



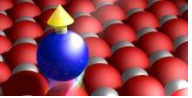
10

Magnetism at the nanoscale

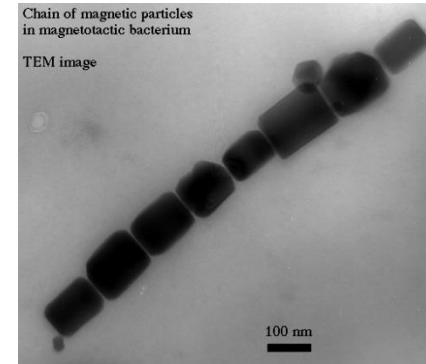


Magnetism in Condensed Matter
S. Blundell
Oxford University Press

Magnetism
From Fundamentals to Nanoscale Dynamics
J. Stöhr and H.C. Siegmann
Springer

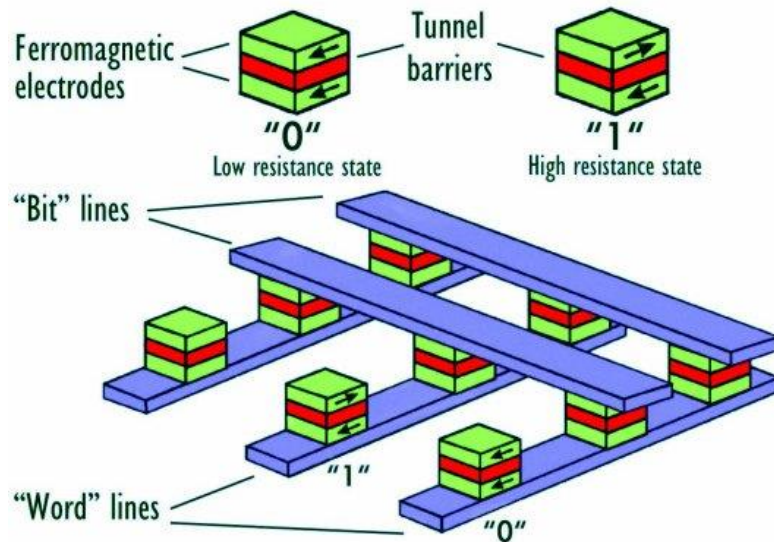


Credit Card



Magnetotactic bacteria synthesize magnetic particles between 30 and 100 nm, big enough to have a permanent magnetic moment, but small enough to be a single domain. [images by R. James, University of Western Australia; see also R. Blakemore. "Magnetotactic Bacteria." *Science* 190, 377 (1975)].

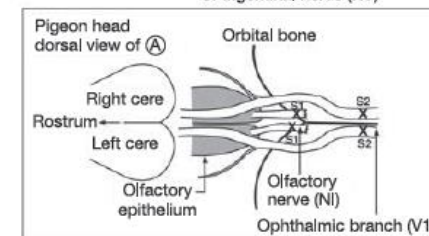
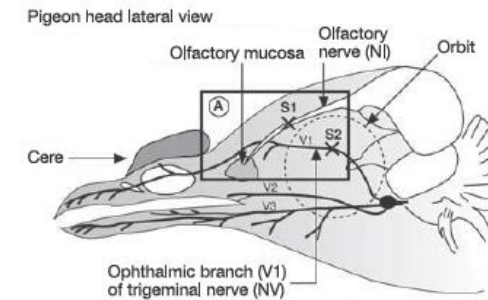
MRAM: Magnetic Random Access Memory

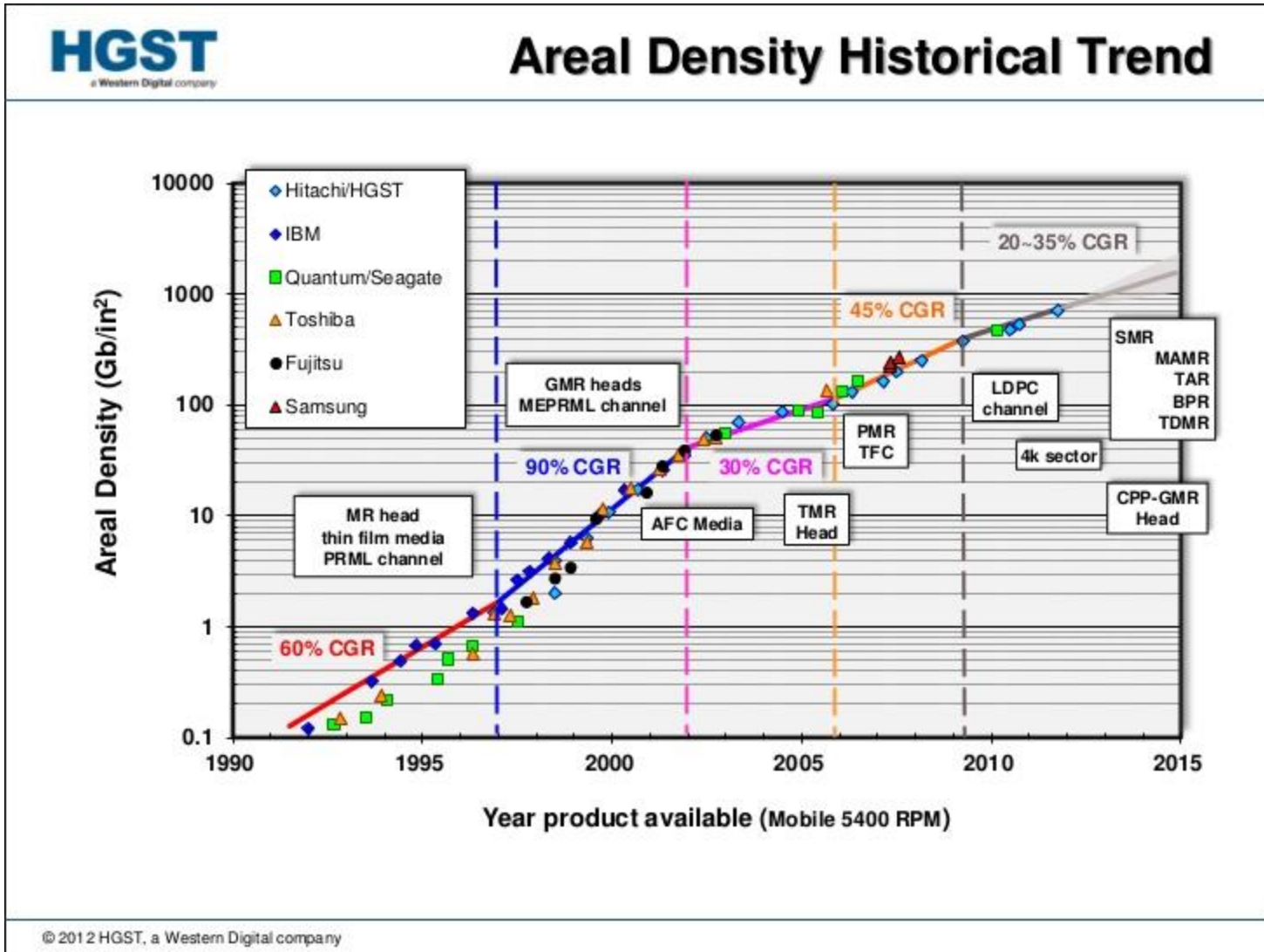


Magnetoreception and its trigeminal mediation in the homing pigeon

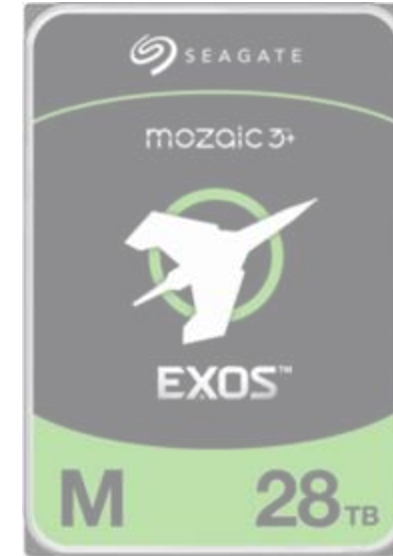
Cordula V. Mora^{1*}, Michael Davison², J. Martin Wild³ & Michael M. Walker¹

