

# Physics of Nuclear Reactors

Nuclear Fission

Daniele Timpano

Milica Krstovic

<b>3.</b>	<b>NUCLEAR FISSION .....</b>	<b>2</b>
<b>3.1</b>	<b>An example nuclear power plant .....</b>	<b>2</b>
<b>3.2</b>	<b>Activity of fission products .....</b>	<b>3</b>
<b>3.3</b>	<b>Energy released by a fission chain .....</b>	<b>4</b>
<b>3.4</b>	<b>Concentration of a fission product .....</b>	<b>5</b>
<b>3.5</b>	<b>Fission Fragments Escape Probability .....</b>	<b>6</b>

***Divide in groups of 5:***

- you will solve the exercises in group*
- we will correct them together at the board*
- We split exercise 1 in (a) – (b-c) – (d-e)*

# 1. An example nuclear power plant

## Exercise description:

Consider a 900 MWe PWR.

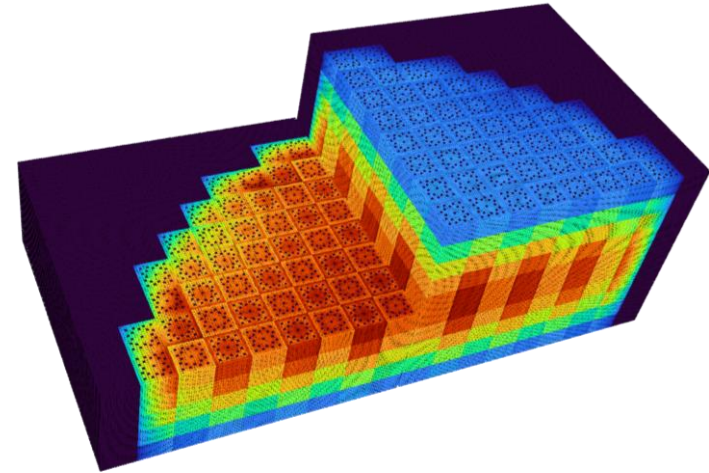
a) The efficiency for the conversion of heat into electricity is equal to 33%; how much is the nominal thermal power? How many fissions per second are necessary to produce this thermal power?

b) The core contains 157 fuel assemblies (0.215 m × 0.215 m square cross-section, and 3.658 m height); what size is its volume and its equivalent radius if it is transformed into a cylinder? It contains 82 t of uranium oxide (density: 10 300 kgm<sup>-3</sup>); how large is the volume of fuel, and what fraction of the total is occupied by the fuel?

c) How large is the power per unit of volume of core, and per unit of volume of fuel?

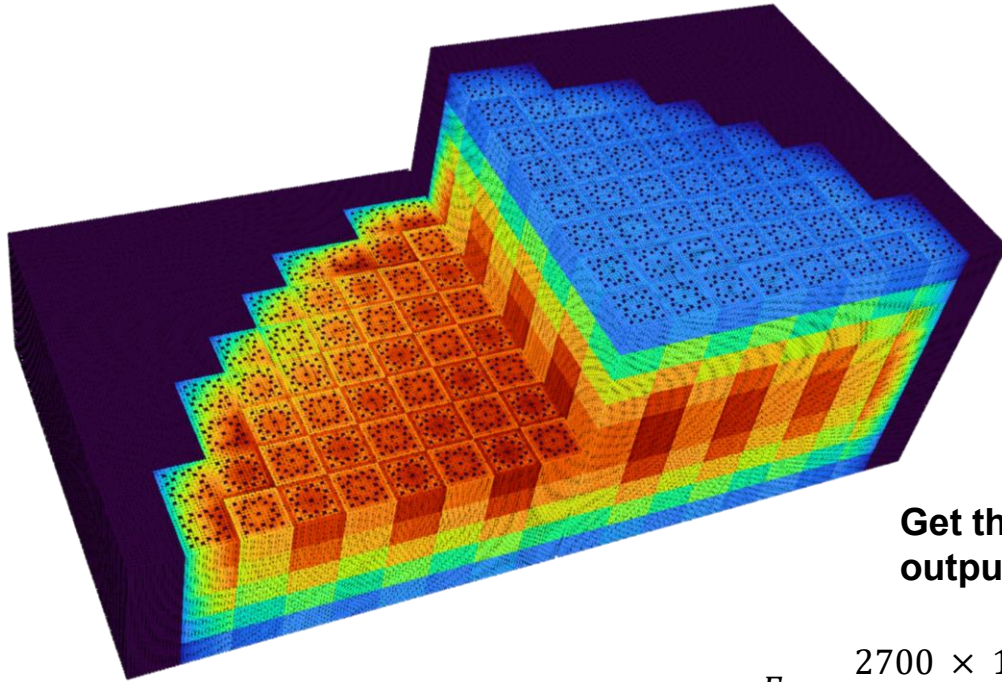
d) The uranium of the fuel is enriched up to 3%. Calculate the number of uranium-235 atoms per unit of volume. Using 582 barns for the uranium-235 fission microscopic cross-section, and neglecting fission of the uranium-238 atoms, calculate the macroscopic fission cross-section of the fuel. Consequently, how large is the mean neutron flux in the fuel?

