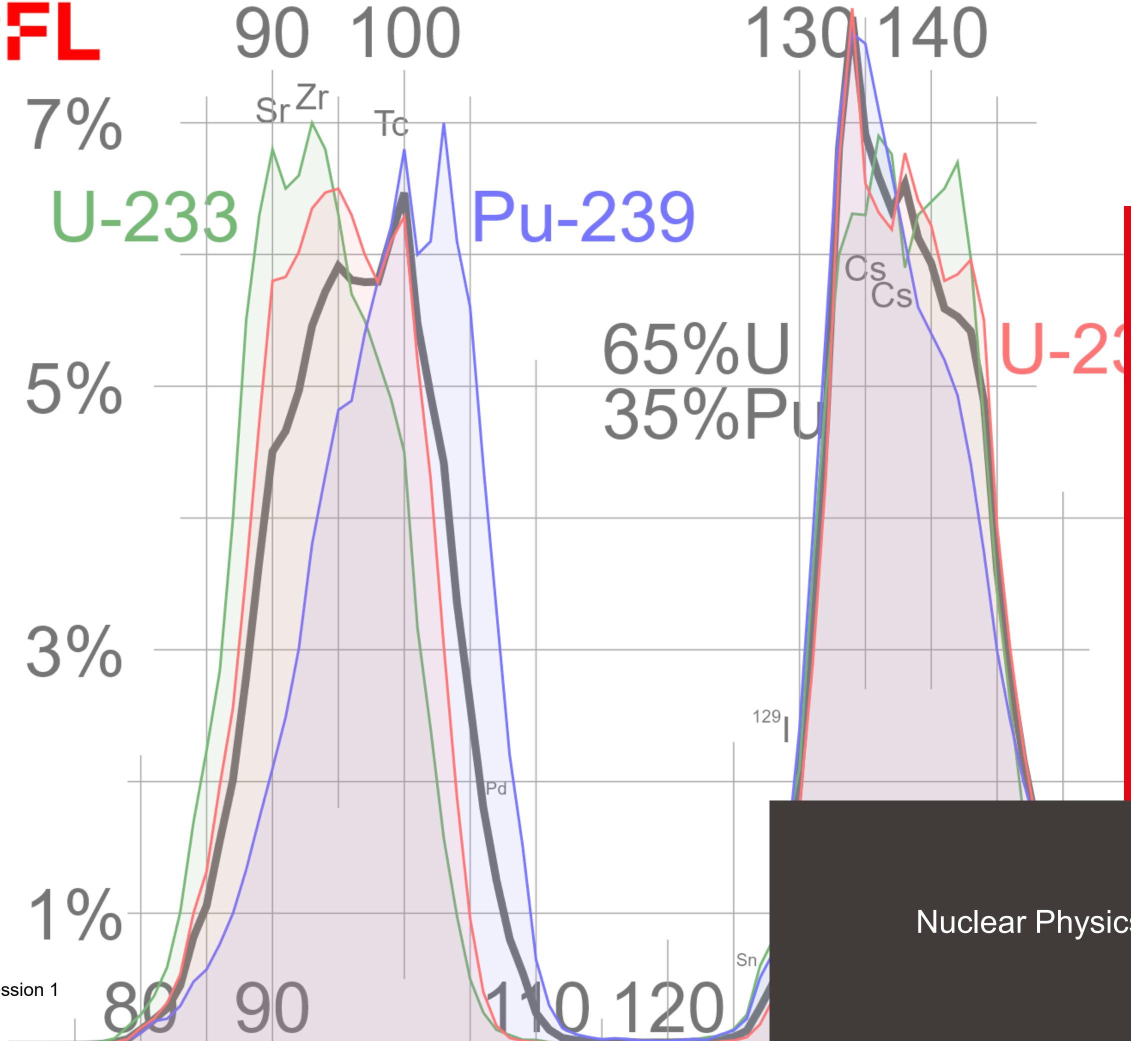

Physics of Nuclear Reactors

Nuclear Physics

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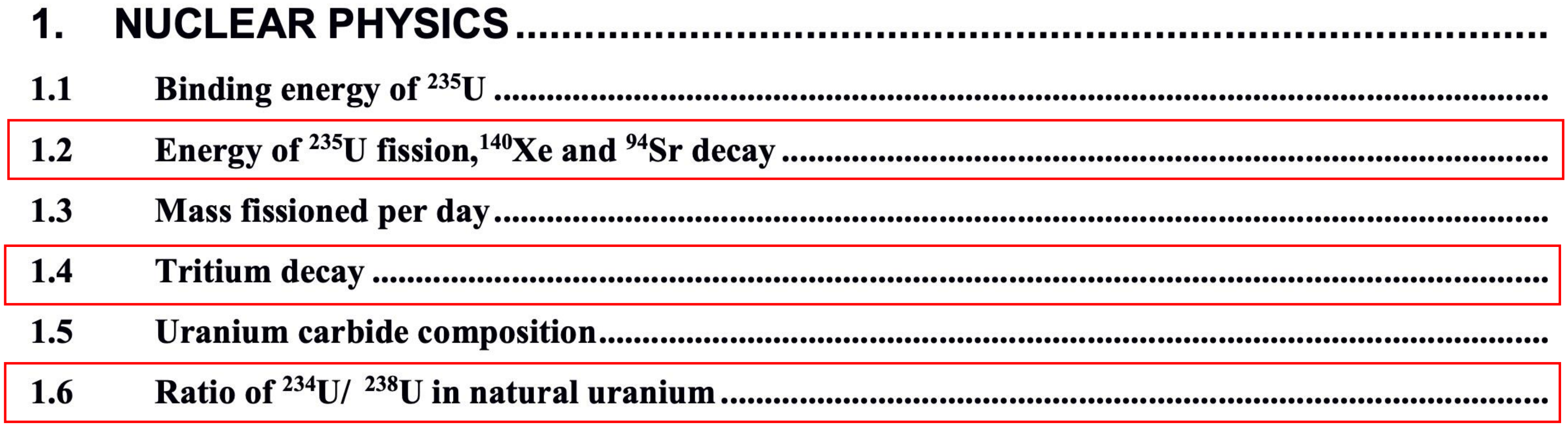
Exercises of today



Divide in groups of 5:

- *you will solve the exercises in group*
- *we will correct them together at the board*

2. Energy of ^{235}U fission, ^{140}Xe and ^{94}Sr decay

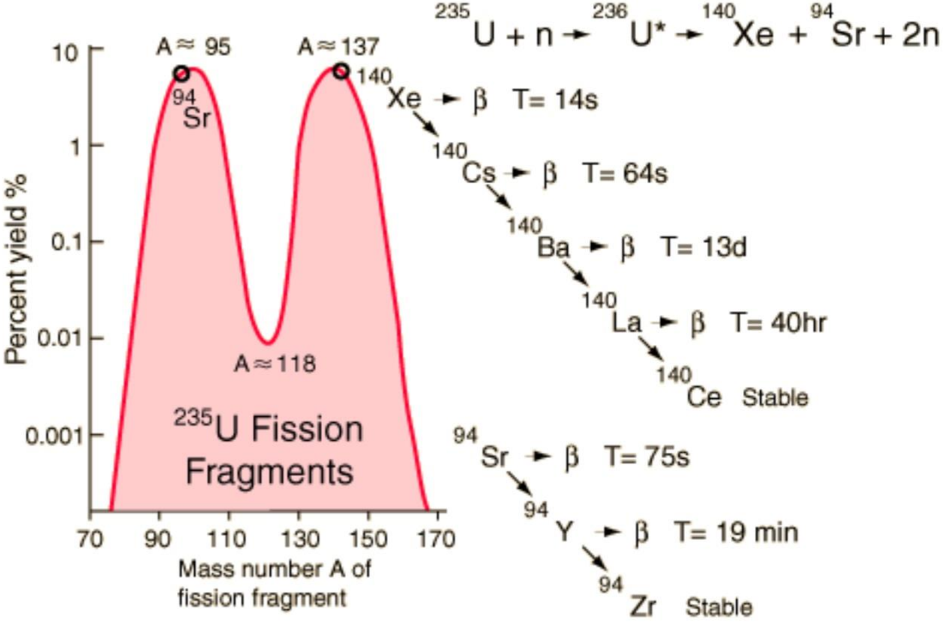
Exercise description:

a) Calculate the energy of the fission reaction $^{235}_{92}U + n \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2n$, $^{140}_{54}\text{Xe}$ mass is m = 139.92164u and $^{94}_{38}\text{Sr}$mass is m = 93.91536u. $^{235}_{92}U$mass is m=235.04393u; proton and neutron masses: 1.00728 and 1.008665 u, where $1u = 1.66054 \times 10^{-27}$kg $= 931.5 \times 10^6\ \text{eV}/c^2$.

b) The fission product $^{140}_{54}\text{Xe}$ and $^{94}_{38}\text{Sr}$, are radioactive. They will decay after certain period to stabile $^{140}_{58}\text{Ce}$ and $^{94}_{40}\text{Zr}$, with the masses 139.90544u and 93.9063u. Calculate the energy released by all these decays.

2. Energy of ^{235}U fission, ^{140}Xe and ^{94}Sr decay

Knowledge to be applied: $Q = \Delta mc^2$, $\Delta m = m_u + m_n - m_{\mathrm{Xe}} - m_{\mathrm{Sr}} - 2m_n$,
$1u = 1.66054 \times 10^{-27}\,\mathrm{kg} = 931.5 \times 10^6\ \mathrm{eV}/c^2$



$^{235}\mathrm{U} + n \rightarrow {}^{236}\mathrm{U}^* \rightarrow {}^{140}\mathrm{Xe} + {}^{94}\mathrm{Sr} + 2n$

$^{140}\mathrm{Xe} \rightarrow \beta$ T= 14s
$^{140}\mathrm{Cs} \rightarrow \beta$ T= 64s
$^{140}\mathrm{Ba} \rightarrow \beta$ T= 13d
$^{140}\mathrm{La} \rightarrow \beta$ T= 40hr
$^{140}\mathrm{Ce}$ Stable

$^{94}\mathrm{Sr} \rightarrow \beta$ T= 75s
$^{94}\mathrm{Y} \rightarrow \beta$ T= 19 min
$^{94}\mathrm{Zr}$ Stable

http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/fisfrag.html

Expected results: a)$Q = 184.7$MeVb) $Q = 15.1$MeV, $Q = 8.4$MeV

2. Energy of ^{235}U fission, ^{140}Xe and ^{94}Sr decay

Calculate** the mass defect **and multiply by** conversion factor **:

$$^{235}_{92}U + n \rightarrow {}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2n$$

$$\Delta m = \underline{m_u + m_n - m_{\mathrm{Xe}} - m_{\mathrm{Sr}} - 2m_n} = 235.04393 + 1.008665 - 139.92164 - 93.9153 - 2 \cdot 1.008665$$

$$Q = 0.19833 \cdot \underline{931.5 \cdot 10^6} = 184.7\mathrm{MeV}$$

The fact that the Q-value of the reaction is positive signifies that nucleons in the product nuclei are stronger bound than those in the reacting nuclei.

Calculate** the mass defect **and multiply by** conversion factor **:

b) Transmutation $^{140}_{54}Xe \rightarrow {}^{140}_{58}Ce$ is represented by 4 β^- decays

$$\Delta m = \underline{m_{\mathrm{Xe}} - m_{\mathrm{Ce}}} = 139.92164\text{-}139.90544 = 0.0162\mathrm{u}$$

$$Q = 0.0162 \times \underline{931.5 \times 10^6} = 15.1\mathrm{MeV}$$

Transmutation $^{94}_{38}Sr \rightarrow {}^{94}_{40}Xe$ is represented by 2 β^- decays

$$^{94}_{38}Sr \rightarrow {}^{94}_{40}Zr$$

$$\Delta m = m_{\mathrm{Sr}} - m_{\mathrm{Zr}} = 93.9153\text{-}93.9063 = 0.009\mathrm{u}$$

$$Q = 0.009 \times 931.5 \times 10^6 = 8.4\mathrm{MeV}$$

4. Tritium decay

Exercise description:

Tritium (^{3}H) decays by negative beta decay with a half-life of 12.26 years. Its molar mass is 3.016g.