Nature 432, 508 (2004)





2025



28TB on 10 platters
→ 1.0-1.5 Tb/in²

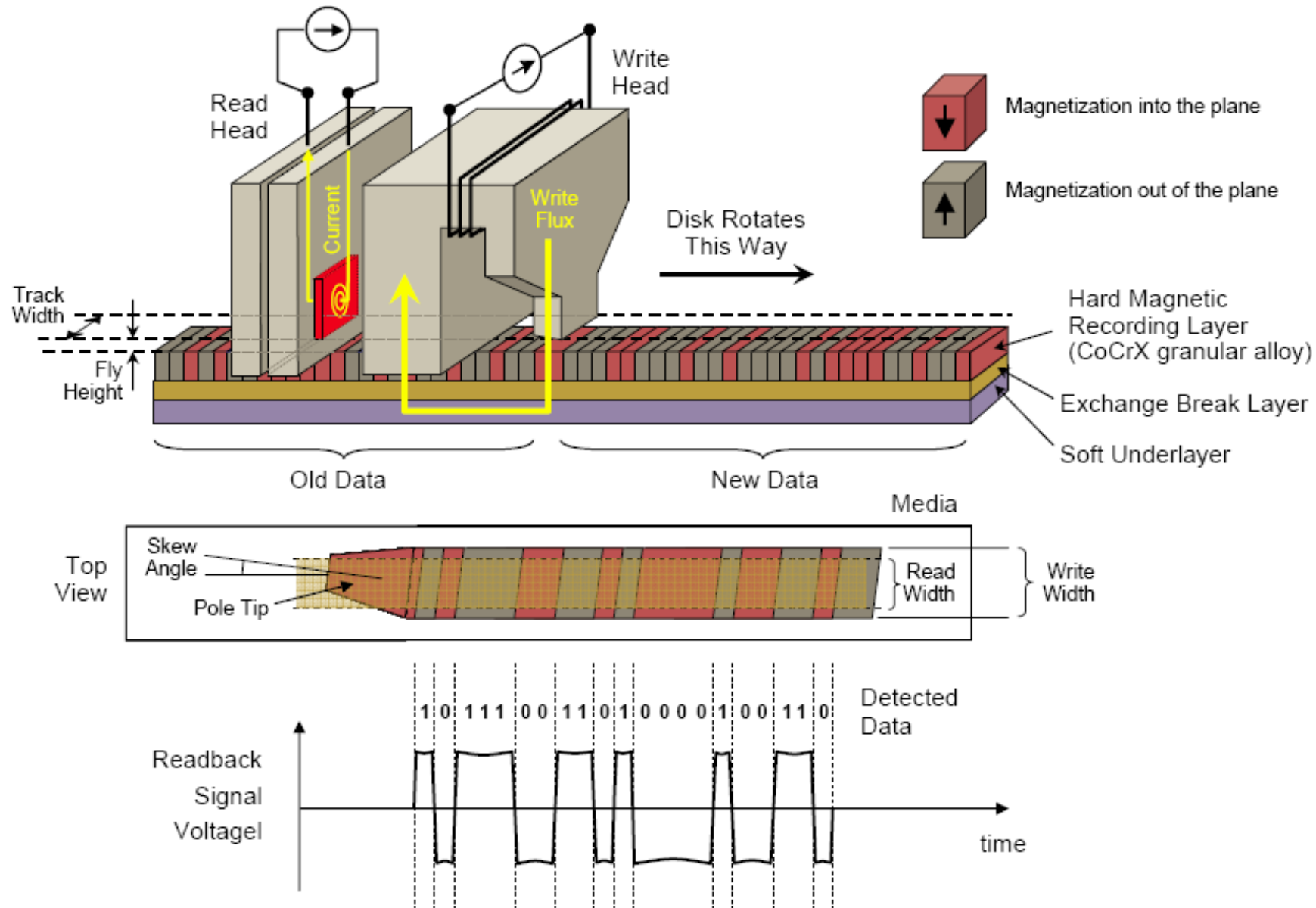


20-25 nm pitch

For large scale storage, SSDs are four to five times more expensive per gigabyte than HDDs
HDDs are more reliable for long-term data recovery

