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Swiss Plasma Center
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Plasma I **Winter semester 2025**

Formula sheet

Below is a list of key equations from the course. Students will have access to this formula sheet while presenting at the blackboard during the oral examination.

Maxwell–Boltzmann distribution:

$$f_\alpha(\vec{v}) = \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2} \exp\left(-\frac{m_\alpha}{2T_\alpha}\vec{v}^2\right)$$

Key equations related to Coulomb collisions:

$$\tan\frac{\theta}{2} = \frac{b_{90}}{b} \qquad b_{90} \equiv b_{90}(v) = \frac{q_1 q_2}{4\pi\varepsilon_0 \mu v^2} \qquad \mu = \frac{m_1 m_2}{m_1+m_2}$$

$$\nu_p^{e/i}(v_e) = n_i \frac{Z^2 e^4}{4\pi\varepsilon_0^2} \frac{\ln\Lambda}{m_e^2 v_e^3} \qquad \nu_{E_k}^{j/k} \sim n_k \frac{Z_j^2\, Z_k^2 e^4}{2\pi\varepsilon_0^2} \frac{\ln\Lambda_k}{m_j\, m_k v_j^3}$$

MHD equations:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0; \qquad \nabla\cdot\mathbf{J} = 0;$$

$$\rho\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathbf{J}\times\mathbf{B} - \nabla p; \qquad \mathbf{E} + \mathbf{u}\times\mathbf{B} = \begin{cases} 0 & \text{“ideal” MHD} \\ \eta\mathbf{J} & \text{“resistive” MHD} \end{cases}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(p\rho^{-\gamma}) = 0; \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J};$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}; \qquad \nabla\cdot\mathbf{B} = 0;$$

Linearized MHD equations, with $\mathbf{E}_1$ and $\mathbf{j}_1$ eliminated:

$$\frac{\partial\rho_1}{\partial t} + \rho_0\nabla\cdot\mathbf{u}_1 = 0$$

$$\rho_0\frac{\partial\mathbf{u}_1}{\partial t} = -\nabla p_1 + \frac{1}{\mu_0}(\nabla\times\mathbf{B}_1)\times\mathbf{B}_0$$

$$\frac{\partial\mathbf{B}_1}{\partial t} = \nabla\times(\mathbf{u}_1\times\mathbf{B}_0)$$

$$\frac{\partial p_1}{\partial t} = c_s^2\frac{\partial\rho_1}{\partial t}$$

After FT and elimination of p_1:

$$-\omega\rho_1 + \rho_0\mathbf{k}\cdot\mathbf{u}_1 = 0$$

$$-\omega\rho_0\mathbf{u}_1 = -\mathbf{k}\rho_1 c_s^2 + \frac{1}{\mu_0}(\mathbf{k}\times\mathbf{B}_1)\times\mathbf{B}_0$$

$$-\omega\mathbf{B}_1 = \mathbf{k}\times(\mathbf{u}_1\times\mathbf{B}_0)$$

Wave equation:

$$\left\{N^2\left[\frac{\mathbf{kk}}{k^2}-\mathbb{1}\right]+\underline{\boldsymbol{\varepsilon}}\right\}\cdot\mathbf{E}=0 \qquad \underline{\boldsymbol{\varepsilon}}\equiv\frac{\mathrm{i}\underline{\boldsymbol{\sigma}}}{\varepsilon_0\omega}+\mathbb{1} \qquad N^2\equiv\frac{k^2c^2}{\omega^2}$$

Dielectric tensor for cold plasma in two-fluid model (no background flow, uniform B-field along z):

$$\epsilon_1=1+\sum_\alpha\frac{\omega_{\mathrm{p}\alpha}^2}{\Omega_\alpha^2-\omega^2} \qquad \epsilon_2=-\sum_\alpha\frac{\Omega_\alpha}{\omega}\frac{\omega_{\mathrm{p}\alpha}^2}{\Omega_\alpha^2-\omega^2} \qquad \epsilon_3=1-\sum_\alpha\frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2}$$

With k in the yz-plane:

$$N^2\left[\frac{\mathbf{kk}}{k^2}-\mathbb{1}\right]+\underline{\boldsymbol{\varepsilon}} \;=\; \begin{pmatrix}-N^2+\epsilon_1 & -\mathrm{i}\epsilon_2 & 0\\ \mathrm{i}\epsilon_2 & -N^2\cos^2\theta+\epsilon_1 & N^2\sin\theta\cos\theta\\ 0 & N^2\sin\theta\cos\theta & -N^2\sin^2\theta+\epsilon_3\end{pmatrix}$$

Dielectric function, Vlasov-Poisson system (unmagnetized plasma):

$$\epsilon(\mathrm{i}p,k) \;=\; 1-\sum_\alpha\frac{\omega_{\mathrm{p}\alpha}^2}{n_\alpha k^2}\int_{\mathcal{L}}\frac{\frac{\mathrm{d}F_{\alpha 0}}{\mathrm{d}u}}{u-\frac{\mathrm{i}p}{k}}\mathrm{d}u \qquad\qquad \mathrm{i}p=\omega$$

Landau's rule:

$$\int_{\mathcal{L}}\frac{h(u)}{u-\frac{\mathrm{i}p}{k}}\mathrm{d}u=\begin{cases}\displaystyle\int_{-\infty}^{\infty}\frac{h(u)}{u-\mathrm{i}p/k}\mathrm{d}u & \Re\{p\}>0\\ \displaystyle\mathrm{P.V.}\int\frac{h(u)}{u-\mathrm{i}p/k}\mathrm{d}u+\mathrm{i}\pi\,h(\frac{\mathrm{i}p}{k}) & \Re\{p\}=0\\ \displaystyle\int_{-\infty}^{\infty}\frac{h(u)}{u-\mathrm{i}p/k}\mathrm{d}u+2\pi\mathrm{i}\,h(\frac{\mathrm{i}p}{k}) & \Re\{p\}<0\end{cases}$$