e) Assuming that the neutrons are monokinetic, with a velocity equal to 3100 ms<sup>-1</sup>, calculate the mean neutron density in the fuel.



**Expected results:** (a)  $8.4 \times 10^{19}$  fissions; (b) 1.52m; 8 m<sup>3</sup>(30%); (c) 100 MWm<sup>-3</sup> ( 340 MWm<sup>-3</sup>); (d)  $\Sigma_f = 45 \text{ m}^{-1}$ ;  $\Phi = 2.33 \times 10^{17} \text{ m}^{-2} \text{ s}^{-1}$ ; (e)  $0.75 \times 10^{14} \text{ m}^{-3}$

# 1. An example nuclear power plant



Divide the electric output by the efficiency to get the **thermal output**:

$$P_{th} = \frac{900 \text{ MWe}}{0.33} \sim = 2700 \text{ MWth}$$

Consider 200 MeV as **energy released per fission**, convert this to Joules:

$$1 \text{ MeV} = 1.602176633999999 \text{E-13 J}$$

Get the **fission rate** by dividing the thermal output by the energy released by one fission

$$F = \frac{2700 \times 10^6 \text{ J/s}}{200 * 1.60\text{E-13 J}} = 8.4 \times 10^{19} \frac{\text{fissions}}{\text{s}}$$

(a) With 200 MeV per fission, the thermal power (2700 MWth) is generated by  $8.4 \times 10^{19}$  fissions

# 1. An example nuclear power plant

(b) The volume of the core is  $27 \text{ m}^3$  which represents an equivalent radius of 1.52 m when in the form of a cylinder:

$$R = \sqrt{\frac{A * N_{ASSEMBLIES} * H}{\pi * H}}$$

The volume  $V$  of the fuel is  $8 \text{ m}^3$ , where the mass is  $M$  and the density is  $D$ :

$$V = M/D$$

i.e. **30% of the core volume.**

**Fast forward question:** a typical W2LOOP PWR of the same size has 55% water, 21% fuel, the rest is zirconium and structural materials. Would this reactor have a higher or lower thermal utilization factor  $f$ ?