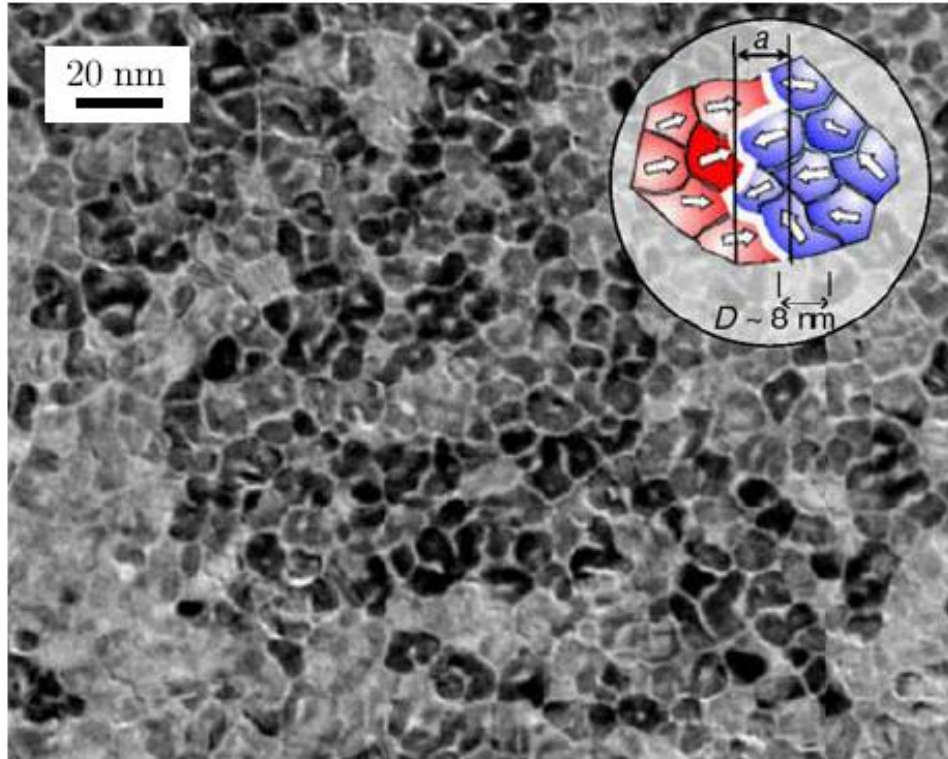


The recording mechanism in an HDD: a condensate of concepts





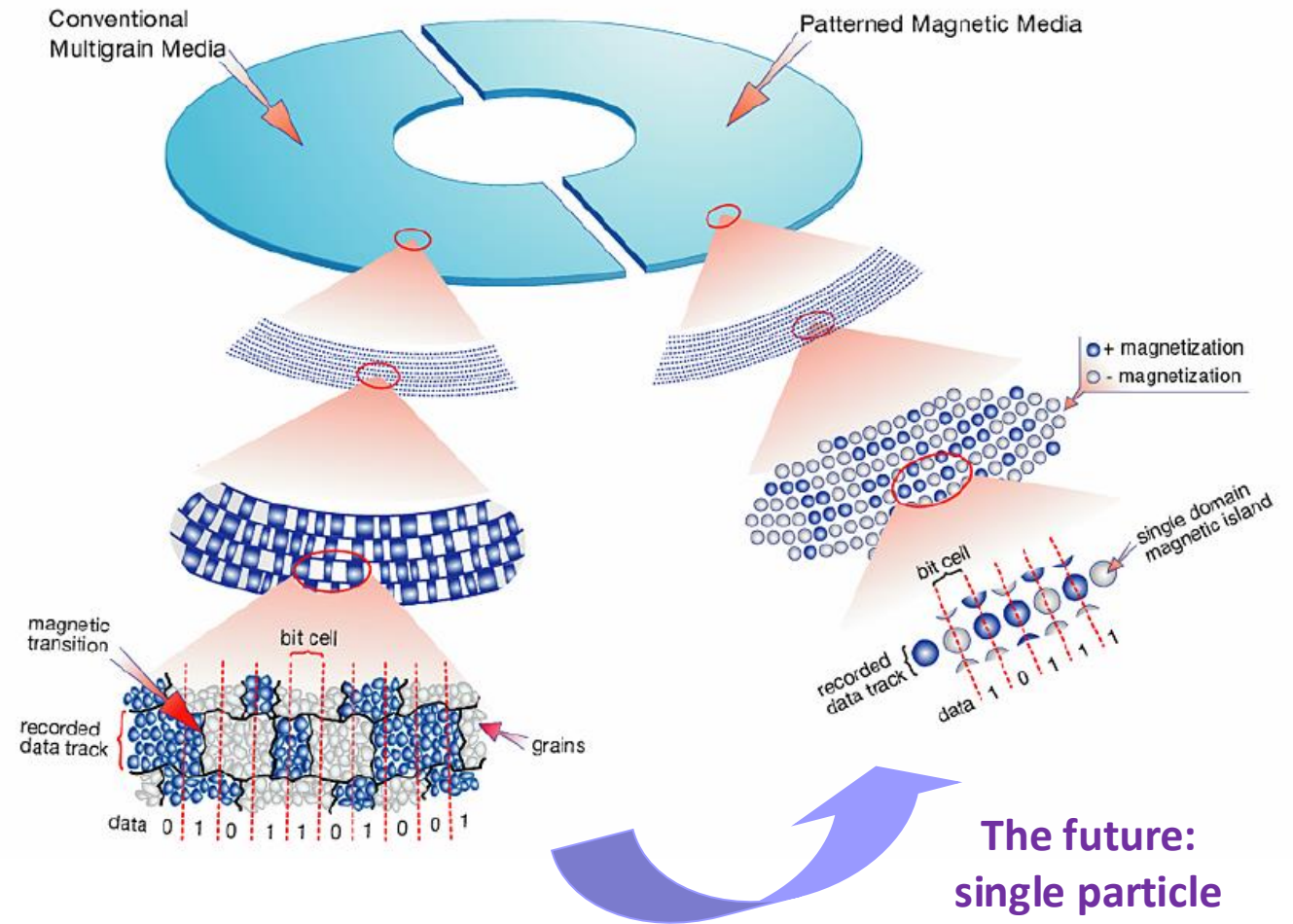
TEM images of CoCrPt recording layer with in-plane magnetization.



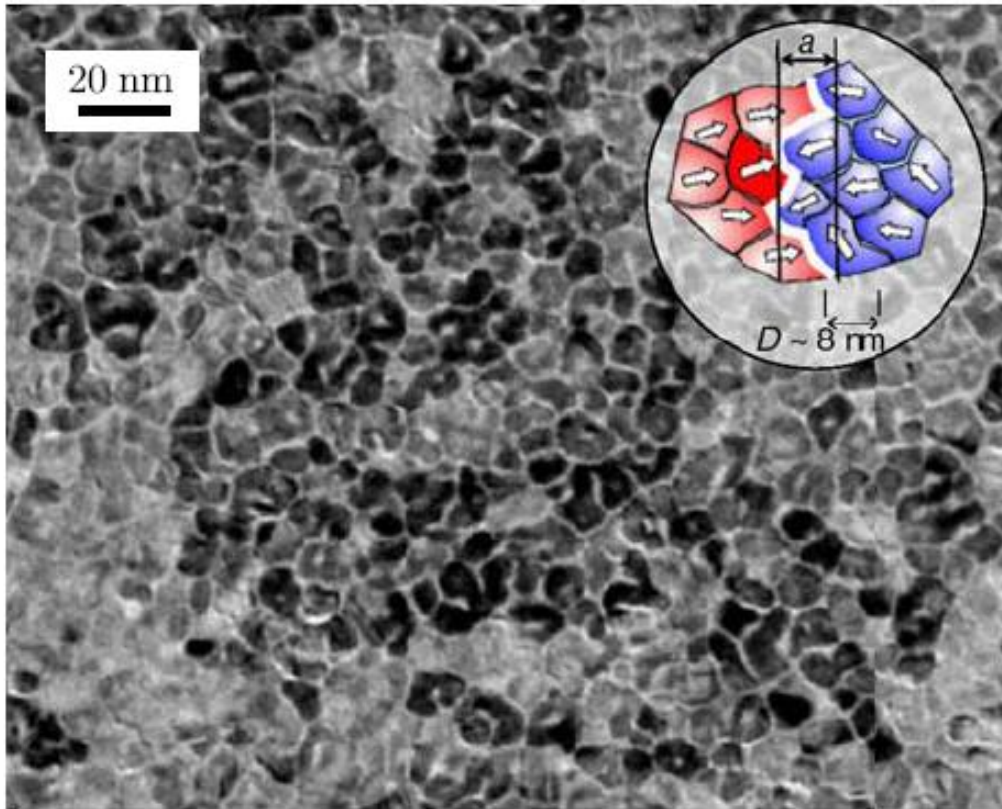
The inset sketches two bits. Each bit consists of several tens of grains. The bit size and shape is defined during the head writing process

1 bit \approx 50 grains

Conventional Media vs. Patterned Media

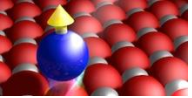


The future:
single particle
per bit



CoCrPt recording layer

- 1) Why every grain has a magnetic moment?
- 2) How is the material chosen?
- 3) Why is every grain magnetically decoupled from the neighbors?
- 4) Why is the grain magnetization pointing in one specific direction?
- 5) Why does the magnetization direction change from grain to grain?
- 6) Why does the grain magnetization not fluctuate in time?
- 7) ...



All the electrons interact with one another, analytical solution not possible
 Orbital approximation, single electron Hamiltonian \rightarrow electron configuration (n, l)
 Pauli exclusion principle \rightarrow max two electrons per orbital
 \mathcal{H}_0 from hydrogenic Hamiltonian, putting Z electrons

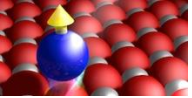
Additional terms:

- V_{ee} Coulomb interaction that describes the repulsion between electrons
- V_{so} spin-orbit coupling, orbital and spin angular momentum are not independent
- V_{zee} Zeeman interaction if a magnetic field is applied

$$\mathcal{H}_{atom} = \sum_{i=1}^Z \left(\frac{p_i^2}{2m_e} + eV(r_i) \right) + \sum_{i < j}^Z \frac{e^2}{|r_i - r_j|} + \sum_{i=1}^Z \xi_{nl}(r_i) \mathbf{l}_i \cdot \mathbf{s}_i + \frac{\mu_B}{\hbar} (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B} = \mathcal{H}_0 + V_{ee} + V_{so} + V_{zee}$$

$$\mathbf{S} = \sum_{i=1}^Z \mathbf{s}_i \quad \mathbf{L} = \sum_{i=1}^Z \mathbf{l}_i$$

$$V_{so} = \lambda \mathbf{L} \cdot \mathbf{S} \quad \lambda = \pm \frac{\zeta_{nl}}{2S}$$



