properties)

What's missing for the criticality condition?

1. Critical Pu Mass (1/2)

The **molar mass of plutonium** in the sphere is:

$$M_{Pu} = 239 \times 0.95 + 0.05 \times 240 \text{ g /mol}$$

The **atomic density of the plutonium** is:

$$N_{\text{Pu}} \simeq \frac{\rho N_A}{\text{M}_{Pu}} = \frac{19 \times 6.022 \times 10^{23}}{239.05} = 4.786 \times 10^{22} \text{cm}^{-3}$$

so that $\Sigma_t = N\sigma_t = 0.281\text{cm}^{-1}$, $\Sigma_a = 0.0866 \text{ cm}^{-1}$, $\nu\Sigma_f = 0.2642 \text{ cm}^{-1}$

yielding $k_\infty = \frac{\bar{\nu}\Sigma_f}{\Sigma_a} = \eta_c = \frac{0.2642}{0.0866} = 3.051$

(**N.B.** The utilization factor f=1, since there is no parasitic absorption in the sphere, the entire material being purely fuel. The resulting k_∞ value is a virtual "record" for multiplying media, related to the very high η-value of ^{239}Pu for neutrons of ~ 1 MeV.)

1. Critical Pu Mass (2/2)

Compute the **diffusion area**:

From Σ_t, one calculates: $\lambda_t = \frac{1}{\Sigma_t} = 3.56\text{cm},\ D = \frac{\lambda_t}{3} = 1.186\text{cm}$

Thus: $L^2 = \frac{D}{\Sigma_a} = \frac{1.186}{0.0866} = 13.70\ \text{cm}^2$ and $B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{2.051}{13.70} = 0.1497\ \text{cm}^{-2}$

Compute the material buckling and **apply criticality condition ($B^2 = B_m^2$):**

The criticality condition $B_m^2 = \frac{k_\infty - 1}{L^2}$ (slide 11 of lecture 4) for a spherical reactor gives:

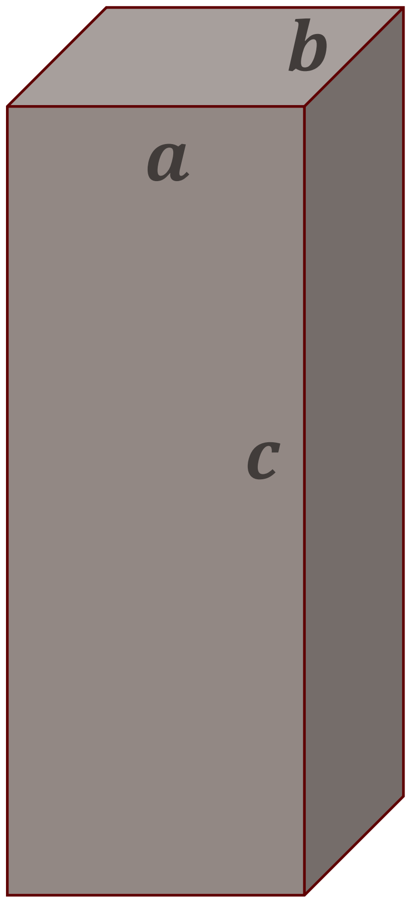
$$R + d = \frac{\pi}{B_m} = 8.12\text{cm}$$

Compute the extrapolation distance to finally get R

Finally, using the extrapolation distance, **d = 0.71 λ_t**, one obtains the critical radius as **R_c=5.59 cm**, i.e. a critical mass of ~ 13.9 kg.

What are the limits of this approach?

2. k_∞ for parallelepiped reactor



- Given material properties: Σ_a , D
- Given dimensions: a, b, c

What is k_∞

1.1078

$$B^2 = \left(\frac{\pi}{a'}\right)^2 + \left(\frac{\pi}{b'}\right)^2 + \left(\frac{\pi}{c'}\right)^2 = B_m^2 = \frac{k_\infty - 1}{L^2}$$

