

Modeling and design of experiments

Chapitre 3: Analysis of variance

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Analysis of variance

- Analysis of variance of a model as a whole

- Anova of the coefficients of a model

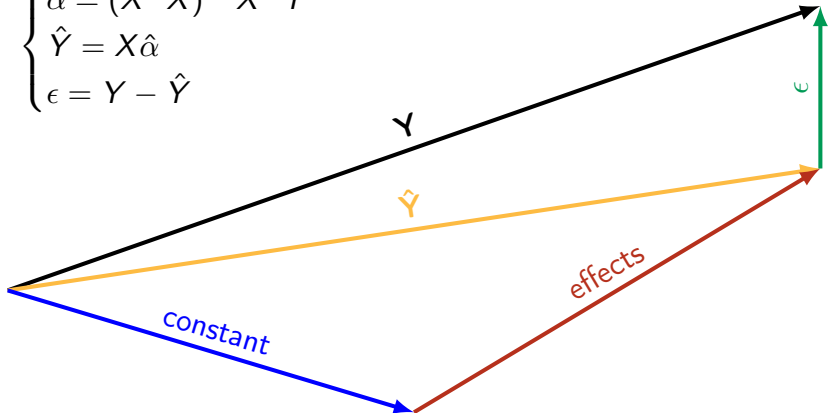
- The concept of alias

3.1.1 Pedagogical objectives

1. Understand how an ANOVA table is structured
2. Understand the consequences of non orthogonality
3. Being able to compute an ANOVA table for orthogonal and non-orthogonal designs
4. Being able to use and interpret the Matlab routine *fitlm()*
5. Being able to interpret the output of a regression

3.1.2 Analysis of variance (geometric perspective)

$$\begin{cases} \hat{\alpha} = (X^T X)^{-1} X^T Y \\ \hat{Y} = X \hat{\alpha} \\ \epsilon = Y - \hat{Y} \end{cases}$$



3.1.3 ANOVA of a model as a whole

Sources	SS	DF	MS	F	p
Model	$SS_{\hat{Y}}$	P	$MS_{\hat{Y}} = \frac{SS_{\hat{Y}}}{P}$	$x = \frac{MS_{\hat{Y}}}{MS_{\epsilon}}$	$F(x, P, N - P)$
Residue	SS_{ϵ}	$N - P$	$MS_{\epsilon} = \frac{SS_{\epsilon}}{N - P}$		
Total	SS_Y	N	–		

N is the number of runs and P , the number of coefficients in the model

3.1.4 ANOVA of the Young modulus model

Sources	SS	DF	MS	F	p
Model	396 990.2	4	99 247.55	$4.8 \cdot 10^6$	$1.22 \cdot 10^{-16}$
Residue	0.1	5	0.02		
Total	396990.3	9	–		

Model : $E = 210 + 0.24x_C - 0.63x_S - 0.053x_T$ avec $x_i \in [-1, 1]$
 99% of the SS comes from the constant : So this table does not give interesting information on the quality of the model.

3.1.5 ANOVA without the constant

Sources	SS	DF	MS	F	p
Model (without const.)	2.00	3	0.67	32.2	0.11%
Residue	0.1	5	0.02		
Total	2.10	9	–		

The analysis without the constant is *sharper*, indicating clearly that the experiments have put effects in evidence

It would be also interesting to know which coefficients of the model are significant and which one could be neglected (parsimony principle).

3.2.1 How to decompose a model

At the level of the linear system, the parting of a model in two sub-models is done that way :

$$\begin{aligned}\hat{Y} &= \hat{Y}_1 + \hat{Y}_2 \\ X\hat{\alpha} &= [X_1 \ X_2] \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = X_1\hat{\alpha}_1 + X_2\hat{\alpha}_2\end{aligned}$$