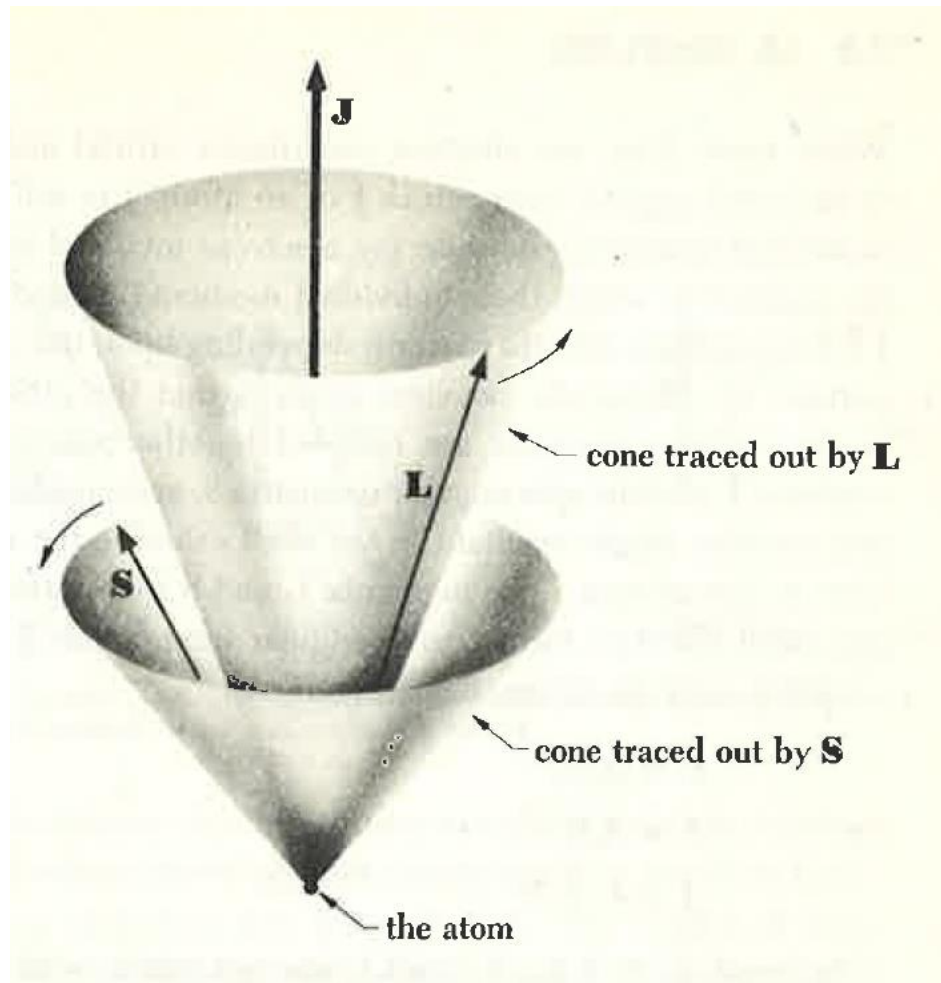
$$\mathcal{H}_{atom} = \sum_{i=1}^Z \left(\frac{p_i^2}{2m_e} + eV(r_i) \right) + \sum_{i<j}^Z \frac{e^2}{|r_i - r_j|} + \sum_{i=1}^Z (l_i \cdot s_i) \xi_{nl}(r_i) + \frac{\mu_B}{\hbar} (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B} = H_{Coul} + V_{ee} + V_{so} + V_{Zee}$$

interaction term	3d transition metals	4f rare earths
V_{ee}	1 eV	1 eV
V_{so}	50 - 100 meV	300 - 500 meV
$V_{Zee} (B = 1\text{T})$	0.1 - 0.2 meV	0.1 - 0.6 meV

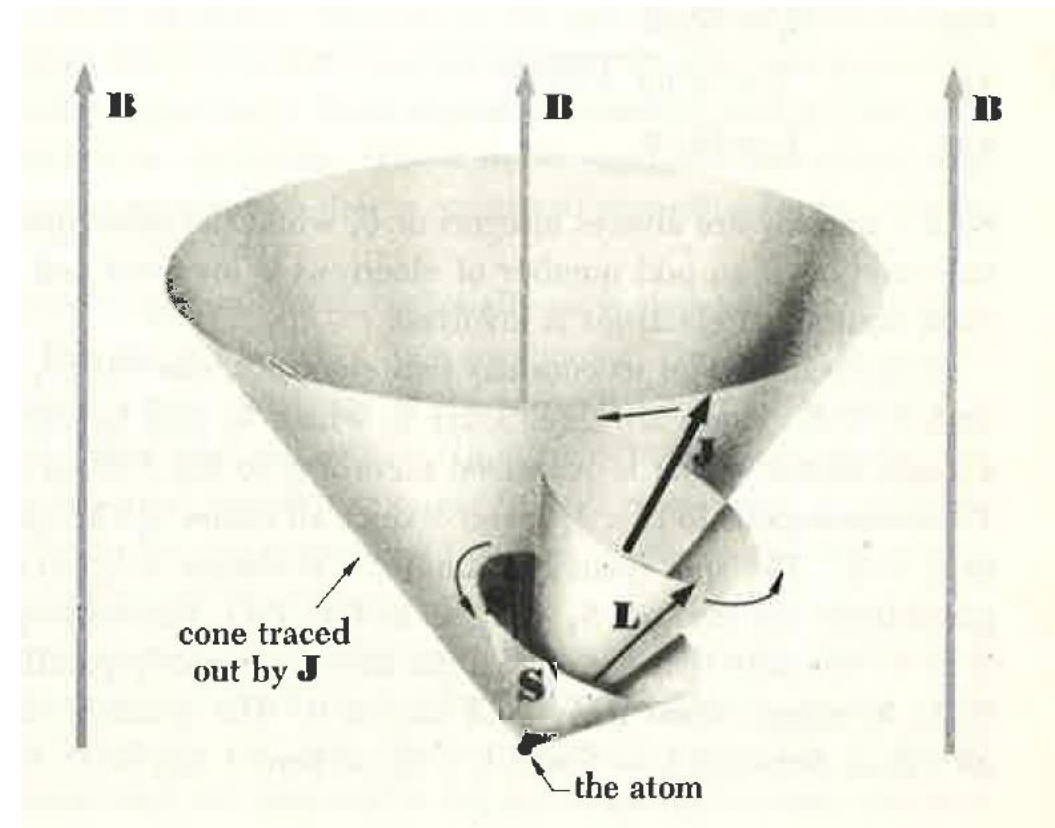


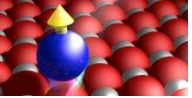
L and S are coupled

No external B field: the total J is conserved in magnitude and direction, L and S precess around the the direction of the resultant J



External B field applied: J precesses about the direction of B





Exercise 10.1

Total magnetic moment operator

$$\boldsymbol{\mu} = \boldsymbol{\mu}_L + \boldsymbol{\mu}_S$$

$$\boldsymbol{\mu} = -\frac{\mu_B}{\hbar} (g_L \mathbf{L} + g_S \mathbf{S}) \quad g_L = 1 \quad g_S = 2$$

g-factors for orbital and spin magnetic moments (similar to classical gyromagnetic ratio)

With spin-orbit coupling, \mathbf{S} and \mathbf{L} don't correspond to good quantum numbers anymore, but \mathbf{J} does. In the presence of a magnetic field, the component of the magnetic moment parallel to \mathbf{J} is conserved, but the component perpendicular to \mathbf{J} is not.