$$L^2 = \frac{D}{\Sigma_a}$$

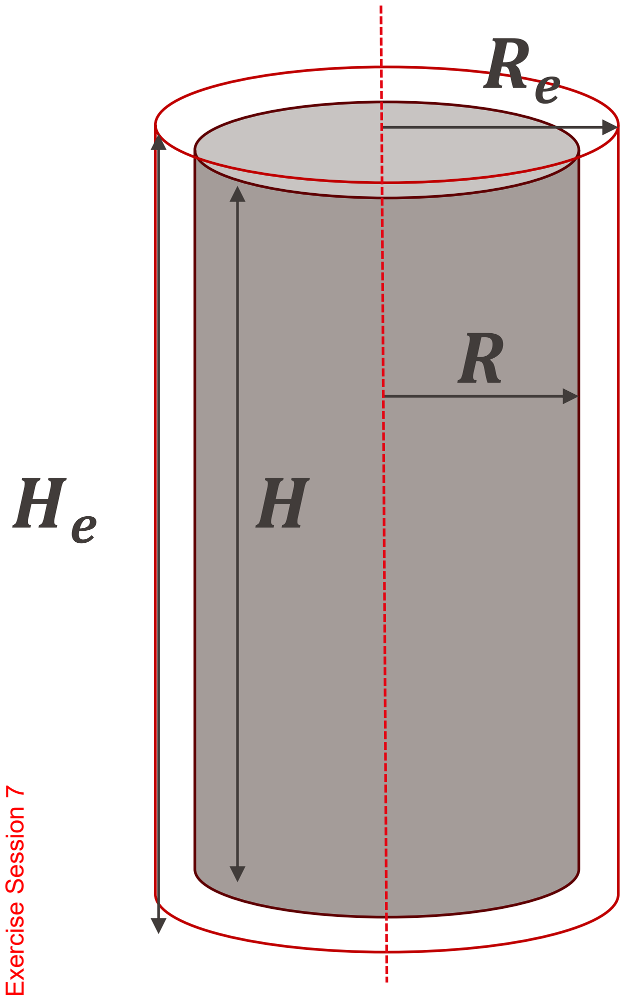
$$a' = a + 2d = 60 + (2 \times 1.62) = 63.24\text{cm}$$

$$b' = b + 2d = 83.24\text{cm}$$

$$c' = c + 2d = 123.24\text{cm}$$

$$d = 0.71\lambda_t = \frac{0.71}{\Sigma_{tr}} = 0.71 \times 3D$$

3. Critical cylinder radius



- Given material properties: Σ_a , Σ_{tr}, $\bar{\nu}\Sigma_f$
- Given height of the cylinder: $\boldsymbol{H}$

What is the critical radius R of the reactor?

55.9 cm

$$B^2 = \left(\frac{\pi}{H_e}\right)^2 + \left(\frac{2.405}{R_e}\right)^2 = B_m^2 = \frac{k_\infty - 1}{L^2}$$

$$k_\infty = \frac{\bar{\nu}\Sigma_f}{\Sigma_a} = \frac{0.0843}{0.082} = 1.0280$$

$$H_e = H + 2d$$

$$L^2 = \frac{D}{\Sigma_a} = \frac{1}{3\Sigma_{\text{tr}}\Sigma_a} = \frac{1}{3 \times 0.342 \times 0.082} = 11.89\text{cm}^2$$

$$R_e = R + d$$

$$d = 0.71\lambda_{tr}$$

$$\lambda_{tr} = \left(\frac{1}{\Sigma_{tr}}\right)$$

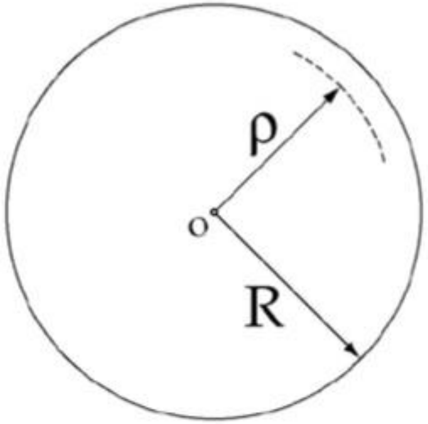
5. Flux normalization

Exercise description:

Find a relationship between the coefficient A, which determines the absolute value of the neutron flux, and the thermal power P of a reactor of the following shape:

(a) spherical, (b) cylindrical, (c) rectangular parallelepiped.

You may neglect the extrapolation distance.