(c) Divide this power by the volumes. The power is  **$100 \text{ MW}/\text{m}^3$  of core**, or  **$340 \text{ MW}/\text{m}^3$  of fuel.**

# 1. An example nuclear power plant

(d) The molar mass of  $\text{UO}_2$  with 3 at% U-235 is given by :

$$M_{\text{UO}_2} = 0.03 \times 235 + 0.97 \times 238 + 2 \times 16 \text{ g/mol}$$

The number of moles of  $\text{UO}_2$ :

$$n = \frac{\rho_{\text{UO}_2}}{M_{\text{UO}_2}} = 3.816 \times 10^4 \text{ mol/m}^3$$

There is one uranium atom in every mole of  $\text{UO}_2$

$$N_U = n N_A$$

Three percent of these atoms are U235, from which we can compute the macroXS and the flux

$$N_{\text{U}235} = 0.03 N_U = 6.89 \times 10^{26} \text{ atoms/m}^3$$

$$\Sigma_f = N_{\text{U}235} \sigma_f \approx 6.89 \times 10^{26} \times 582 \times 10^{-28} \approx 40.1 \text{ m}^{-1}$$

$$\phi = \frac{q'''}{\Sigma_f * E_f} \approx 3.0 \times 10^{17} \text{ m}^{-2}\text{s}^{-1}$$

(e) For monokinetic neutrons,  $\phi = nv$  hence  $n = \Phi/v$

# 3. Energy released by a fission chain

## Exercise description:

---

- (a) Show that the energy released in the  $n^{\text{th}}$  generation of a fission chain reaction initiated by one fission, is given by:

$$E_n = k^{n-1} E_R$$

Where  $k$  is the multiplication factor and  $E_R$  is the recoverable energy per fission.

- (b) Show that the total energy released up to and including the  $n^{\text{th}}$  generation is given by:

$$E = \frac{k^n - 1}{k - 1} E_R$$

---

### 3. Energy released by a fission chain

- (a) The energy released at the first generation is  $E_R$  since there is one fission.  
The energy released at the second generation is  $k.E_R$  since there is  $k$  fissions  
The energy released at the third generation is  $k^2.E_R$  since there is  $k^2$  fissions

$$\text{So at the } n\text{th generation, } E_n = k^{n-1}E_R$$

- (b) The total energy released is the sum of the energy released at each generation, so

$$E = E_R + kE_R + k^2E_R + \dots + k^{n-1}E_R$$

Which is a geometric progression whose sum is:

$$E = \frac{k^n - 1}{k - 1} E_R$$



**In case the criticality condition is met, what is the limit of this sum?**

# 4. Concentration of a fission product

## **Exercise description:**

---

The fission product of  $^{131}\text{I}$  has a half-life of 8.05 days and is produced in fission with a yield of 2.9% (that is, 0.029 atoms of  $^{131}\text{I}$  are produced per fission). Calculate the equilibrium activity (in Ci) of this radionuclide in a reactor operating at 3300 MW.

**Expected results:**  $A_{\text{equilibrium}} = 8.07 \times 10^7 \text{ Ci}$

---

# 4. Concentration of a fission product

## Exercise solution:

The fission rate is defined as

$$FR = 3300 \text{ MW} \left( \frac{10^6 \text{ joules}}{\text{MW} \cdot \text{sec}} \right) \left( \frac{1 \text{ fission}}{200 \text{ MeV}} \right) \left( \frac{1 \text{ MeV}}{1.60 \times 10^{-13} \text{ joule}} \right) = 1.031 \times 10^{20} \frac{\text{fissions}}{\text{sec}}$$

At equilibrium

$$0 = \frac{dI}{dt} = \gamma_I FR - \lambda_I I$$

$$A_{\text{equilibrium}} = (0.029)(1.031 \times 10^{20}) = 2.990 \times 10^{18} \left[ \frac{\text{deintegration}}{\text{sec}} \right] = \frac{2.990 \times 10^{18}}{3.7 \times 10^{10}} = 8.07 \times 10^7 \text{ Ci}$$

## 2. Activity of fission products

$$A_{fp} = 1.4 \times 10^6 P (t^{-0.2} - (t + T)^{-0.2}) \text{ [Ci]}$$

- P - nominal power of the reactor [MWth]
  - T – operation time [days]
  - t – time following the shutdown [days]
- 
- Reactor loaded with 120 fuel elements
  - Operation of 1 year at 100MWth → fuel discharge

$$\text{Ci} = 3.7 \times 10^{10} \text{Bq}$$

## 2. Activity of fission products

$$A_{fp} = 1.4 \times 10^6 P (t^{-0.2} - (t + T)^{-0.2}) \text{ [Ci]}$$

- P - nominal power of the reactor [MWth]
  - T – operation time [days]
  - t – time following the shutdown [days]
- Reactor loaded with 120 fuel elements
- Operation of 1 year at 100MWth → fuel discharge