At the level of the sum of squares it gives :

$$\begin{aligned}\hat{Y}^2 &= (\hat{Y}_1 + \hat{Y}_2)^2 \\ &= \hat{Y}_1^2 + 2\hat{Y}_1 \cdot \hat{Y}_2 + \hat{Y}_2^2 \\ &= \hat{Y}_1^2 + \hat{Y}_2^2 \quad \text{if and only if } \hat{Y}_1 \cdot \hat{Y}_2 = 0\end{aligned}$$

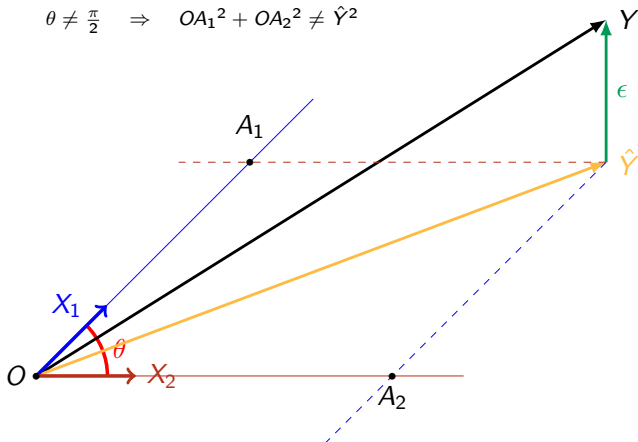
3.2.2 ANOVA for two orthogonal parts

Source	SS	DF	MS	F	p
Partie 1	$SS_{\hat{Y}_1}$	P_1	$\frac{SS_{\hat{Y}_1}}{P_1}$	$x_1 = \frac{MS_{\hat{Y}_1}}{MS_{\epsilon}}$	$F(x_1, P_1, N - P)$
Partie 2	$SS_{\hat{Y}_2}$	P_2	$\frac{SS_{\hat{Y}_2}}{P_2}$	$x_2 = \frac{MS_{\hat{Y}_2}}{MS_{\epsilon}}$	$F(x_2, P_2, N - P)$
Résidu	SS_{ϵ}	$N - P$	$\frac{SS_{\epsilon}}{N - P}$		
Total	SS_Y	N	-		

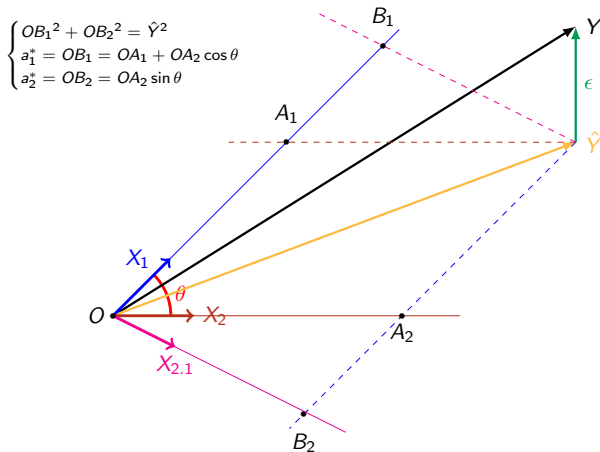
N is the number of runs and P_1 et P_2 , the number of coefficients of the parts 1 and 2 respectively, $P = P_1 + P_2$

3.2.3 Orthogonal decomposition

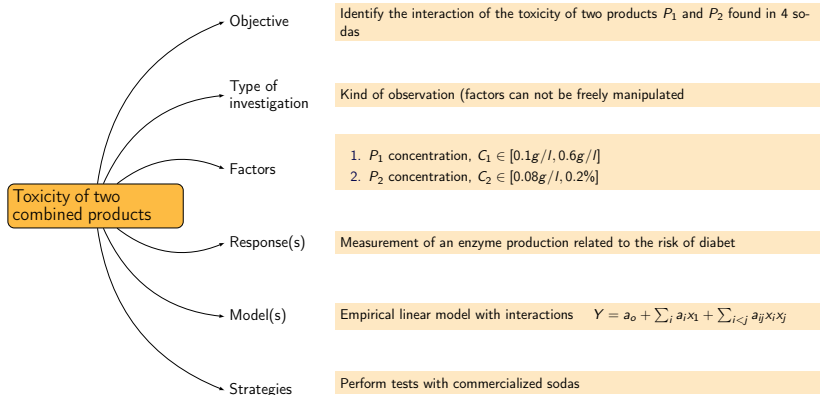
$$\theta \neq \frac{\pi}{2} \Rightarrow OA_1^2 + OA_2^2 \neq \hat{Y}^2$$