Knowledge to be applied: $\Phi(r) = \frac{A\sin\left(\frac{\pi r}{R}\right)}{r}$, $\Phi(r,z) = AJ_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$, $\Phi(x,y,z) = A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi x}{b}\right)\cos\left(\frac{\pi z}{c}\right)$, $P = \int_V E_f\Sigma_f\Phi(r)\mathrm{dr}$

Expected results: (a) $A = \frac{1}{4R^2}\frac{P}{E_f\Sigma_f}$, (b) $A = \frac{2.405\pi}{4}\frac{P}{VE_f\Sigma_f}$ with $V = \pi R^2 H$, (c) $A = \frac{\pi^3}{8}\frac{P}{VE_f\Sigma_f}$ with $V = abc$

5. Flux normalization

$$\boldsymbol{A} = ?$$

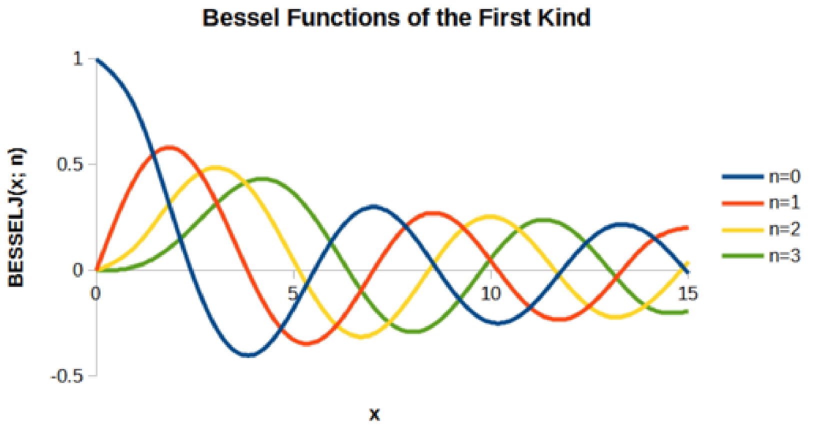
If you know:

1) Power of the reactor **P**
2) Size of the reactor
3) Energy per fission $\boldsymbol{E_f}$
4) Fission cross section $\boldsymbol{\Sigma_f}$

5. Flux normalization

What is the link between P and Φ?

$$P = \int_V \phi \Sigma_f E \, dV$$



(a) Spherical

$$\Phi(r) = A\frac{\sin\left(\frac{\pi r}{R}\right)}{r}$$

(b) Cylindrical

$$\Phi(r,z) = AJ_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

(c) Rectangular parallelepiped

$$\Phi(x,y,z) = A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$$

(a) Spherical

$$dV = 4\pi r^2 dr$$

(b) Cylindrical

$$dV = 2\pi r dr dz$$

(c) Rectangular parallelepiped

$$dV = dx\, dy\, dz$$

Property of Bessel Functions

$$\frac{d}{dx}(xJ_1(x)) = xJ_0(x)$$

5. Flux normalization

What is the link between P and Φ?

$$P = \int_V \phi \Sigma_f E \, dV$$

(b) For the cylindrical reactor

$$P = 2\pi A E \Sigma_f \int_0^R r\, J_0\left(\frac{2.405 r}{R}\right) dr \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{H}\right) dz$$

$$P = 2\pi A E \Sigma_f \cdot \frac{R^2}{2.405} J_1(2.405) \cdot 2H$$

$$A = \frac{2.405\, P}{4\pi H R^2 A E \Sigma_f \, J_1(2.405)}$$

6. Flux peaking factors

Exercise description:

Calculate the values of flux peaking factor $\Phi_{\max}/\bar{\Phi}$ for the spherical, rectangular and cylindrical bare homogeneous reactors of Ex. 7.5.

Knowledge to be applied: $\bar{\Phi} = \frac{\int_V \Phi \mathrm{dv}}{\int_V \mathrm{dv}}$, $\Phi_{\max} = \Phi(\text{centre})$

Expected results: (a) $\frac{\Phi_{\max}}{\bar{\Phi}} \simeq 3.29$ (b) $\frac{\Phi_{\max}}{\bar{\Phi}} \simeq 3.64$ (c) $\frac{\Phi_{\max}}{\bar{\Phi}} \simeq 3.88$

6. Flux peaking factors

What is Φ_{max} and $\bar{\Phi}$?