We can write the operator total magnetic moment operator as

$$\boldsymbol{\mu} = -\frac{\mu_B}{\hbar} g_J \mathbf{J} \quad g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

g_J is the Landé g-factor
it describes the projection of $\mathbf{L} + 2\mathbf{S}$ onto \mathbf{J}

$$\mu_B = \frac{e \hbar}{2 m_e} = 0.058 \text{ meV T}^{-1} \quad \text{Bohr magneton}$$

In the limit of a weak magnetic field \mathbf{B} , defining the quantization axis z , the magnetic moment along z is

$$\begin{aligned} \mu_z &= \frac{1}{B} \langle L S J M_J | (\boldsymbol{\mu}_L + \boldsymbol{\mu}_S) \cdot \mathbf{B} | L S J M_J \rangle \\ &= -\frac{\mu_B}{\hbar} \langle L S J M_J | L_z + 2S_z | L S J M_J \rangle = -\frac{\mu_B}{\hbar} g_J \langle L S J M_J | J_z | L S J M_J \rangle = -\mu_B g_J M_J \end{aligned}$$

frequently indicated simply as μ



Hund's rules to find the **ground state**:

1) Total spin $\mathbf{S} = \sum_i \mathbf{s}_i$ maximized ($\Rightarrow S = M_S$ max)

2) Total orbital momentum $\mathbf{L} = \sum_i \mathbf{l}_i$ maximized ($\Rightarrow L = M_L$ max)

3) Total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$

\mathbf{L} and \mathbf{S} parallel ($J = L + S$) if subshell more than half filled

\mathbf{L} and \mathbf{S} antiparallel ($J = |L - S|$) if subshell less than half filled



Coulomb repulsion V_{ee}



Spin-orbit interaction V_{SO}

Ti: [Ar] 3d² 4s²

Only the 3d subshell is partially occupied

m_l	+2	+1	0	-1	-2
m_s	↑	↑			

$$M_S = 1 \rightarrow S = 1$$

$$M_L = 3 \rightarrow L = 3$$

subshell less than half-filled $\rightarrow J = L - S = 2$

$$\mu_z = -g_J M_J \mu_B = -\frac{2}{3} 2 \mu_B = -\frac{4}{3} \mu_B$$

Fe: [Ar] 3d⁶ 4s²

Only the 3d subshell is partially occupied

m_l	+2	+1	0	-1	-2
m_s	↑↓	↑	↑	↑	↑

$$M_S = 2 \rightarrow S = 2$$

$$M_L = 2 \rightarrow L = 2$$

subshell more than half-filled $\rightarrow J = L + S = 4$

$$\mu_z = -g_J M_J \mu_B = -\frac{3}{2} 4 \mu_B = -6 \mu_B$$



Atom described by quantum numbers $|L S J M_J\rangle$ with M_J assuming $2J + 1$ values between $-J$ and $+J$

At $B = 0$ T the $2J + 1$ values are degenerate in energy

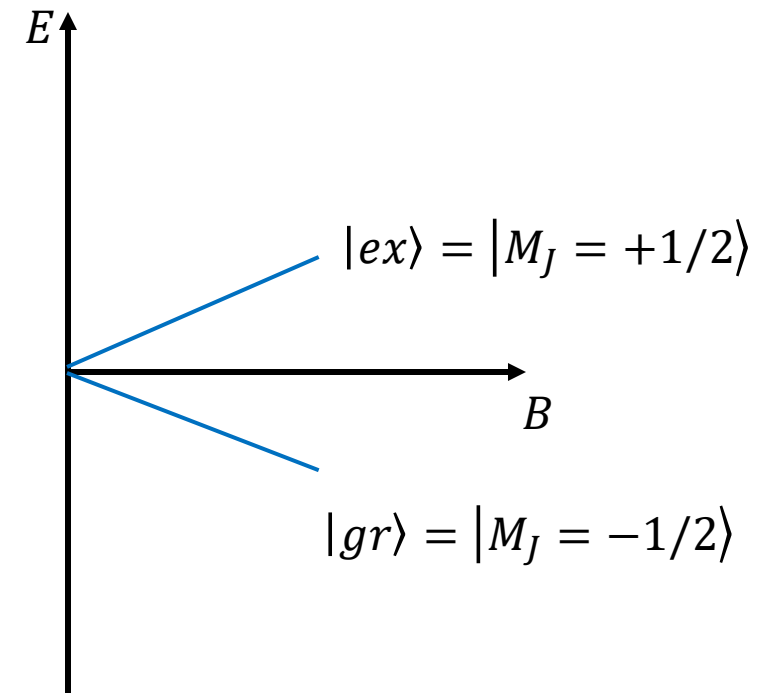
At $B \neq 0$ T the $2J + 1$ states split: Zeeman splitting

$$H_{Zee} = g_J \mu_B \mathbf{J} \cdot \mathbf{B} / \hbar$$

$$E(M_J) = g_J \mu_B M_J B$$

Example: atom with $J = 1/2$

two energy levels: $\pm g_J \mu_B M_J B$





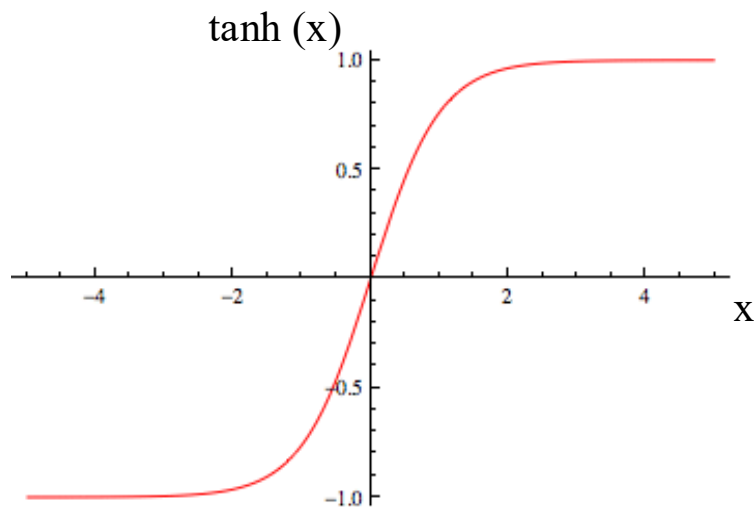
Boltzmann factor: expresses the relative probability that a quantum state of energy E to be occupied at temperature T

$$\exp(-E_{M_J}/k_B T) = \exp(-M_J g_J \mu_B B / k_B T)$$

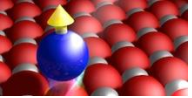
The net magnetic moment is given by the contribution of all occupied states: in this case with only two states of opposite magnetization we have:

$$\mu(B, T) = \mu^\uparrow - \mu^\downarrow = |M_J| g_J \mu_B \left(\frac{e^x}{e^x + e^{-x}} - \frac{e^{-x}}{e^x + e^{-x}} \right) = |M_J| g_J \mu_B \tanh(x) \quad x = \frac{M_J g_J \mu_B B}{k_B T}$$

$$\mu(B, T) = \mu_B \tanh\left(\frac{\mu_B B}{k_B T}\right) \quad \text{with} \quad |M_J| = \frac{1}{2} \quad \text{and} \quad g_J = 2$$



	$\tanh(x)$	Occupation gr	Occupation ex
$T = 0$ or $B = \infty$	1	100%	0%
$B = 0$ or $T = \infty$	0	50%	50%
$B = 0.55 k_B T / \mu_B$	0.5	75%	25%



