ÉCOLE POLYTECHNIQUE
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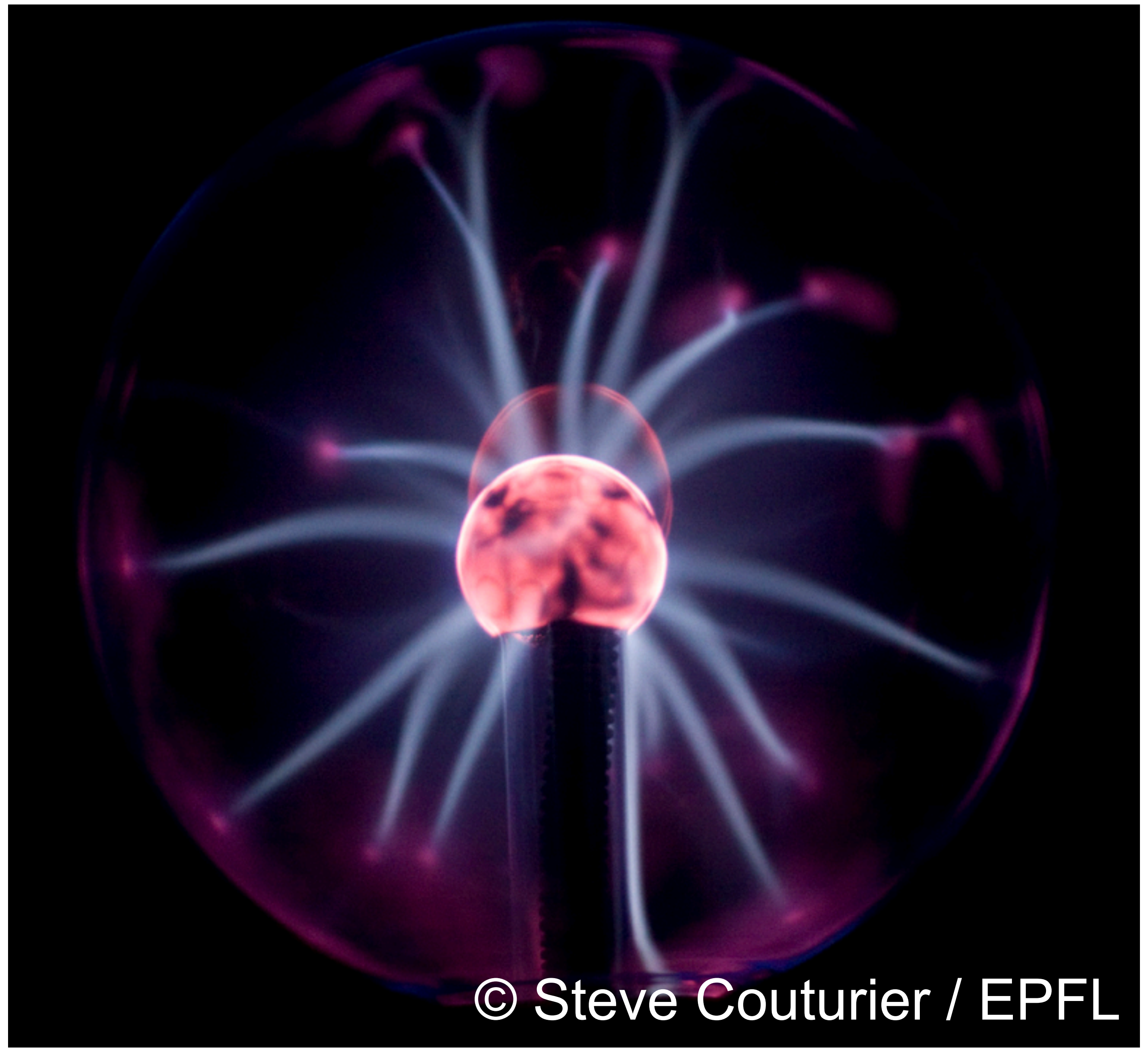