(a) To what nucleus does ^{3}H decay?
(b) What is the mass of 1mCi of tritium?

Knowledge to be applied:

$1\text{Ci} = 3.7 \times 10^{10}\text{Bq} = 37\text{GBq}$, $A_x = \lambda_x N_x$, $N_A = 6.022 \times 10^{23}\ \text{mol}^{-1}$, $\lambda_x = \frac{\ln(2)}{T_{1/2}}$

Expected results: (b) $m_{3H} = 1.03 \cdot 10^{-7} g$

4. Tritium decay

Tritium is a beta minus emitter. It consists in the transformation of a neutron into a proton and an electron. The product is ^3_2He:

$$^3_1H \rightarrow \, ^3_2He + e^-$$

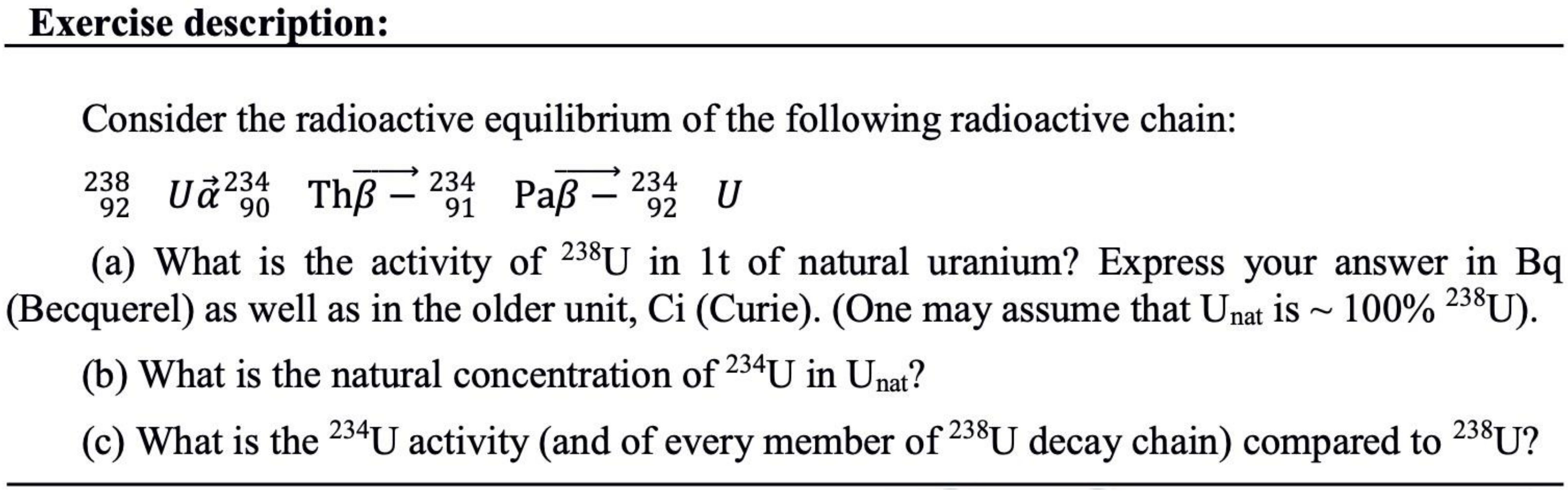
Convert Ci in Bq and **divide by tritium decay constant**:

$$N = \frac{A}{\lambda} = \frac{1 \cdot 10^{-3} Ci \cdot 3.7 \cdot 10^{10} Bq/Ci}{ln2/(12.26y \cdot 3.154 \cdot 10^{7} s/y)} = 2.06 \cdot 10^{16}$$