Isolated atoms vs. bulk

Bulk (at room T): only a few elements (Fe, Co, and Ni) have a magnetic moment

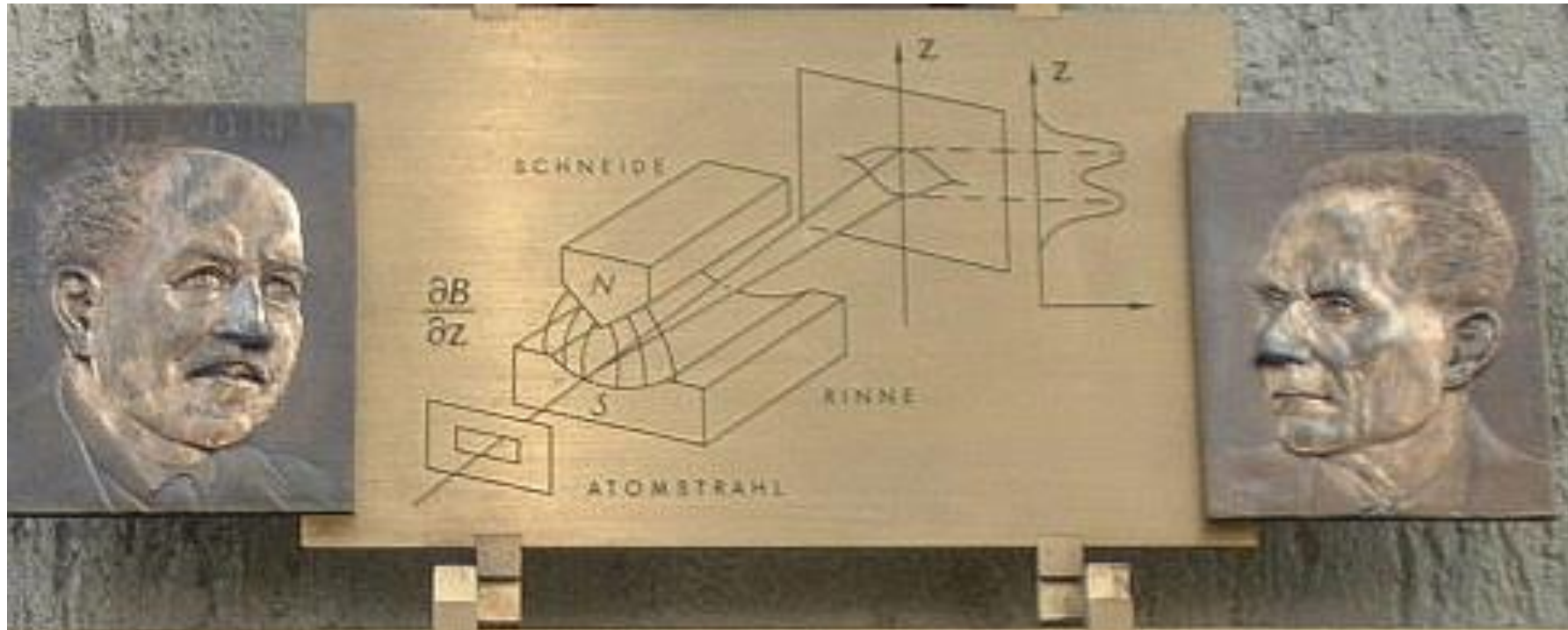
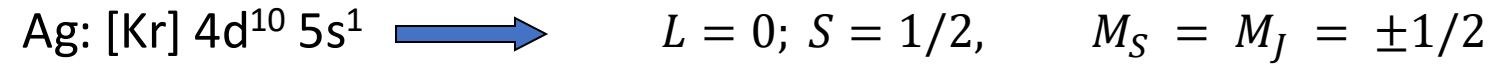
Atoms: all atoms except noble gas have a magnetic moment

Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
↓ Period																		
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo
Lanthanides				57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides				89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

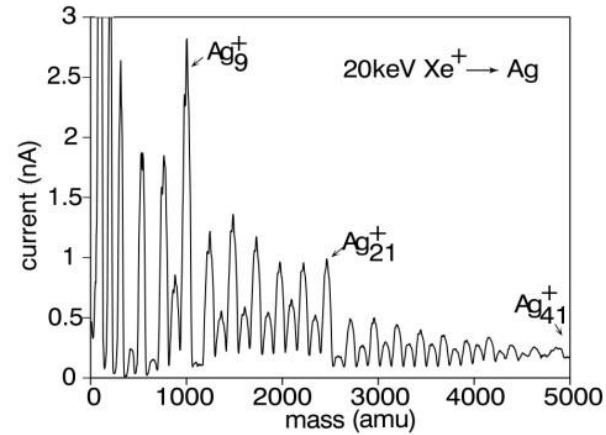
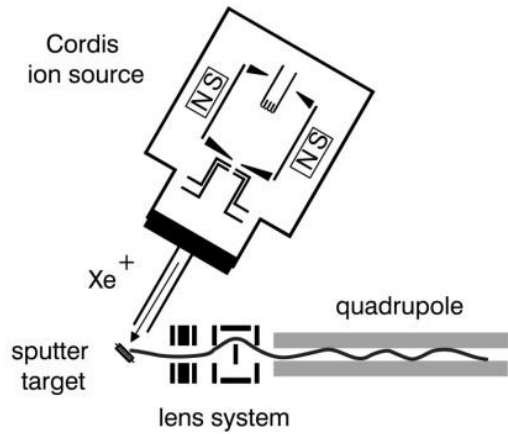


Stern-Gerlach experiment (1922)

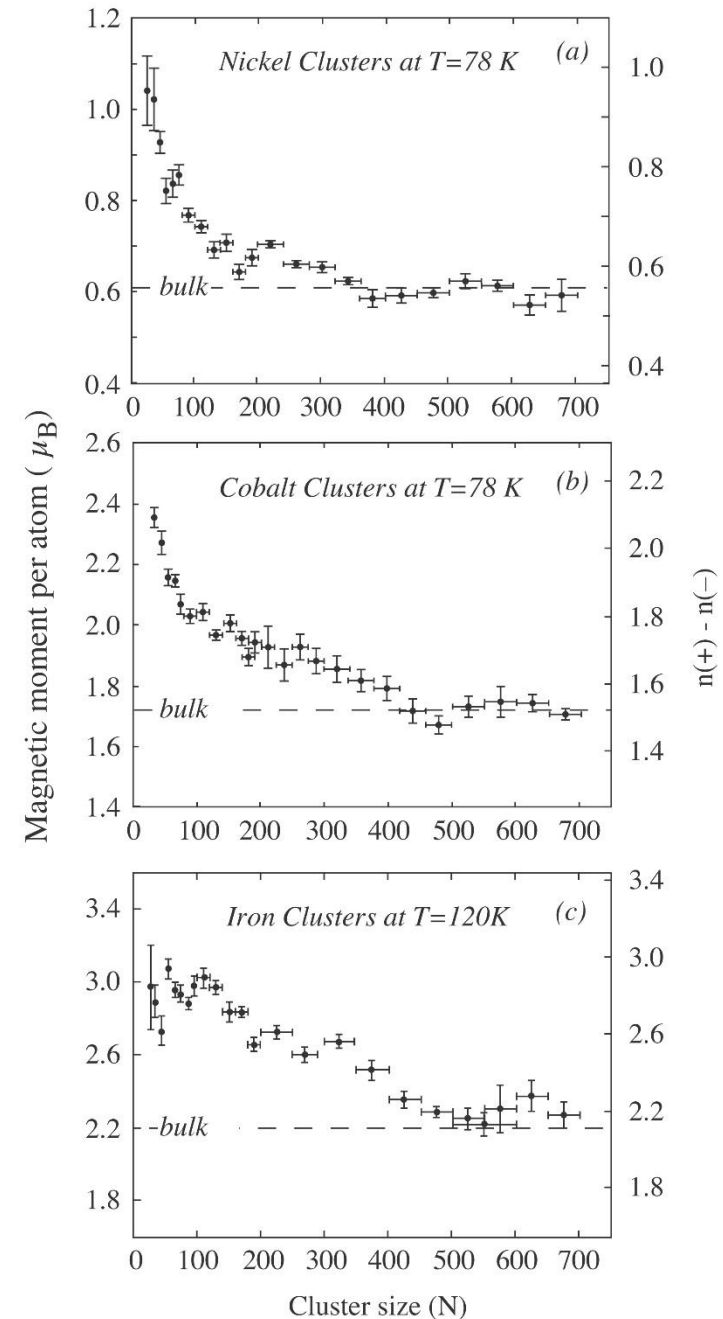
Beam of Ag atoms through a non-uniform magnetic field