For a bare homogeneous reactor, the maximum value of the flux Φ_{max} is to be found at the centre of the system. Thus, for each of the 3 geometries in Ex. 7.5

$$\boldsymbol{\Phi_{max} = \lim_{V\to 0} \Phi}$$

The average flux value $\bar{\Phi} = \frac{P}{V E_f \Sigma_f}$ where V is the volume of the reactor.

6. Flux peaking factors

What is Φ_{max} and $\bar{\Phi}$?

(a) For the spherical reactor:

$$\frac{\Phi_{\max}}{\bar{\Phi}} = \frac{\lim_{r\to 0}\left(\frac{A}{r}\sin\left(\frac{\pi r}{R}\right)\right)}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{\frac{\pi A}{R}}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{\frac{1}{4R^2}\frac{P}{E_f\Sigma_f}\frac{\pi}{R}}{\frac{3P}{4\pi R^3 E_f\Sigma_f}} = \frac{\pi^2}{3} \simeq 3.29$$

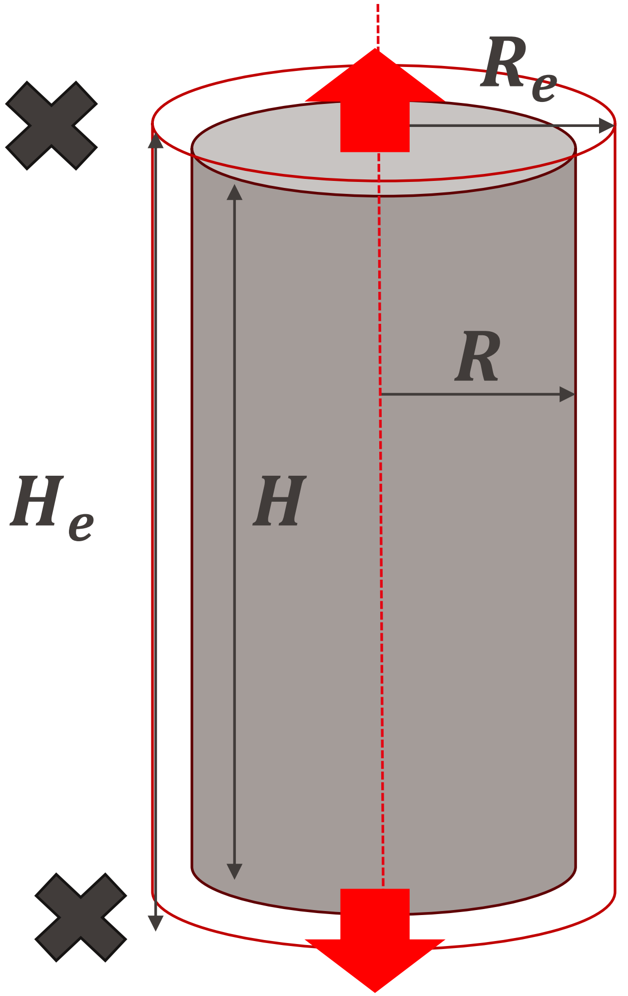
(b) For the cylindrical reactor:

$$\frac{\Phi_{\max}}{\bar{\Phi}} = \frac{\lim_{r\to 0, z\to 0}\left(\mathrm{A}J_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)\right)}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{A}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{\frac{2.405\pi}{4J_1(2.405)}\frac{P}{\mathrm{V}E_f\Sigma_f}}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{2.405\pi}{4J_1(2.405)} \simeq 3.64$$

(c) For the rectangular parallelepiped reactor:

$$\frac{\Phi_{\max}}{\bar{\Phi}} = \frac{\lim_{x\to 0, y\to 0, z\to 0}\left(A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{c}\right)\cos\left(\frac{\pi z}{c}\right)\right)}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{A}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{\frac{\pi^3}{8}\frac{P}{\mathrm{V}E_f\Sigma_f}}{\frac{P}{\mathrm{V}E_f\Sigma_f}} = \frac{\pi^3}{8} \simeq 3.88$$

4. Leakage as absorption



What is the critical radius R of the reactor, if we consider the axial leakage as the additional absorption?