Plasma definition: frequencies and parameters

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 1c

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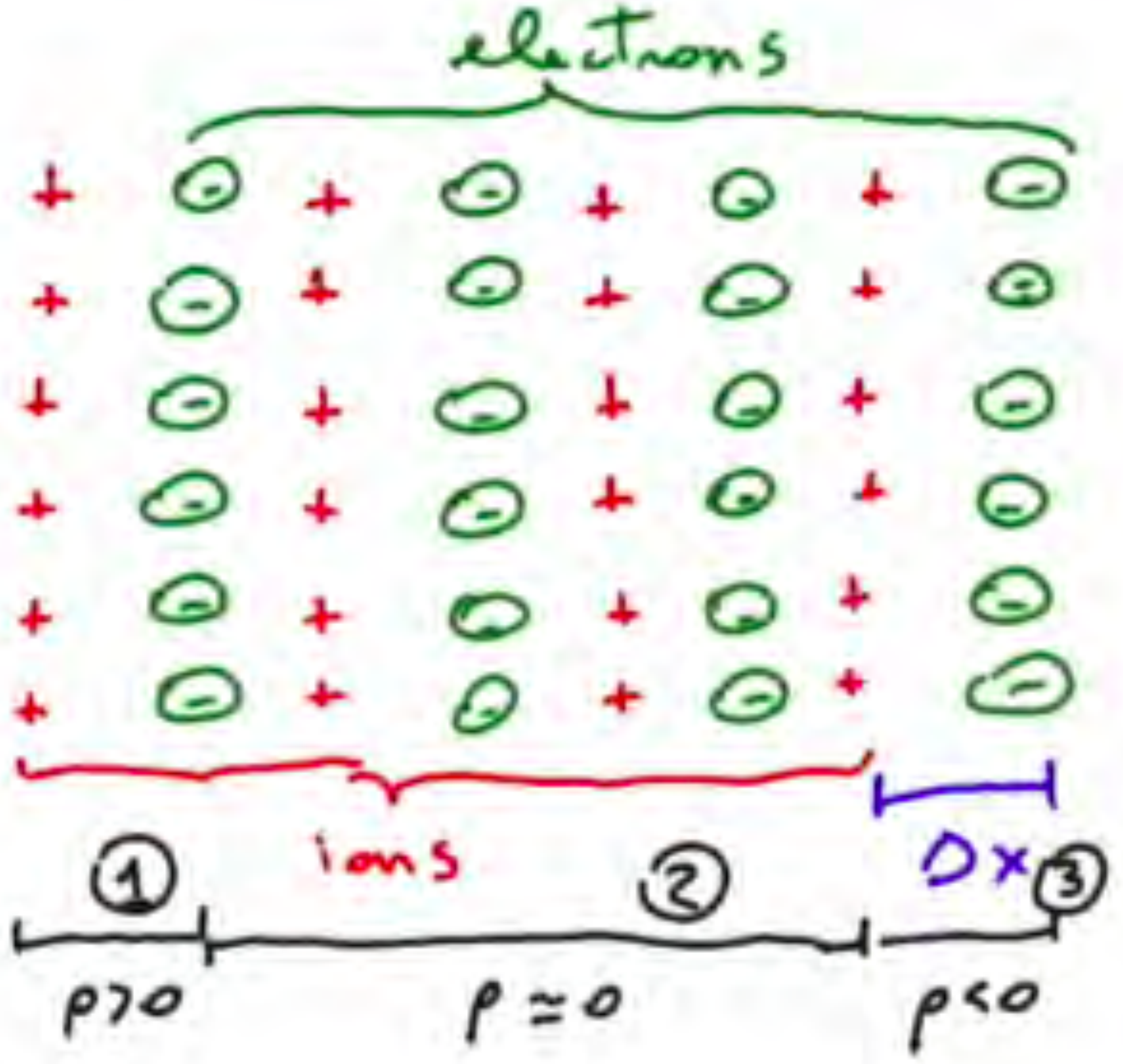
Introduction

- Rigorous definition of plasma
- Some definitions and estimates: Debye length, plasma frequency, collision frequency
- The parameter space of plasmas

Plasma frequency

A simple scenario
→ Plasma, slab of electrons with number density n_0
→ Ions, fixed background, same density
→ Displace the electrons by Δx
→ Frequency of the resulting oscillations?