**What is the average  
FP activity in  
becquerels of a  
single fuel  
element 1 day after  
discharge?**

$$\text{Ci} = 3.7 \times 10^{10} \text{Bq}$$

## 2. Activity of fission products

$$A_{fp} = 1.4 \times 10^6 P (t^{-0.2} - (t + T)^{-0.2}) \text{ [Ci]}$$

- P - nominal power of the reactor [MWth]
- T – operation time [days]
- t – time following the shutdown [days]

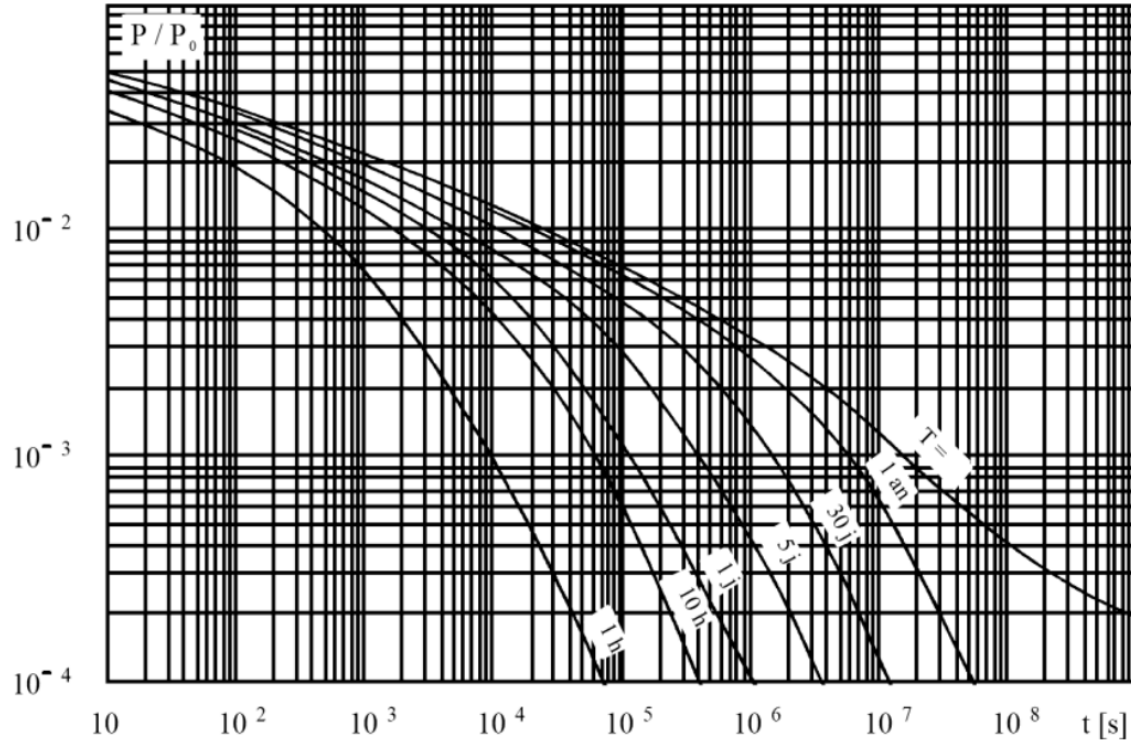
- Operation of 1 year

**What is (usually) the thermal power of a commercial LWR?**

**What is its FP activity 1 day after the shutdown (order of magnitude)?**

$$\text{Ci} = 3.7 \times 10^{10} \text{Bq}$$

## 2. Activity of fission products

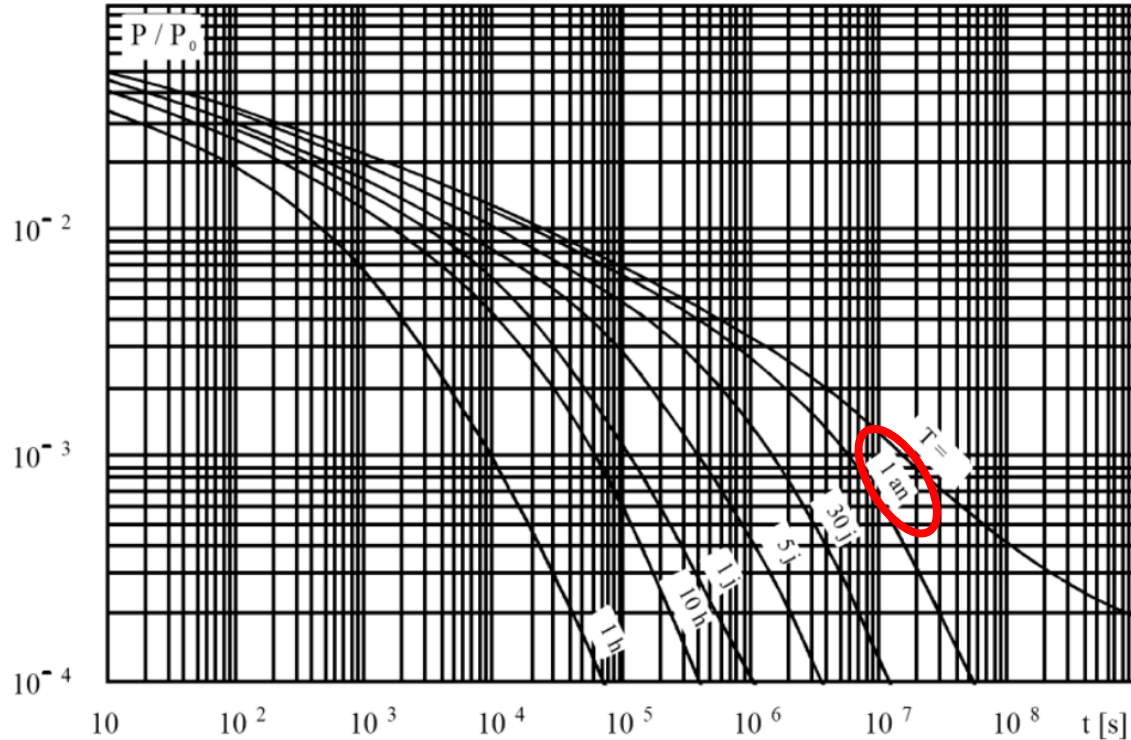


Why does the residual thermal power depend on the operation time before shutdown?