3.2.3 Orthogonal decomposition (2)

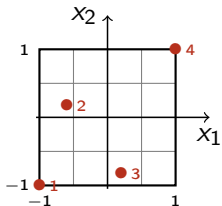


3.2.4 Determination of a cocktail effect



3.2.5 Design of experiments

- Points of measurement :



- Model matrix :

$$X = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -0.6 & 0.17 & -0.1 \\ 1 & 0.2 & -0.83 & -0.17 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -0.6 & 0.17 & -0.1 \\ 1 & 0.2 & -0.83 & -0.17 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

- Dispersion matrix :

$$(X'X)^{-1} = \begin{pmatrix} 0.22 & -0.01 & 0.07 & -0.19 \\ -0.01 & 0.39 & -0.25 & 0.02 \\ 0.07 & -0.25 & 0.36 & -0.08 \\ -0.19 & 0.02 & -0.08 & 0.41 \end{pmatrix}$$

- Variance inflation factors :

-	a_0	a_1	a_2	a_{12}
VIF	1.7	1.9	2	1.7

The analysis shows that the design is applicable

3.2.6 Inference of the coefficients

After the experiments

Experimental data :

Expériences	1	2	3	4
Y(set 1)	80.4	70.8	67.1	270.0
Y(set 2)	89.7	58.9	53.7	275.3

Model coefficients :

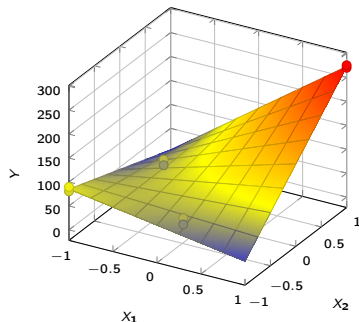
-	a_0	a_1	a_2	a_{12}
α_j	97.7	52.6	41.2	81.2
α_j/α_0	-	54%	42%	83%

Estimator :

$$\hat{\alpha} = (X'X)^{-1} X'Y$$

Model :

$$Y = 97.7 + 52.6x_1 + 41.2x_2 + 81.2x_1x_2$$



3.2.7 Angles between regressors and SS

- ▶ The information matrix ($X'X$) gives the product of the regressors 2 by 2

- ▶ The scalar product is defined as $\vec{x}_i \cdot \vec{x}_j = \|\vec{x}_i\| \|\vec{x}_j\| \cos \phi_{ij}$

- ▶ The angles between the regressors can then be computed by

$$\phi_{ij} = \arccos \left(\frac{k_{ij}}{\sqrt{k_{ii}} \sqrt{k_{jj}}} \right) \text{ if } k_{ij} \text{ are the element of the matrix of information}$$

-	x_1	x_2	x_{12}
I	97°	102°	53°
x_1	-	47°	89°
x_2	-	-	87°

Regressor	I	x_1	x_2	x_{12}
$SS(a_i x_i)$	76 366	13 291	9 231	26 842
$\sum SS(a_i x_i)$		125 730		
$SS(Y)$		178 863	$R = 1.42$	

3.2.8 Sequential orthogonalisation

1. Compute the half effect, the estimate and the residue for a model with 1 regressor (let's say a_0)
2. Compute the sum of squares $SS(a_0)$ for this model and $SS(\epsilon_0)$ for the corresponding residue
3. Compute the half effects, the estimates and the residue for a model with 2 regressors (let's say a_0 et a_1)
4. Compute the sum of squares $SS(a_1|a_0)$ by subtracting $SS(a_0)$ from the sum of squares of the model with two regressors (or from the difference between the sums of squares of the two residues)
5. etc.