Use the **molar mass of tritium** and the **Avogadro constant** to recover the mass of the sample:

$$m = \frac{NM}{N_A} = \frac{2.06 \cdot 10^{16} \cdot 3g/mol}{6.022 \cdot 10^{23} 1/mol} = 1.03 \cdot 10^{-7} g$$

6. Ratio of ^{234}U/ ^{238}U in natural uranium



Exercise description:

Consider the radioactive equilibrium of the following radioactive chain:

$$^{238}_{92}\ U\vec{\alpha}\,^{234}_{90}\ \mathrm{Th}\overrightarrow{\beta -}\,^{234}_{91}\ \mathrm{Pa}\overrightarrow{\beta -}\,^{234}_{92}\ U$$

(a) What is the activity of ^{238}U in 1t of natural uranium? Express your answer in Bq (Becquerel) as well as in the older unit, Ci (Curie). (One may assume that U_{nat} is ~ 100% ^{238}U).

(b) What is the natural concentration of ^{234}U in U_{nat}?

(c) What is the ^{234}U activity (and of every member of ^{238}U decay chain) compared to ^{238}U?

6. Ratio of ^{234}U / ^{238}U in natural uranium

Knowledge to be applied:

The half-lives $t_{1/2}$ are:

^{238}U: $4.5 \cdot 10^9$ y, ^{234}Th : 27 d, ^{234}Pa: 1.2 m,

^{234}U : $2.5 \cdot 10^5$ y

$$1\text{Ci} = 3.7 \times 10^{10}\text{Bq} = 37\text{GBq},$$

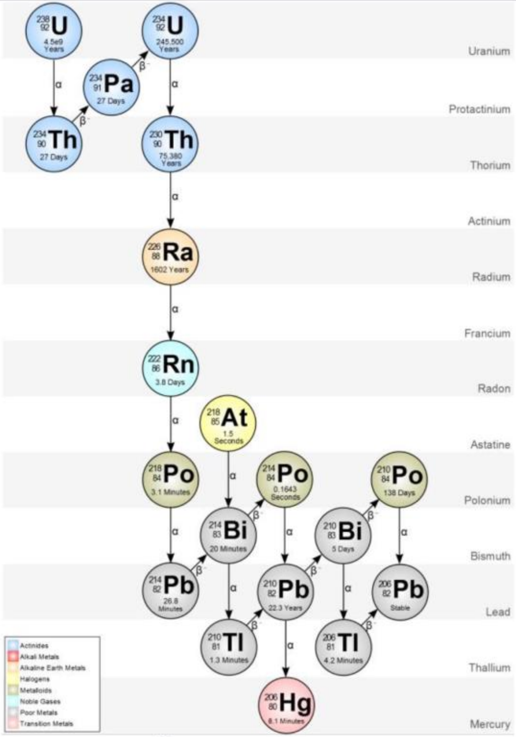
$$\frac{\mathrm{dN}_x(t)}{\mathrm{dt}} = -\lambda_x N_x(t) + S_x(t),$$

$$\frac{\mathrm{dN}_x(t)}{\mathrm{dt}}_{\text{equilibrium}} = 0,$$

$$A_x = \lambda_x N_x,$$

$$N_A = 6.022 \times 10^{23}\ \text{mol}^{-1},$$

$$\lambda_x = \frac{\ln(2)}{T_{1/2}},$$



http://metadata.berkeley.edu/nuclear-forensics/Decay%20Chains.html

Expected results: (a) $A_{U238} = 1.235 \times 10^{10}\text{Bq} = 0.334\text{Ci}$, (b) $\frac{N_{U234}}{N_{U238}} = 0.0056$, (c) the same

6. Ratio of ^{234}U/ ^{238}U in natural uranium

Assuming 100% ^{238}U one can determine the amount of atoms in 1 ton of U:

$$1\text{ton}(U) = 10^6/238 \times 6.02214129 \times 10^{23} = 2.53 \times 10^{27}\text{atoms}$$