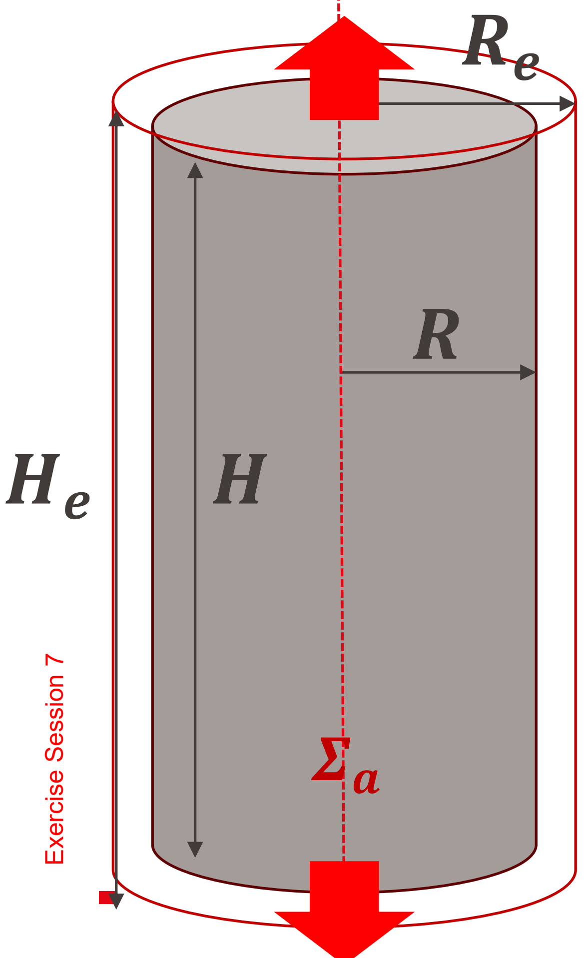
$$B^2 = B_z^2 + B_r^2 = \left(\frac{\pi}{H_e}\right)^2 + \left(\frac{2.405}{R_e}\right)^2$$

$$B_z^2 = 0 \qquad B^2 = B_r^2$$

4. Leakage as absorption

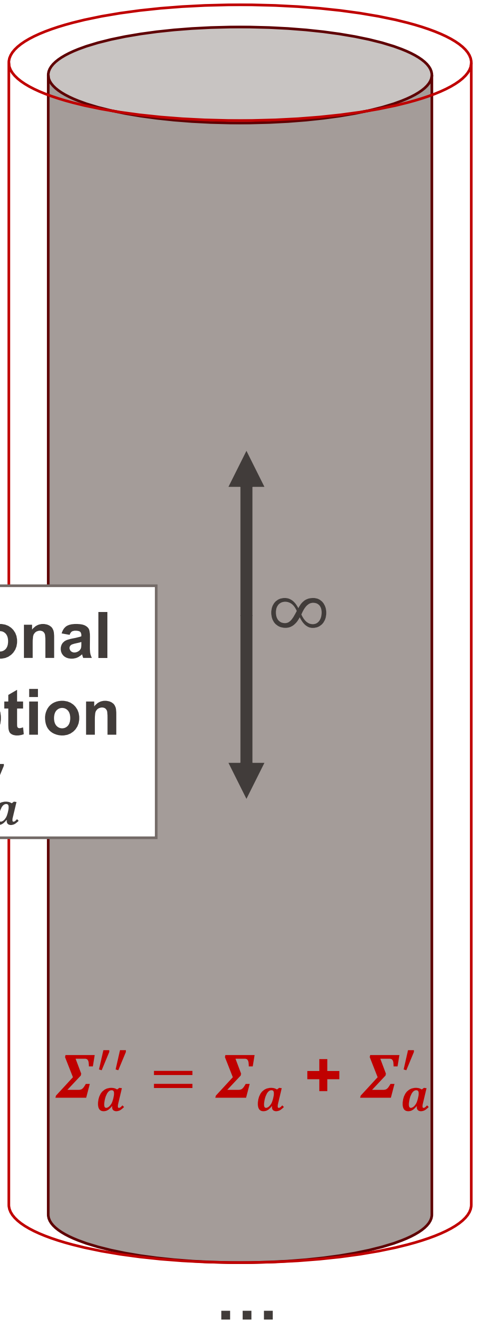
$$B^2 = \left(\frac{\pi}{H_e}\right)^2 + \left(\frac{2.405}{R_e}\right)^2$$

$$\Phi(\mathrm{r,z}) = \mathrm{AJ_0}\left(\frac{2.405r}{R_e}\right) cos\left(\frac{\pi z}{H_e}\right)$$



