Plasma Physics I

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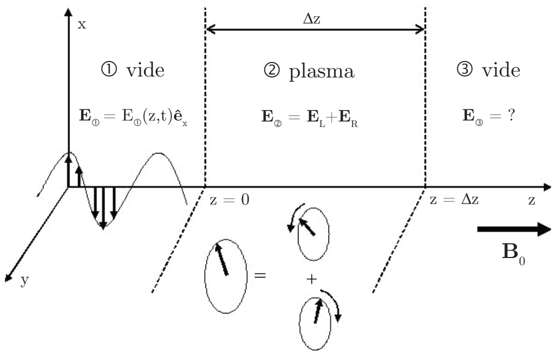
Exercise 1



Consider the situation displayed above. An electromagnetic wave propagating along the magnetic field B_0 crosses a portion of plasma. In the plasma, consider the dispersion relation of an electromagnetic wave in a cold uniform plasma :

$$N^2 = \frac{k^2c^2}{\omega^2} = \frac{(\omega \mp \omega_R)(\omega \pm \omega_L)}{(\omega \pm \Omega_i)(\omega \mp |\Omega_e|)} \approx 1 - \frac{\omega_{pe}^2/\omega^2}{1 \mp |\Omega_e|/\omega}; \quad (\omega \gg \Omega_i)$$

The upper sign is related to the right-handed wave (R) and the lower sign to the left-handed wave (L).

a.) Show that the rotation (polarization) angle α, when the wave exits the plasma, is equal to half of the phase difference between the two waves. Find a relation for α as a function of the distance travelled, ω, Ω_e and ω_{pe}. Consider the limit:

$$\frac{\omega_{pe}^2/\omega^2}{1 \mp |\Omega_e|/\omega} \ll 1$$

b.) The Faraday rotation of a micro-wave beam ($\lambda = 8\,\text{mm}$) in a uniform plasma with a magnetic field $B_0 = 0.1\,\text{T}$ is measured. When the beam propagates through 1 m of plasma, the polarization direction turns of $90°$. Find the plasma density.

Exercise 2

Show that in a plasma described by the *Vlasov* equation:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{x}} + \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

the evolution of the energy of the particles, E_p, and of the fields, E_f, is described by:

$$\frac{dE_p}{dt} = -\frac{dE_f}{dt} = \int d^3x(\boldsymbol{j} \cdot \boldsymbol{E})$$

and the total energy is conserved.