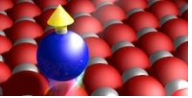


Ag atoms deviate depending on their spin moment



Below about 200 atoms per cluster, the magnetic moment strongly increases with respect to bulk values and tends to the gas-phase atom value



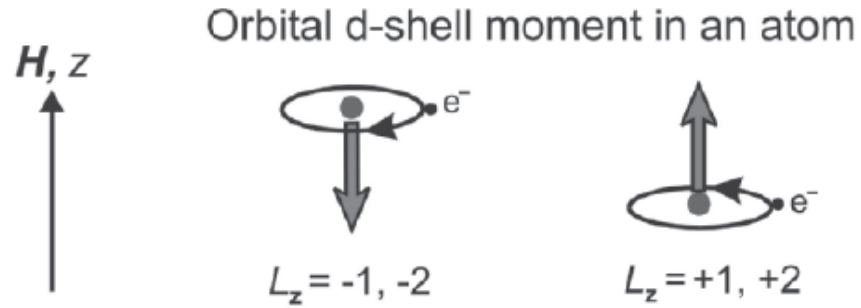


Suppression of the orbital contribution to the magnetic moment

Origin:

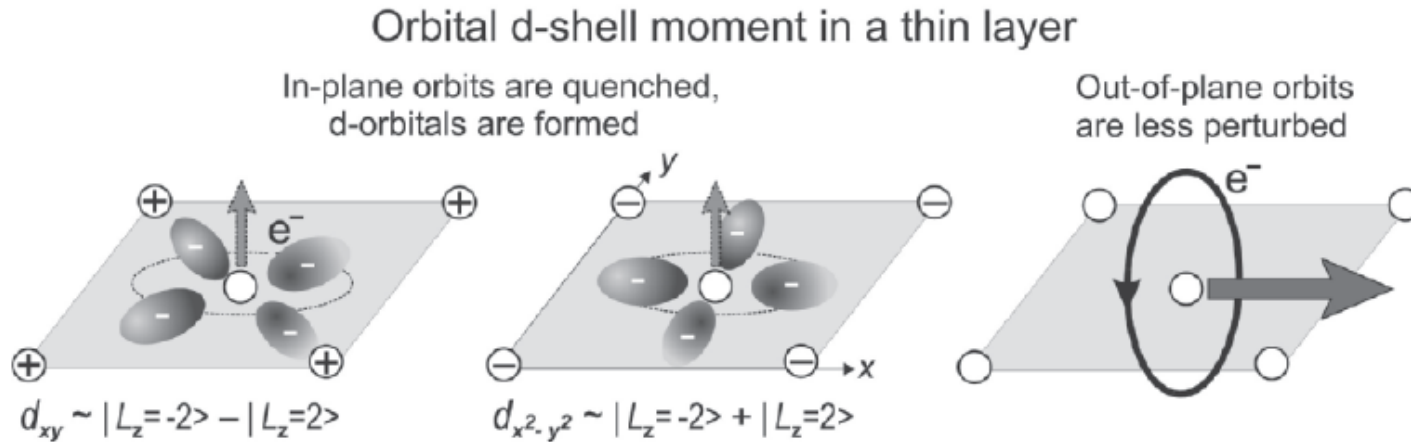
- crystal field and covalent bonds (main effect in semiconductors, insulators, oxides)
- electron delocalization and band formation (in metals)

This effect reduces the orbital angular momentum so that it does not contribute to the overall magnetism, which is then dominated by the spin magnetic moment.



the orbital moment L arises from the electron precession (Y_l^m)

bond formation stops the precession



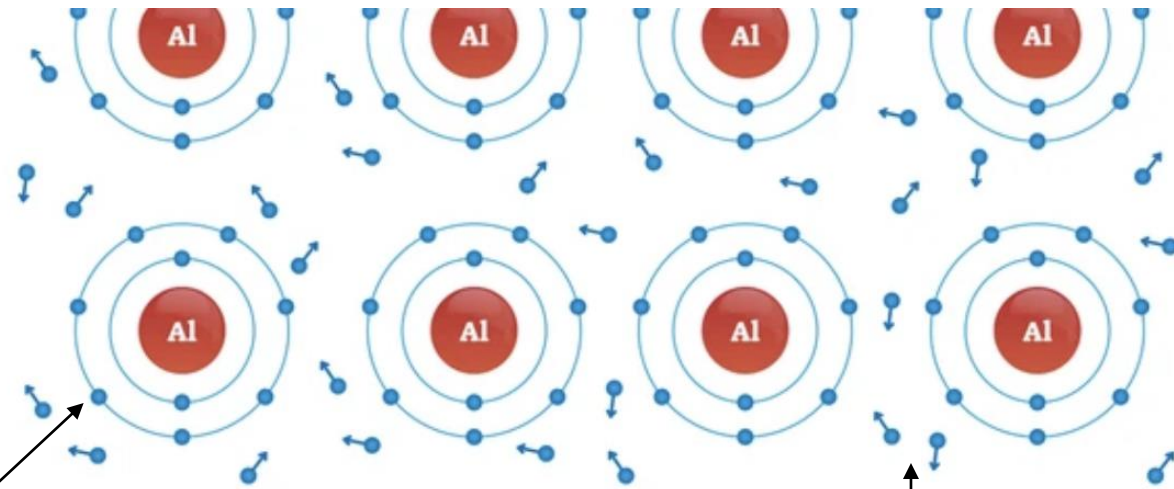
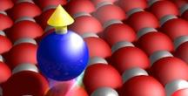
a strong directional bonding generates a reduction in the component of L perpendicular to the bond direction

The central atom binds to four atoms in a plane.

a) The d electron will form a standing wave with a spatial shape depending on the distribution of the electronic charge on the neighbouring atoms (i.e. orbital motion frozen by the formation of bonds with the neighbouring atoms) → **out-of-plane orbital moment is quenched**

b) The orbital motion perpendicular to the bonding plane is less perturbed by the bonds → **in-plane orbital moment will stay unquenched**

symmetry breaking implies **reduced** and **anisotropic** orbital moment



core electrons
similar to free atom case, full shells

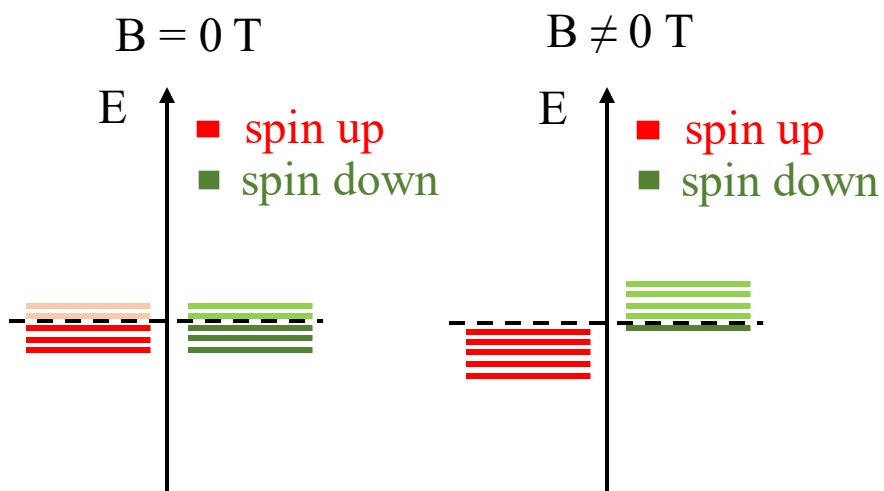
conduction electrons
delocalized, "sea" of electrons

The delocalization significantly alters the electronic wave functions, and there are no more "circular" orbits → the orbital magnetic moment is strongly suppressed.