3.2.9 ANOVA with SS of type I

	Source	SS	SS*	DF	MS	F	P
▶ $SS(\hat{Y})$	Model	178 863		4	44 716	818	$1.1 \cdot 10^{-8}$
	Residue 1	218		4	55		
▶ $SS(a_0)$	a_0		116 634	1			
	Residue 2		62 447	7			
	a_1		37 034	1			
▶ $SS(a_1 a_0)$	a_1		37 034	1			
	a_2		9 139	1			
	Residue 3		25 414	6			
	a_2		9 139	1			
▶ $SS(a_2 a_0, a_1)$	a_2		9 139	1			
	Residue 4		16 274	5			
	a_{12}		16 056	1			
	Residue 5		219	4			
	Total	179 082	179 082	8			

3.2.10 Types de SS

$$SS(A|B) = SS(A, B) - SS(A)$$

Type I (sequential)

$SS(a_0)$ for a_0

$SS(a_1|a_0)$ for a_1

$SS(a_2|a_0, a_1)$ for a_2

$SS(a_{12}|a_0, a_1, a_2)$ for a_{12}

Type II

$SS(a_0|a_1, a_2, a_{12})$ for a_0

$SS(a_1|a_0, a_2, a_{12})$ for a_1

$SS(a_2|a_0, a_1, a_{12})$ for a_2

$SS(a_{12}|a_0, a_1, a_2)$ for a_{12}

3.2.11 Overview of Sum of Squares (SS)

In ANOVA, different types of sum of squares (SS) are used to assess the significance of factors and their interactions. The three main types are :

- ▶ **Type I (Sequential) SS**
- ▶ **Type II SS**
- ▶ **Type III SS**

These types differ in how they handle the order of terms and interactions in the model.

3.2.12 Type I (Sequential) SS

- ▶ **Sequential Analysis** : Tests factors in the order they are entered into the model.
- ▶ **Interpretation** : Each factor is tested after adjusting for the preceding factors, but not for later factors.
- ▶ **Usage** :
 - ▶ Suitable for balanced designs or when factors are naturally ordered.
 - ▶ Useful when you want to assess the incremental effect of each factor.

Example : In a model with factors A , B , and $A \times B$, Type I SS tests A , then tests B after adjusting for A , and then the interaction term $A \times B$.

3.2.13 Type II SS

- ▶ **Adjusts for Other Main Effects** : Tests the main effects while ignoring interactions.
- ▶ **Interpretation** : Each factor is tested after adjusting for other main effects, but not for interactions.
- ▶ **Usage** :
 - ▶ Suitable for **balanced designs** or when interactions are not significant.
 - ▶ Useful when you want to test main effects independently of other factors.

Example : In a model with factors A , B , and $A \times B$, Type II SS tests the main effects of A and B while ignoring the interaction term.

3.2.14 Type III SS

- ▶ **Adjusts for All Terms** : Tests each factor after adjusting for all other factors and their interactions.
- ▶ **Interpretation** : Each factor is tested after considering all other factors, including interactions.
- ▶ **Usage** :
 - ▶ Preferred for **unbalanced designs** and when **interactions are included**.
 - ▶ Ensures that the effect of each factor is evaluated fairly, regardless of the order of terms.

Example : In a model with factors A , B , and $A \times B$, Type III SS tests A while adjusting for B and $A \times B$, and similarly for other terms.

3.2.15 Criteria for Choosing the Type of SS

- ▶ **Type I (Sequential) SS :**
 - ▶ Use when the order of factors matters or for **balanced** designs.
 - ▶ Appropriate when factors are added sequentially, and you want to test their incremental effect.
- ▶ **Type II SS :**
 - ▶ Use for **balanced** designs and when you are not interested in interactions.
 - ▶ Suitable when the main effects are the focus and interactions can be ignored.
- ▶ **Type III SS :**
 - ▶ Use for **unbalanced** designs or when **interactions** are included in the model.
 - ▶ Best when you want to ensure that each factor effect is tested after considering all other factors.

