The Schwarzschild Solution

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1 Gargantua

This exercise discusses certain aspects of the black hole *Gargantua* in the movie *Interstellar* by C. Nolan. We will neglect the spin of the black hole and assume that it can be described by the Schwarzschild geometry.

A particularly impressive moment of the movie is the visit of a planet (*Miller*'s planet), orbiting around Gargantua, where one hour correspond to seven years for an observer far away from the black hole. We will assume for simplicity that the trajectory of the planet is circular.

Q1. How close should Miller's planet be to the Schwarzschild radius of the black hole? Is that actually possible?

Let us overlook this issue, and turn to tidal forces. In the movie, Gargantua is supposed to be a supermassive black hole of mass $M = 10^8 M_\odot$.

Q2. Calculate the tidal effect of Gargantua on Miller's planet, assuming that the latter is comparable to the Earth. Estimate the height of tides from a simple model, and compare to the 1.2 km given in the movie.

Q3. In fact, there should not be tides on this planet. Any guess why?
Hint: If the Moon had oceans, would there be tides due to the Earth on it?

2 Black hole shadow

Consider the Schwarzschild metric of a spherically symmetric black hole,

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{dr^2}{1 - r_s/r} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right). \tag{1}$$

Q1. Show that a photon moving in the equatorial plane $\theta = \pi/2$ follows a trajectory $r(\phi)$ obeying

$$\left(\frac{dr}{d\phi}\right)^2 + V(r) = 0, \tag{2}$$

and determine $V(r)$.

Q2. Show that $V(r)$ can be expressed solely in terms of r, r_s and the impact parameter b.
Hint: You may study (16) at large r to relate b to other constants of motion.

Q3. Photons sent from infinity towards the black hole may or may not fall through the horizon depending on the impact parameter b. Compute the critical impact parameter b_c such that all photons with $b < b_c$ fall through the horizon.

Q4. What is the impact parameter b_1 such that the incoming photon goes around the black hole once and then continues exactly in the same direction? You may define b_1/r_s implicitly in a form that could easily be given to a computer (for example, as a solution of an equation, possibly involving integrals).