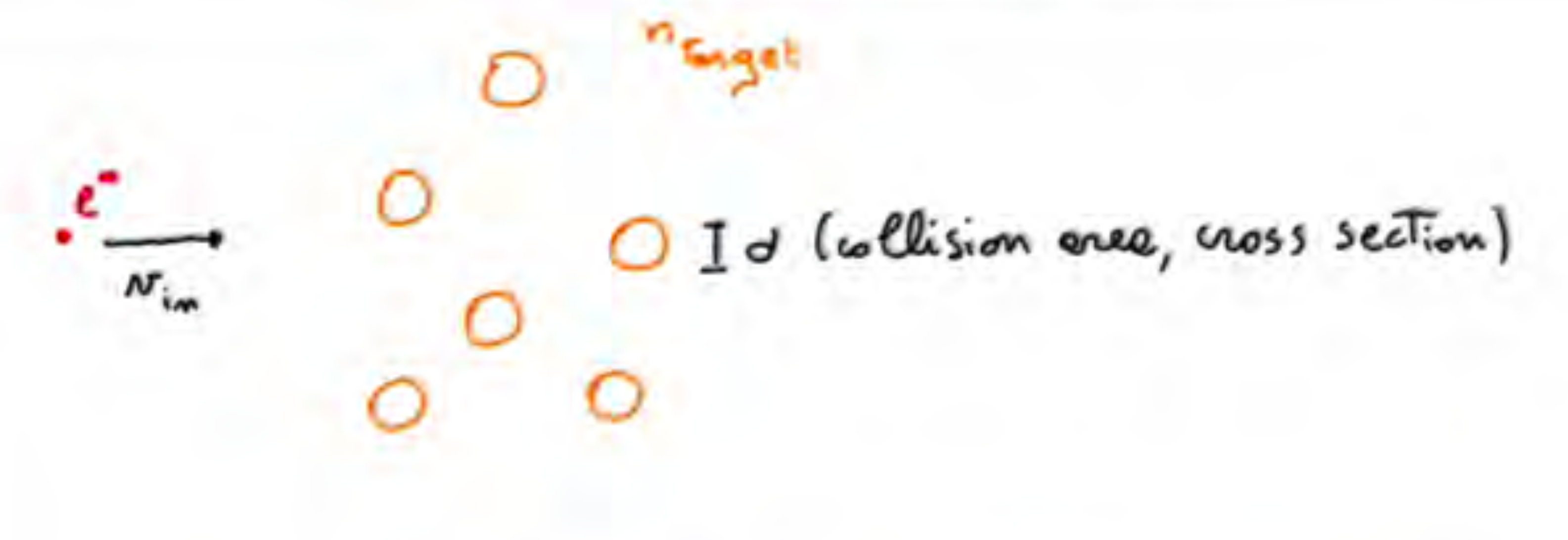
We calculate the electric field
$\frac{\partial E}{\partial x} = \frac{\rho}{\epsilon_0}$, if $E = 0$ at the left end, than in ② $E = \frac{n_0 e \Delta x}{\epsilon_0}$

Equation of motion for the electrons

$$m_e \frac{\mathrm{d}^2 \Delta x}{\mathrm{d}t^2} = -eE = -\frac{n_0 e^2}{\epsilon_0} \Delta x \quad \Rightarrow \quad \frac{\mathrm{d}^2 \Delta x}{\mathrm{d}t^2} + \underbrace{\left(\frac{n_0 e^2}{\epsilon_0 m_e}\right)}_{\omega_{pe}^2 \text{electron plasma frequency}} \Delta x = 0$$

Observations
→ $\omega_{pe} = \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}} = \sqrt{\frac{n_0 e^2}{\epsilon_0 T_e} \frac{T_e}{m_e}} = \sqrt{\frac{n_0 e^2}{\epsilon_0 T_e}} \sqrt{\frac{T_e}{m_e}} = \frac{v_{th,e}}{\lambda_{De}}$
→ $f_{pe} = 8980 \sqrt{n_0}$ Hz, n_0 is electron density in cm^{-3}