One can use **the decay constant of** ^{238}U to recover **the activity** of the sample:

$$\lambda_{U238} = \ln(2)/T_{1/2,U238} = \ln(2)/(4.5 \times 10^9 \times 365 \times 24 \times 3600) = 4.88 \times 10^{-18} s^{-1}$$

$$A_{U238} = \lambda_{U238} N_{U238} = 4.88 \times 10^{-18} \times 2.53 \times 10^{27} = 1.225 \times 10^{10}\text{Bq}$$

$$A_{U238} = 1.235 \times 10^{10}\text{Bq} = 1.235 \times 10^{10}/3.7 \times 10^{10}\text{Ci} = 0.334\text{Ci}$$

6. Ratio of ^{234}U/ ^{238}U in natural uranium

The second part of the exercise requires expliciting **a set of ODE:**

$$ {}^{238}_{92}U\vec{\alpha}\,{}^{234}_{90}\mathrm{Th}\overrightarrow{\beta -}\,{}^{234}_{91}\mathrm{Pa}\overrightarrow{\beta -}\,{}^{234}_{92}U $$

Equilibrium condition

$$0 = \frac{dN_{234Th}}{dt} = -\lambda_{234Th}N_{234Th} + \lambda_{238U}N_{238U}$$

$$0 = \frac{dN_{234Pa}}{dt} = -\lambda_{234Pa}N_{234Pa} + \lambda_{234Th}N_{234Th} \longrightarrow \lambda_{234U}N_{234U} = \lambda_{234Pa}N_{234Pa} = \lambda_{234Th}N_{234Th} = \lambda_{238U}N_{238U}$$

$$0 = \frac{dN_{234U}}{dt} = -\lambda_{234U}N_{234U} + \lambda_{234Pa}N_{234Pa}$$

The activities of all the members of the decay chain are the same.

1. Binding Energy of ^{235}U

Exercise description:

Calculate the "binding energy" of Uranium (100% $^{235}_{92}U$) in MeV per atom E_b, in MeV per nucleon $E_{b,A}$, and in MJ per kg E_{1kg}. $^{235}_{92}U$ mass is m=235.04393u; proton and neutron masses: 1.00728 and 1.008665 u, where u is unified atomic mass unit defined as 1/12 weight of $^{12}_{6}C$.

Knowledge to be applied: $E_b = \Delta mc^2$, $E_{b,A} = E_b/A$, $\Delta m = Zm_p + (A - Z)m_n - m$, $1u = 1.66054 \times 10^{-27}\text{kg} = 931.5 \times 10^6\ \text{eV}/c^2$, $1\text{eV} = 1.6022 \times 10^{-19}$ J

Expected results: $E_b = 1737.8\text{MeV}$, $E_{b,A} = 7.395\text{MeV}$, and $E_{1\text{kg}} = 7.134 \cdot 10^8\text{MJ}$

1. Binding Energy of ^{235}U

$$\Delta m = \mathrm{Z}\mathrm{m}_p + (A - Z)m_n - m = 92 \times 1.00728 + (235 - 92) \times 1.00867 - 235.04393 = 1.86564u$$

$$E_b = \Delta \mathrm{mc}^2 = 1.86564 \times 931.5 \times 10^6 = 1737.8\mathrm{MeV}$$