- 120 fuel elements
- Operation of 1 year at 100MWth

$$C_i = 3.7 \times 10^{10} \text{ Bq}$$

## 2. Activity of fission products

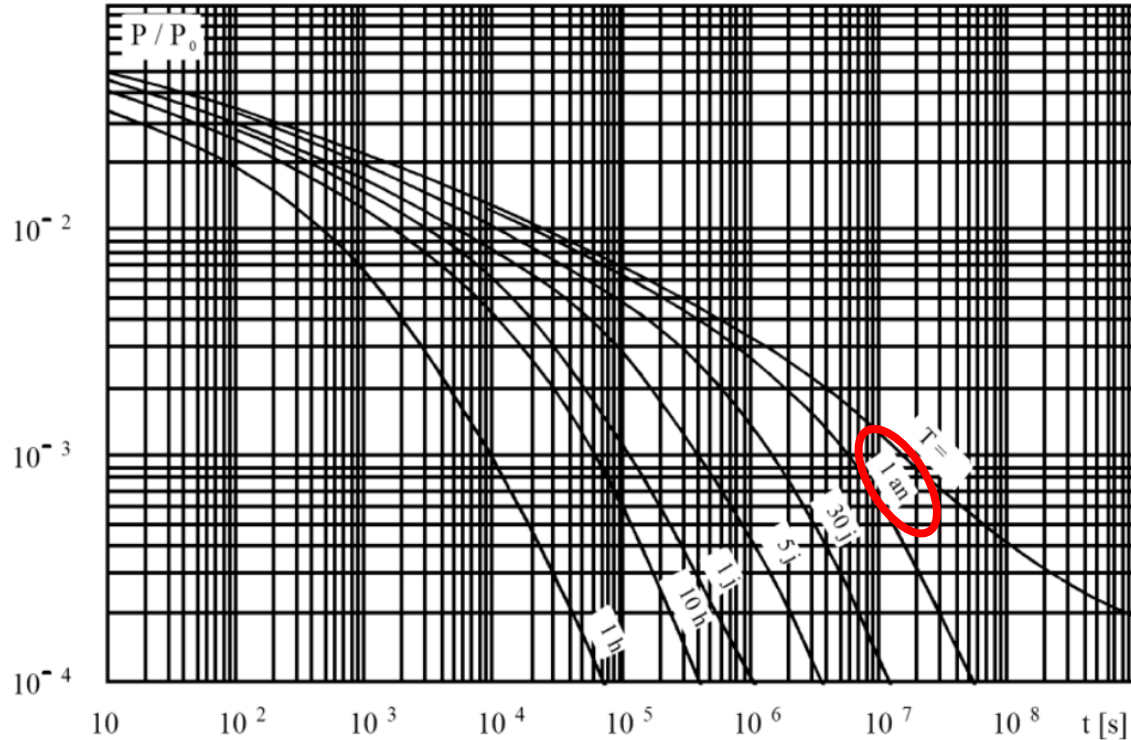


Give a coarse estimate of fraction of power still produced, in the seconds following the reactor shutdown.

- 120 fuel elements
- Operation of 1 year at 100MWth

$$C_i = 3.7 \times 10^{10} \text{ Bq}$$

## 2. Activity of fission products



Estimate the average residual thermal power of single fuel element using the graph (1 day after discharge)

- 120 fuel elements
- Operation of 1 year at 100MWth

$$C_i = 3.7 \times 10^{10} \text{ Bq}$$

# 5. Fragments Escape Probability

Near the surface of a flat fuel element in an operating reactor, fissions are occurring at the constant rate of  $S$  fissions/cm<sup>3</sup>-s. Given that the average range of the fission fragments is  $R$ , show that the rate at which such fragments would escape per cm<sup>2</sup>/sec from the surface of the fuel if it were not clad is equal to  $SR/2$ .

# 5. Fragments Escape Probability

## Exercise solution:

Assume that all fission fragments have the same range  $R$  in moving through the fuel.

Then any fragment whose track to the surface is less than  $R$  will escape.

Using the figure below, the number of fragments emanating from the unit area of the fuel of thickness  $dx$  which will escape, is

$$dN_E = S \cdot dx \cdot \frac{\alpha}{4\pi}$$

Where  $\alpha$  is the solid angle for which fragments escape from the fuel.

$\alpha$  is given by:

$$\alpha = 2\pi \int \sin v dv = 2\pi(1 - \cos v_{max})$$

The maximum polar angle for which fragments escape from the fuel is given by:

$$\cos v_{max} = \frac{x}{R}$$

So finally,  $\alpha = 2\pi \left(1 - \frac{x}{R}\right)$ .

Integrating over the full range of fission fragments, the total number of escape  $N_E$  per  $\text{cm}^2$  over one side of the fuel plate is then:

$$N_E = \int \frac{1}{4\pi} S \alpha dx = \frac{1}{2} S \int \left(1 - \frac{x}{R}\right) dx = \frac{1}{4} SR$$

Because of escape through both sides of the fuel plate, the total number of escape per  $\text{cm}^2$  is  $SR/2$