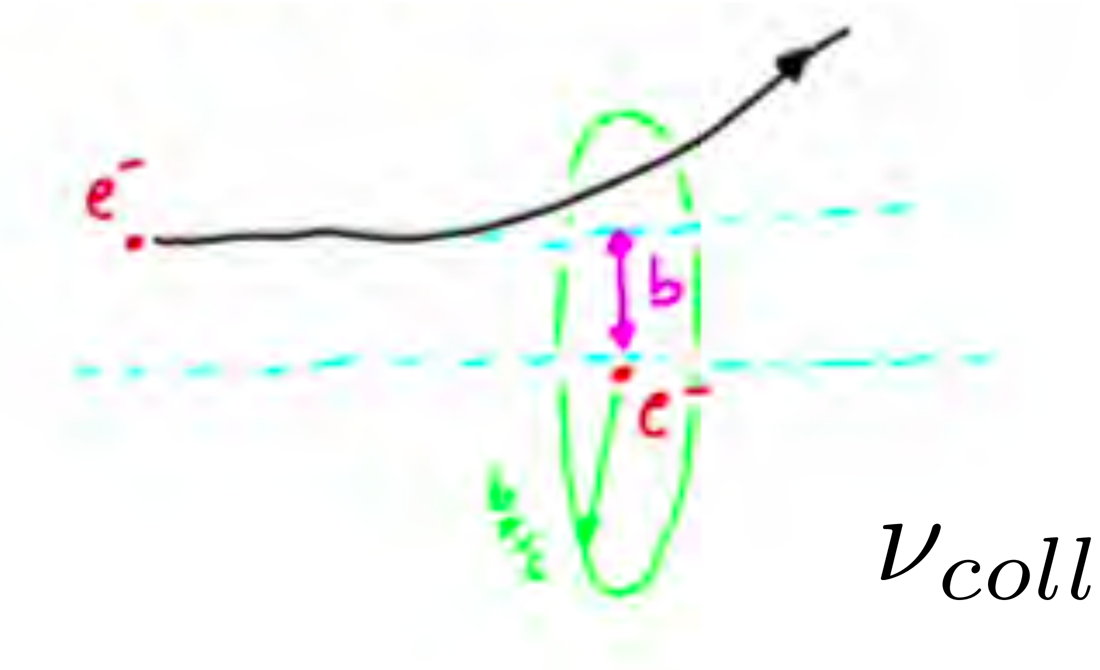
Collision frequency



$$\nu_{coll} = n_{target} \cdot \sigma \cdot v_{in}$$

Collisions with neutrals, $\nu_{coll} = n_n \cdot v_{th,e} \cdot \pi a_0^2$ $\qquad \pi a_0^2 \sim 10^{-20} \mathrm{m}^2$

Coulomb collisions



$$\frac{\text{Coulomb interaction energy}}{\text{Kinetic energy}} \sim \frac{\frac{e^2}{4\pi\epsilon_0 b}}{m_e v_{th,e}^2} \sim 1$$

$$\Rightarrow b \sim \frac{e^2}{4\pi\epsilon_0 m_e v_{th,e}^2} = b_{\frac{\pi}{2}}$$

$$\nu_{coll} = n_e v_{th,e} \pi b_{\frac{\pi}{2}}^2 = \frac{\pi n_0 v_{th,e} e^4}{16\pi^2 \epsilon_0^2 m_e^2 v_{th,e}^4} = \frac{n_0 e^4}{16\pi \epsilon_0^2 m_e^2 v_{th,e}^3}$$