$$E_{b,A} = E_b/A = 1737.8/235 = 7.395\mathrm{MeV}$$

$$m_U = 235.04393u$$

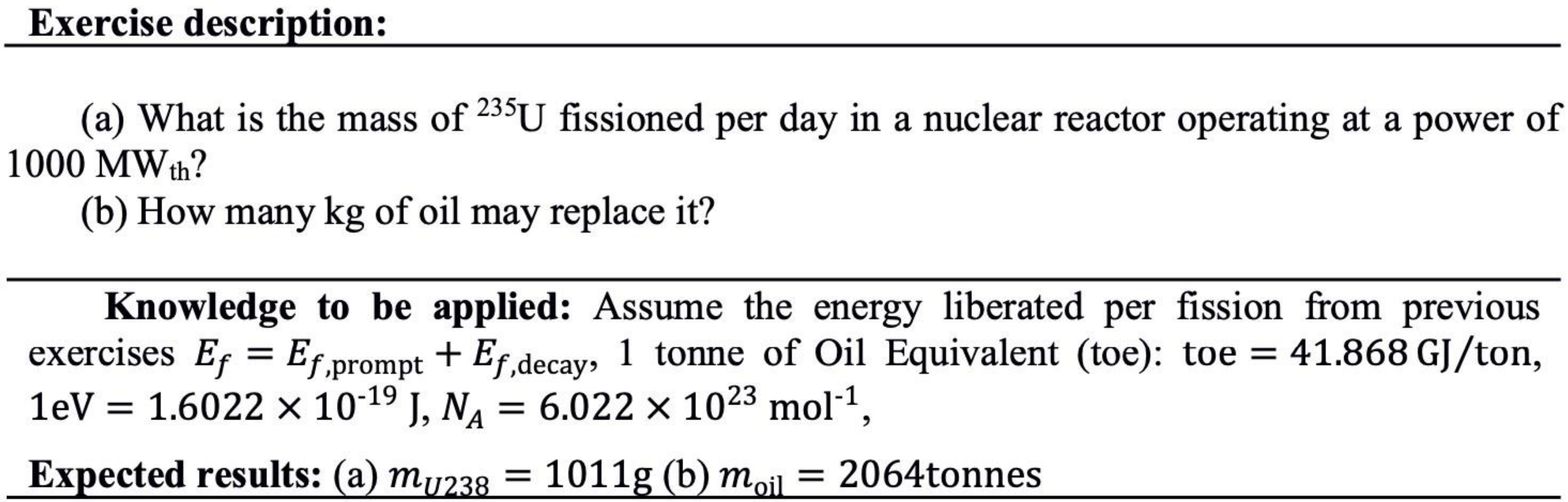
$$m_U = 235.04393 \times 1.66054 \times 10^{-27} = 390.3 \times 10^{-27}\mathrm{kg}$$

$$1\mathrm{kg}(U) \Rightarrow 1/390.3 \times 10^{-27} = 2.562 \times 10^{24}\mathrm{atoms}$$

$$E_{1\mathrm{kg}} = 2.562 \times 10^{24} \times 1737.8 = 4.452 \times 10^{27}\mathrm{MeV}$$

$$E_{1\mathrm{kg}} = 4.452 \times 10^{27} \times 1.6022 \times 10^{-19} = 7.134 \cdot 10^8\mathrm{MJ}$$

3. Mass fissioned per day



Exercise description:

(a) What is the mass of ^{235}U fissioned per day in a nuclear reactor operating at a power of 1000 MW_{th}?

(b) How many kg of oil may replace it?

Knowledge to be applied: Assume the energy liberated per fission from previous exercises $E_f = E_{f,\text{prompt}} + E_{f,\text{decay}}$, 1 tonne of Oil Equivalent (toe): toe $= 41.868\,\text{GJ/ton}$, 1eV $= 1.6022 \times 10^{-19}$ J, $N_A = 6.022 \times 10^{23}$ mol^{-1},

Expected results: (a) $m_{U238} = 1011$g (b) $m_{\text{oil}} = 2064$tonnes

3. Mass fissioned per day

(a)