3.2.16 Comparison between type I and type II

Source	SS*	DF	MS	F	P
a_1	37 033	1	37 033	678	0.001 %
a_2	9 074	1	9 074	166	0.021 %
a_{12}	16 122	1	16 122	295	0.007 %
Résidu 1	219	4	55		
Total	179 082	8			

Source	SS*	DF	MS	F	P
a_1	7 130	1	7 130	130	0.034 %
a_2	4 700	1	4 700	86	0.075 %
a_{12}	16 122	1	16 122	295	0.007 %
Résidu 1	219	4	55		
Total	179 082	8			

3.3.1 What is an alias ?

- ▶ Alias : Zorro and Diego de la Vega
- ▶ The Concept of alias is useful for dealing with non-orthogonal situations
- ▶ It let compute the connection between two parts of a model
- ▶ Examples :
 - ▶ For a given design, what is the consequence of considering or not a regressor ?
 - ▶ For a given design, what is the consequence of considering the second degree coefficients on the first degree coefficients ?
 - ▶ In some situations we will see that the fact of considering the second degree coefficients will change the value of the constant, meaning that the second degree coefficients are aliased with the constant.

3.3.2 The alias matrix

The alias matrix corresponds to the projection of the base vectors of the second subspace on the base vectors of the first sub-space

- ▶ Let's consider a linear model $y = f(x_1, \dots, x_N, a_0, a_1, \dots, a_M)$ and a design with the model matrix X
- ▶ Now let's part the model in two parts $f = f_1 + f_2$ with the corresponding model matrix X_1 et X_2 so that $X = [X_1, X_2]$
- ▶ The **alias matrix** A of X_2 in relation to X_1 is :

$$A = (X_1^T X_1)^{-1} (X_1^T X_2)$$

3.3.3 What is the alias used to ?

To make the link between the two subspace :

$$\hat{Y}_1 = X_1 \hat{\alpha}$$

$$\hat{Y} = [X_1 \ X_2] \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix}$$

Coefficients : $\hat{\alpha} = \hat{\alpha}_1 + A \hat{\alpha}_2$

Orthogonal projection : $X_{2.1} = X_2 - X_1 A$

Orthogonal decomposition :

$$Y = X_1(\hat{\alpha}_1 + A \hat{\alpha}_2) + (X_2 - X_1 A)\alpha_2 + \epsilon$$

$$Y = X_1 \alpha + X_{2.1} \alpha_2 + \epsilon$$

3.3.4 Example of an alias matrix

- ▶ Let's consider a design E and the model

$$y = a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2$$

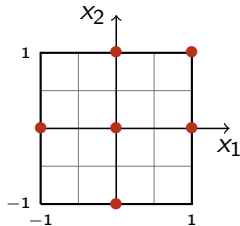
- ▶ the separation is between the linear and the interaction parts :

$$f_1(x) = a_0 + a_1x_1 + a_2x_2$$

$$f_2(x) = a_{12}x_1x_2$$

$$\text{▶ } X_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix} \text{ et } X_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}$$



3.3.5 Example of an alias matrix (2)

$$\begin{aligned}
 A &= (X_1^T X_1)^{-1} (X_1^T X_2) \\
 &= \frac{1}{44} \begin{pmatrix} 8 & -2 & -2 \\ -2 & 17 & -5 \\ -2 & -5 & 17 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix}^T \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
 &= \frac{1}{44} \begin{pmatrix} 8 & -2 & -2 \\ -2 & 17 & -5 \\ -2 & -5 & 17 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/11 \\ 5/22 \\ 5/22 \end{pmatrix} \Rightarrow \begin{cases} l_0 = a_0 + \frac{1}{11} a_{12} \\ l_1 = a_1 + \frac{5}{22} a_{12} \\ l_2 = a_2 + \frac{5}{22} a_{12} \end{cases}
 \end{aligned}$$

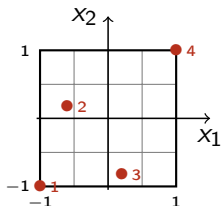
3.3.6 ANOVA table for non orthogonal parts

