

Plasma I

Series 2 (September 17, 2025)

Prof. Christian Theiler

Swiss Plasma Center (SPC)

École Polytechnique Fédérale de Lausanne (EPFL)

Exercise 1

A partially ionized slab of plasma has the following density distribution:

$$n(x) = n_0 \cos\left(\frac{\pi x}{2L}\right), \quad -L \leq x \leq L$$

Diffusion and recombination processes are present and a source term ensures $\partial n / \partial t = 0$. The recombination gives a reduction term equal to $-\alpha n^2$ in the continuity equation. Consider a constant diffusion term $D = 0.1 \text{m}^2/\text{s}$, $\alpha = 10^{-15} \text{m}^3/\text{s}$ and $L = 2 \text{m}$.

- a.) Find an equation expressing the global balance of the total number of plasma particles, taking into account the global losses due to diffusion and the ones due to recombination. Use this equation to find the rate at which particles must be injected by the source to maintain the given density distribution, that is express $\int_{-L}^L S(x) dx$ as a function of D , α , L and n_0 .
- b.) Find the value of the peak density (n_0) that would correspond to having the rate of losses at the wall equal to the recombination rate.
- c.) Estimate the plasma particle mean life-time for that value of density.

Exercise 2

Consider a weakly ionized hydrogen plasma in a long cylindrical vacuum tube (length L , radius r_c , and $L \gg r_c$) with $T_e = T_i$. Two plates of radius $r_s < r_c$ are located at the two ends of the cylinder and biased to generate an RF field, which acts as a source of plasma. The plasma source can be assumed equal to a constant S_0 in the inner column of radius r_s and zero elsewhere. Measurements show that the plasma density drops to zero at the cylinder vessel, while the neutral density is constant. Since $L \gg r_c$ one can neglect the axial losses at the end of the cylinder, and the plasma can be considered axially uniform. Moreover, recombination and turbulent phenomena can be neglected with respect to the dominant diffusion process.

- a.) Write the diffusion coefficient describing the main transport process in such plasma.

- b.) Find the steady state profile of the plasma density as a function of the radius, $n(r)$.
- c.) Estimate the relative ionization degree (i.e. $n(0)/(n_n + n(0))$) of the plasma at the center of the column.

Consider $T_e = T_i = 1$ eV, $\sigma_{i/n} \approx 10^{-18}$ m², $S_0 = 10^{19}$ m⁻³s⁻¹, $r_s = 0.5$ m, $r_c = 1$ m, $L = 10$ m .