$$1d = 24 \times 60 \times 60 = 86400s$$

$$E_{1d} = 1000 \times 10^{6} \times 86400 = 8.64 \times 10^{13} J$$

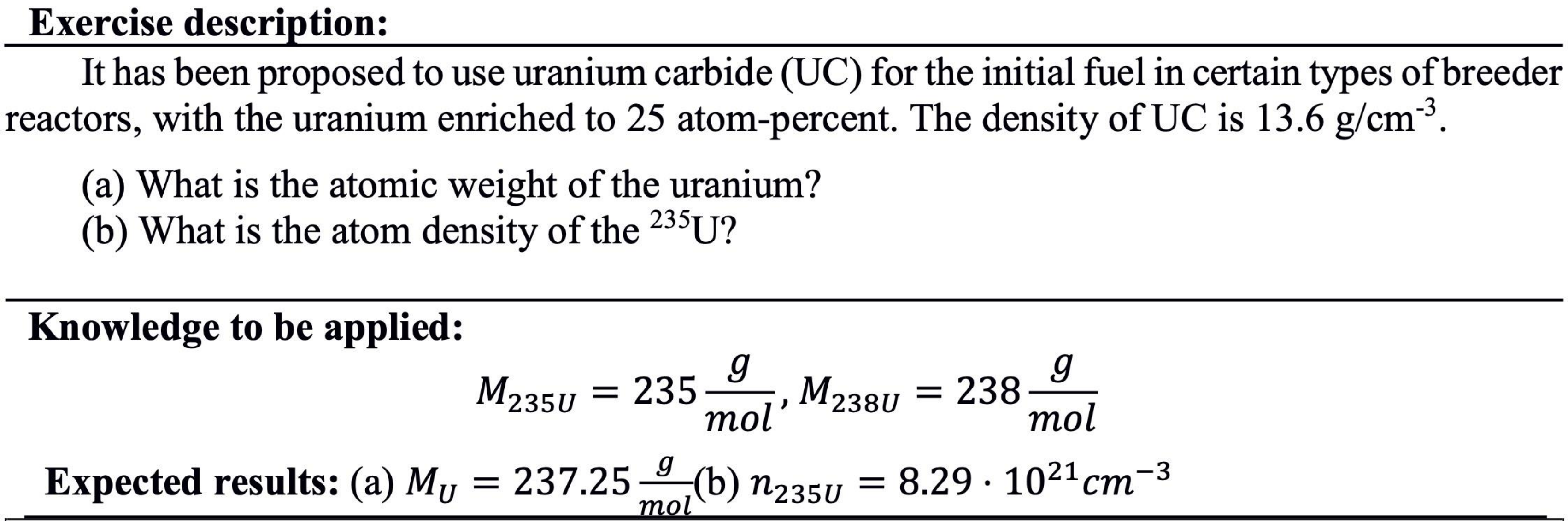
$$E_f = 184.7 + 15.1 + 8.4 = 208.2\text{MeV} = 208.2 \times 10^{6} \times 1.6022 \times 10^{-19} = 3.334 \times 10^{-11} J$$

$$N_{U238} = E_{1d}/E_f = 8.64 \times 10^{13}/3.36510^{-11} = 2.59 \times 10^{24}$$

$$m_{U238} = N_{U238}/N_A * 235 = 2.59 \times 10^{24}/6.022 \times 10^{23}*235 = 1011\text{g}$$

(b) $m_{\text{oil}} = E_{1d}/\text{toe} = 8.64 \times 10^{13}/41.868 \times 10^{9} = 2064\text{tonnes}$

5. Uranium carbide composition



Exercise description:

It has been proposed to use uranium carbide (UC) for the initial fuel in certain types of breeder reactors, with the uranium enriched to 25 atom-percent. The density of UC is 13.6 g/cm^{-3}.

(a) What is the atomic weight of the uranium?
(b) What is the atom density of the ^{235}U?

Knowledge to be applied:

$$M_{235U} = 235\frac{g}{mol}, M_{238U} = 238\frac{g}{mol}$$

Expected results: (a) $M_U = 237.25\frac{g}{mol}$(b) $n_{235U} = 8.29 \cdot 10^{21} cm^{-3}$

5. Uranium carbide composition

Exercise solution:

(a) $M_U = \frac{25}{100} M_{235U} + \frac{75}{100} M_{238U} = 237.25\, g/mol$

(b)Density of U per cm^{-3}:

$$\rho_U = \frac{M_U}{M_U + M_C} \rho_{UC} = 12.95\, g/cm^3$$

Number of atom of ^{235}U per cm^{-3}:

$$n_{235U} = \frac{25}{100} \frac{\rho_U N_A}{M_U} = 8.21 \cdot 10^{21} cm^{-3}$$