Axial leakage

Additional absorption $\sim \Sigma_a'$



$$B^2 = \left(\frac{2.405}{R_e}\right)^2$$

$$\Phi(\mathrm{r}) = \mathrm{AJ_0}\left(\frac{2.405r}{R_e}\right)$$

Derive the expression for the additional absorption coefficient Σ_a'

4. Leakage as absorption

What is the additional absorption coefficient Σ'_a that accounts for axial leakage?

$$\nabla^2\phi + B_m^2\phi = 0$$

$$B^2 = B_m^2$$

Cylinder with axial leakage (finite height)

$$B^2 = B_z^2 + B_r^2$$

Cylinder with no axial leakage (infinite height)

$$B^2 = B_r^2$$

4. Leakage as absorption

$$\nabla^2\phi + B_m^2\phi = 0$$

Multiply both side by D to get the leakage

$$-L + DB^2\,\phi = 0$$

Cylinder with axial leakage (finite height)

$$B_0^2 = B_z^2 + B_r^2$$

$$-(L_z + L_r) + DB_0^2\,\phi = 0$$

Cylinder with no axial leakage (infinite height)

$$B_1^2 = B_r^2$$

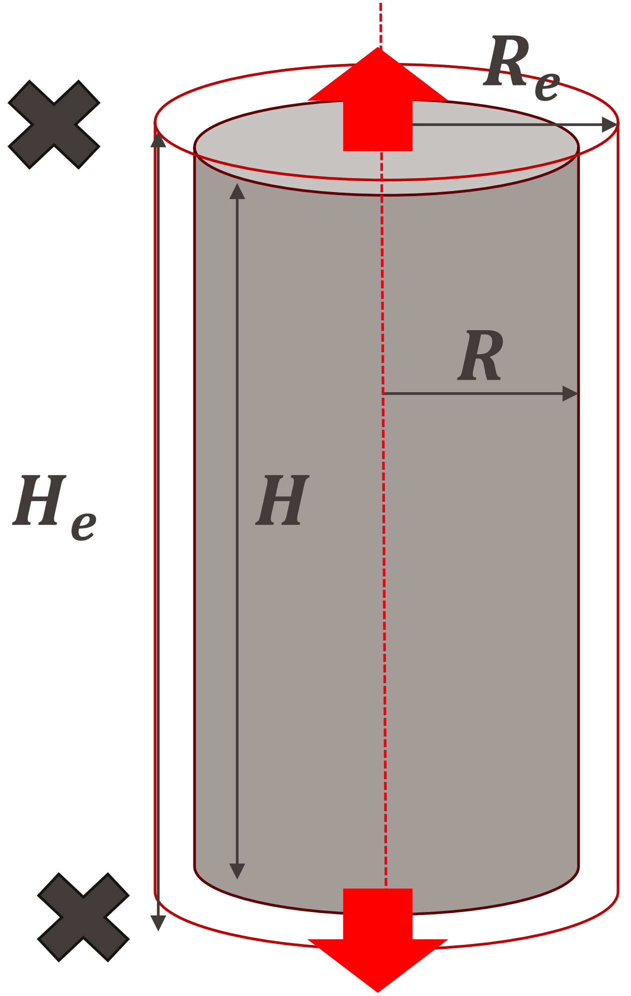
$$-L_r + DB_1^2\,\phi = 0$$

$$L_z = DB_z^2\,\phi$$

$$DB_z^2\,\phi = \Sigma'_a\phi$$

$$\boxed{\Sigma'_a = DB_z^2}$$

4. Leakage as absorption



What is the critical radius R of the reactor, if we consider the axial leakage as the additional absorption?

$$B^2 = \left(\frac{2.405}{R_e}\right)^2 \qquad \Sigma'_a = DB_z^2$$

R=?