Source	SS	SS*
X_1	$\alpha_1' X_1' X_1 \alpha_1$	$\alpha' X_1' X_1 \alpha = (\alpha_1 + A\alpha_2)' X_1' X_1 (\alpha_1 + A\alpha_2)$ $\alpha_2' X_{2.1}' X_{2.1} \alpha_2 = \alpha_2' (X_2 - X_1 A)' (X_2 - X_1 A) \alpha_2$
X_2	$\alpha_2' X_2' X_2 \alpha_2$	
Résidu	$\epsilon' \epsilon$	
Total	SS_Y	–

N is the number of runs, A the alias matrix relative, α the coefficients of the first part of the model when it is inferred alone and $X_{2.1}$ is the model matrix of the second part of the model orthogonal to the first part.

3.3.7 Let's go back to the case of the cocktail

Runs



Model matrix

$$X = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -0.6 & 0.17 & -0.1 \\ 1 & 0.2 & -0.83 & -0.17 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -0.6 & 0.17 & -0.1 \\ 1 & 0.2 & -0.83 & -0.17 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Corrected sum of squares

$$a_0 | a_1, a_2, a_{12} \rightarrow A_0 = (-0.1 \quad -0.165 \quad 0.433)$$

which means that :

$$a_0^* = a_0 - 0.1a_1 - 0.165a_2 + 0.433a_{12}$$

Then :

$$SS(a_0 | a_1, a_2, a_{12}) = \left(a_0 + A_0 \begin{bmatrix} a_1 \\ a_2 \\ a_{12} \end{bmatrix} \right)^T X_0^T X_0 \left(a_0 + A_0 \begin{bmatrix} a_1 \\ a_2 \\ a_{12} \end{bmatrix} \right)$$

And so on for the next steps :

$$a_0, a_1 | a_2, a_{12} \rightarrow A_1 = \begin{pmatrix} -0.09 & 0.44 \\ 0.71 & 0.09 \end{pmatrix}$$

$$a_0, a_1 | a_2, a_{12} \rightarrow A_2 = \begin{pmatrix} 0.46 \\ -0.04 \\ 0.18 \end{pmatrix}$$

Note : Matrices and arrays in Matlab

- ▶ Arrays are the **fundamental data type** used to store collections of data in the form of elements arranged in rows and columns.
- ▶ Arrays can be one-dimensional (vectors) or two-dimensional (matrices), but MATLAB also supports **multidimensional** arrays.
- ▶ Arrays can hold **various types of data**, such as numbers, strings, or even more complex objects.
- ▶ Most operations in MATLAB are **vectorized**, meaning that they are applied element-wise to arrays, which makes computations with arrays fast and efficient.
- ▶ The function `repmat(X,v,l)` create a new array by repetition of X, v times vertically and l times horizontally.

```
>> V=[1,2;3,4]
```

```
V =
```

```
    1    2  
    3    4
```

```
>> U=ones(3)
```

```
U =
```

```
    1    1    1  
    1    1    1  
    1    1    1
```

```
>> I=eye(3)
```

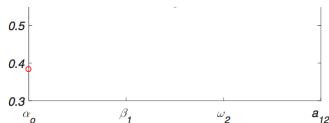
```
I =
```

```
    1    0    0  
    0    1    0  
    0    0    1
```

Note : cell array

- ▶ **Cell arrays** are a type of data structure that allows you to store elements of varying types and sizes.
- ▶ Unlike regular arrays, a **cell array can hold different types of data in each of its cells.**
- ▶ Each element in a cell array is accessed using **curly braces { }** to retrieve the actual content inside the cell.

```
>> label={'\alpha_o' '\beta_1' '\omega_2' 'a_{12}'}  
  
label =  
  
1x4 cell array  
  
    '\alpha_o'    '\beta_1'    '\omega_2'    'a_{12}'  
  
>> plot(1:4,sin([1:4]*pi/8),'or')  
>> set(gca,'XTick',1:4,'XTickLabel',label)  
>> |
```