FOCUS

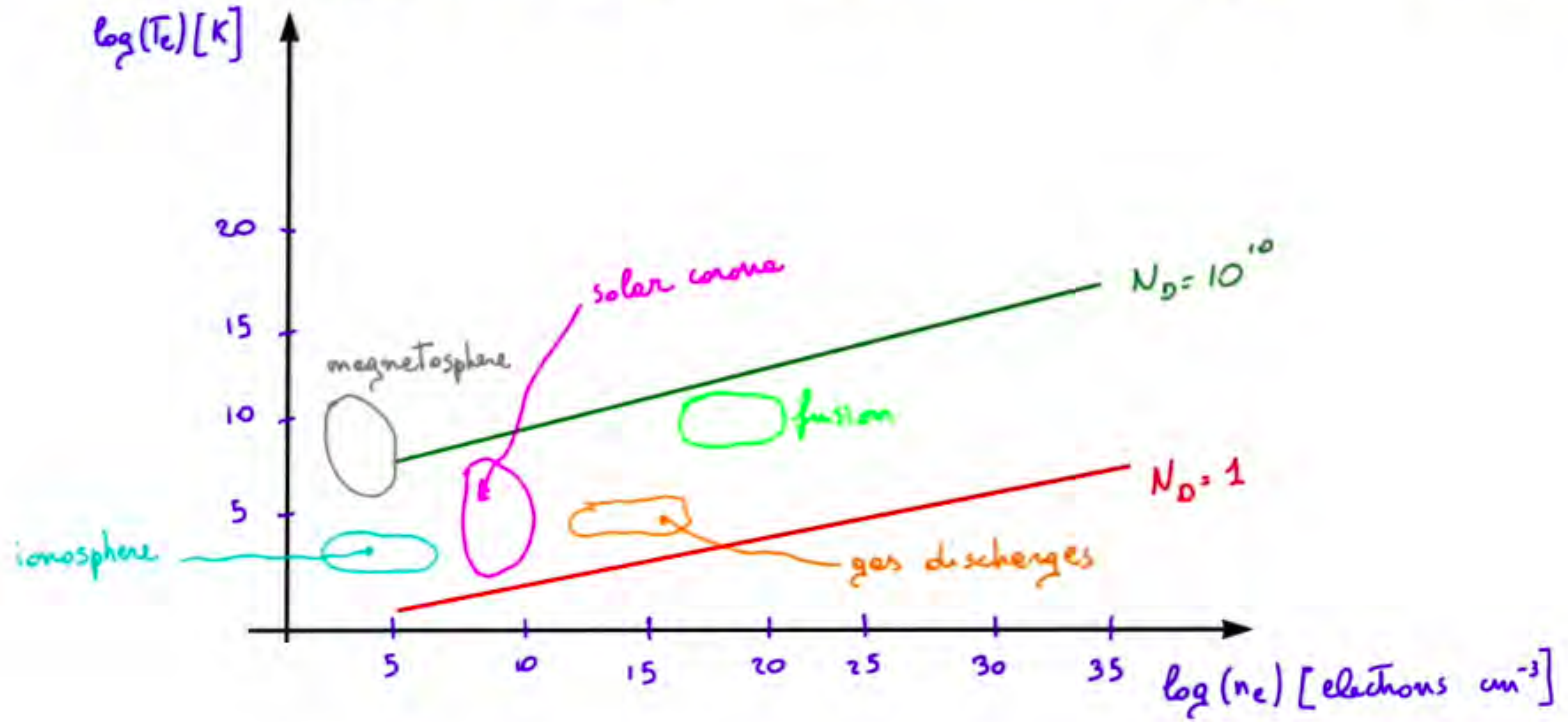
When do we have a plasma?

Plasma: ionized gas, globally neutral, that displays collective effects

- Globally neutral $\Rightarrow$ $\boxed{\text{size plasma } \gg \lambda_{De}}$ (The plasma is quasi-neutral on scale length $\gtrsim \lambda_D$)
- Collective effects $\Rightarrow$ one-to-one interactions weak, $N_D \gg 1$

$$\Rightarrow \quad \frac{\omega_{pe}}{\nu_{coll}} \gg 1 \Rightarrow \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}} \quad \frac{16\pi\epsilon_0^2 m_e^2 v_{th,e}^3}{n_0 e^4} = 16\pi\lambda_{De}^3 n_0 = 16\pi N_D \gg 1 \Big\}$$

$N_D \gg 1$
$g = \frac{1}{N_D} \ll 1$
plasma parameter



In the range of parameters considered, we can ignore
$\rightarrow$ relativistic effects
$\rightarrow$ quantum effects

Summary

- Rigorous definition of plasma
- Introduced the Debye length, plasma frequency, collision frequency
- Parameter space of plasmas