Note : Table in Matlab

- ▶ A table is a data type specifically designed to store and organize heterogeneous data, where **each column can hold a different type of data** (e.g., numerical, text, or categorical).

```
Nom={'Alluminium';'Plomb';'Cuivre';'Fer'};  
E=[0.72E11;0.17E11;1.00E11;2.20E11];  
mu=[.34;.45;.34;.30];  
Symbole={'Al';'Pb';'Cu';'Fe'};
```

```
T=table(E,mu,Symbole,'RowNames',Nom);
```

	E	mu	Symbole
Alluminium	7.2e+10	0.34	'Al'
Plomb	1.7e+10	0.45	'Pb'
Cuivre	1e+11	0.34	'Cu'
Fer	2.2e+11	0.3	'Fe'

Note : Linear model in Matlab

*Linear*model is an object created by routines such as *fitlm* or *stepwiselm* and with the following content

- ▶ experimental data,
- ▶ model description,
- ▶ statistics for a diagnostic,
- ▶ estimated coefficients,
- ▶ residuals.

The object can be reused to predict the responses of the model with the methods *predict* and *feval*

3.3.8 Routines *fitlm* and *stepwiselm*

These MATLAB functions return a linear regression model fit to variables in the table or dataset array.

Matlab

- ▶ `mdl=fitlm(tbl)`
`mdl=fitlm(tbl,modelspec)`
`mdl=fitlm(x,y)`
`mdl=fitlm(x,y,modelspec)`
`mdl=fitlm(...,Name,Value)`
- ▶ `mdl=stepwiselm(tbl,modelspec)`
`mdl=stepwiselm(x,y,modelspec)`
`mdl=stepwiselm(...,Name,Value)`

These routines can be fed by tables or arrays

3.3.9 `fitlm()` output

Linear regression model:

$R \sim 1 + \text{Var1} * \text{Var2}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
	_____	_____	_____	_____
(Intercept)	97.643	3.4385	28.397	9.1511e-06
Var1	52.5	4.5959	11.423	0.00033506
Var2	41.297	4.4527	9.2748	0.00075164
Var1:Var2	81.214	4.7279	17.178	6.7382e-05

Number of observations: 8, Error degrees of freedom: 4

Root Mean Squared Error: 7.39

R-squared: 0.997, Adjusted R-Squared 0.994

F-statistic vs. constant model: 380, p-value = 2.29e-05

3.3.10 Wilkinson's notation

Wilkinson notation is a concise way to specify the terms in a linear model. It describes the relationships between predictors (independent variables) and a response (dependent variable) without explicitly stating the coefficients of the model.

Example : $Y \sim 1 + X_1 * X_2$ represents the model $y = a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2$

Termse of the model	Wilkinson's notation
intercept a_0	1
sans intercept	-1
a_1	X1
a_1, a_2	X1 + X2
a_1, a_2, a_{12}	X1 * X2 ou X1 + X2 + X1 : X2
a_{12}	X1 : X2
a_1, a_{11}	X1 ²
a_{11}	X1 ² - X1

3.3.11 Methods for linear model objects

Matlab offers several methods to be used with *linearmodel* objects

Matlab

- ▶ `anova mdl`
- ▶ `coefCI mdl`
- ▶ `coefTest mdl`, `coefTest mdl, H, C`
- ▶ `plot mdl`
- ▶ `plotAdded mdl, coef`
- ▶ `plotDiagnostics mdl, plotype`
- ▶ `plotResiduals mdl`
- ▶ `plotEffects mdl`
- ▶ `ypred = predict mdl, Xnew`

1. Significance of Coefficients and R-squared

- ▶ **Check p-values :**
 - ▶ Small p-values (< 0.05) indicate that the corresponding predictors are statistically significant.
- ▶ **R-squared :**
 - ▶ Measures the proportion of variance explained by the model.
 - ▶ Ranges from 0 to 1 ; higher values indicate a better fit.
- ▶ **Adjusted R-squared :**
 - ▶ Adjusts for the number of predictors in the model.
 - ▶ Prevents overfitting by penalizing for additional, non-informative variables.

2. Residual Analysis and Normality of Residuals

▶ Residual Plot :

- ▶ Check if residuals are randomly scattered around zero.
- ▶ Patterns (curvature, funnel shape) may indicate misspecification.

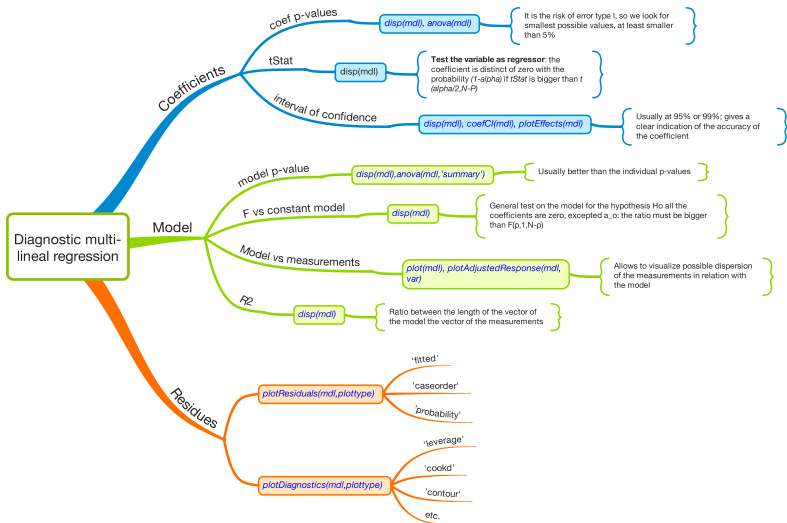
▶ Normality of Residuals :

- ▶ Use a Q-Q plot to check if residuals follow a normal distribution.
- ▶ Apply the Shapiro-Wilk test for a formal test of normality.

3. Homoscedasticity and Leverage & Influence

- ▶ **Homoscedasticity (Constant Variance) :**
 - ▶ Residuals should have constant variance across the range of predictors.
 - ▶ Breusch-Pagan test can formally check for heteroscedasticity.
- ▶ **Leverage and Influence :**
 - ▶ Use leverage statistics to detect points with large influence on the model.
 - ▶ Cook's distance can help identify outliers that may disproportionately affect the model.

3.3.12 Diagnostic of a LSF



3.3.13 *candexch* routine

The *candexch()* function in MATLAB is used to generate optimal experimental designs. It is commonly used when working with a set of candidate points to select the most informative subset for fitting a model.

It selects a subset of points from a candidate set that maximizes the D-optimality criterion, ensuring the most information is gained from the least number of experimental runs. By exchanging points iteratively, it refines the design to provide a robust and efficient design for fitting statistical models.

Matlab

- ▶ *list=candexch(X,nrows)*
- ▶ This routine *candidate exchange* allows the selection of the best *nrows* of a model matrix in the D-optimal perspective,
- ▶ The standard routine proposes duplicated points,

3.3.14 Summary : ANOVA in DOE

- ▶ **Purpose of ANOVA :**
 - ▶ Decomposes total variance into components (Model and Residual).
 - ▶ Tests the significance of factor effects and interactions.

- ▶ **Key Concepts :**
 - ▶ **Sum of Squares (SS)** : Measures variability attributed to factors.
 - ▶ **Mean Squares (MS)** : SS divided by degrees of freedom (DF).
 - ▶ **F-statistic** : Ratio of MS for model terms to MS for residuals.
 - ▶ **p-values** : Indicates significance of factors.

- ▶ **Model Interpretation :**
 - ▶ Significant terms ($p\text{-value} < 0.05$) indicate meaningful effects.
 - ▶ Main effects and interactions are analyzed through ANOVA tables.

- ▶ **Common Applications :**
 - ▶ Factorial experiments : Assess main effects and interactions.
 - ▶ Response surface methodology : Investigate curvature and optimization.