Electrons in d bands are intermediate (delocalized but not as much as s or p electrons)

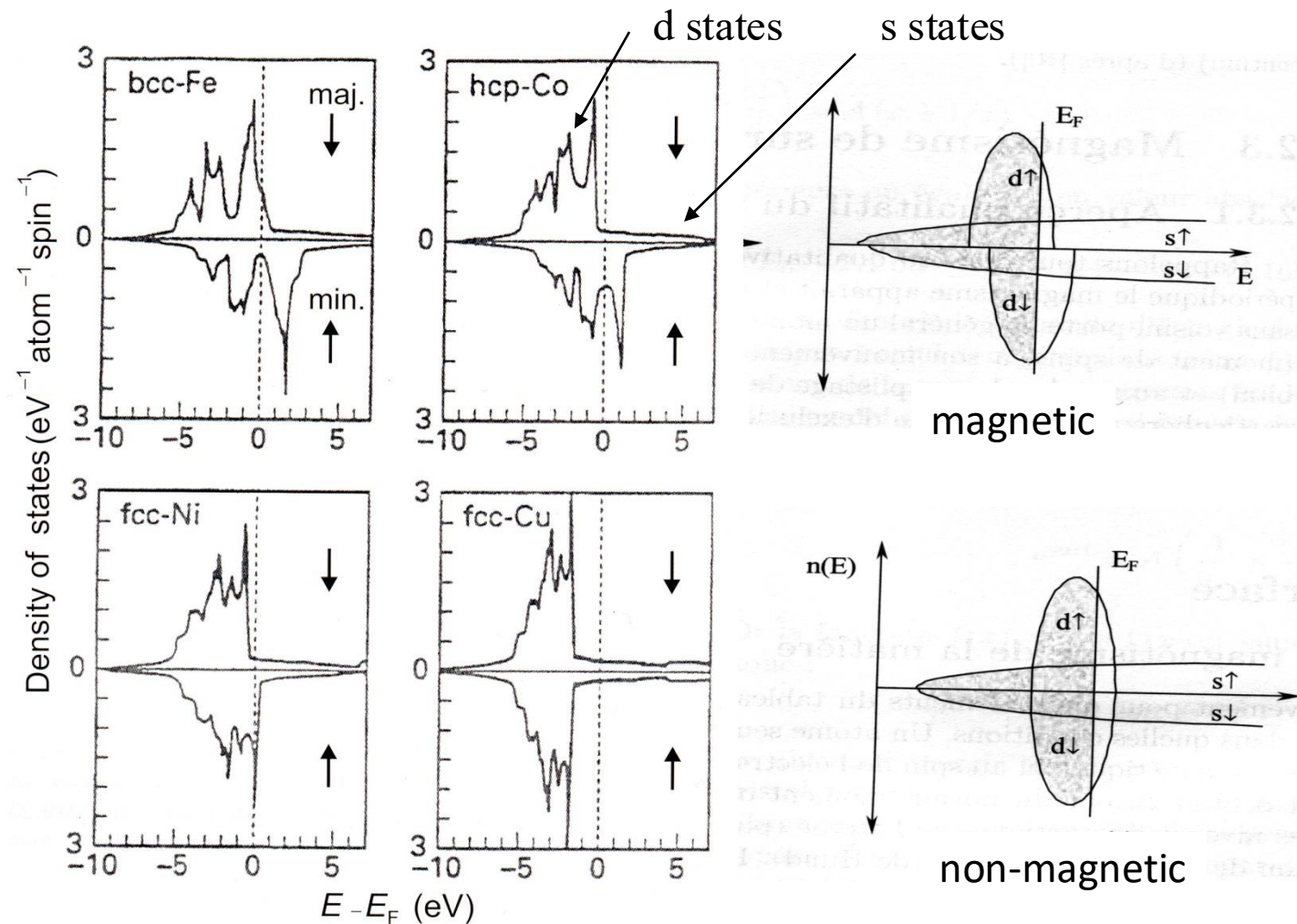


Atom



The orbitals are degenerate. Only when the quantization axis is defined (for example by an external magnetic field) we measure the spin defined by Hund's rules

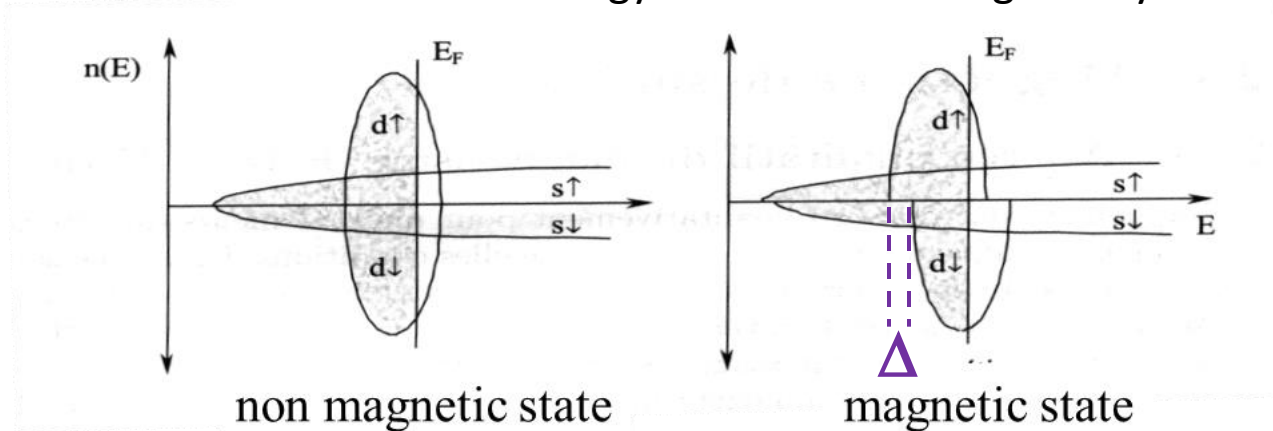
Bulk



Due to electron-electron interaction, spontaneous magnetization can appear even in absence of an external magnetic field.



The total electron energy of N electrons is given by the sum of the energies of all occupied states



non magnetic state

$$N_{up} = N/2$$

$$N_{dw} = N/2$$

magnetic state

$$N_{up} = N/2 + n(E_F)\Delta/2$$

$$N_{dw} = N/2 - n(E_F)\Delta/2$$

$n(E)$: density of states

Coulomb interaction: $V_{ee} N_{up} N_{dw}$

Each band shifts by $\Delta/2$ in opposite directions;
 N_{up} (N_{dw}) increases (decreases) by $dN = n(E_F) \Delta/2$.
 Gain (loss) in energy is $= dN \Delta/2$

$$\text{Kinetic energy variation} = \frac{\Delta}{2} (dN_{up} - dN_{dw}) = \frac{\Delta}{2} \left[n(E_F) \frac{\Delta}{2} - \left(-n(E_F) \frac{\Delta}{2} \right) \right] = \frac{\Delta^2}{2} n(E_F)$$

$$\text{Coulomb (e-e) interaction variation} = V_{ee} \left[\frac{N^2}{4} + n(E_F) \frac{\Delta}{2} \frac{N}{2} - n(E_F) \frac{\Delta}{2} \frac{N}{2} - \frac{\Delta^2}{4} n^2(E_F) \right] - V_{ee} \frac{N^2}{4} = -V_{ee} \frac{\Delta^2}{4} n^2(E_F)$$

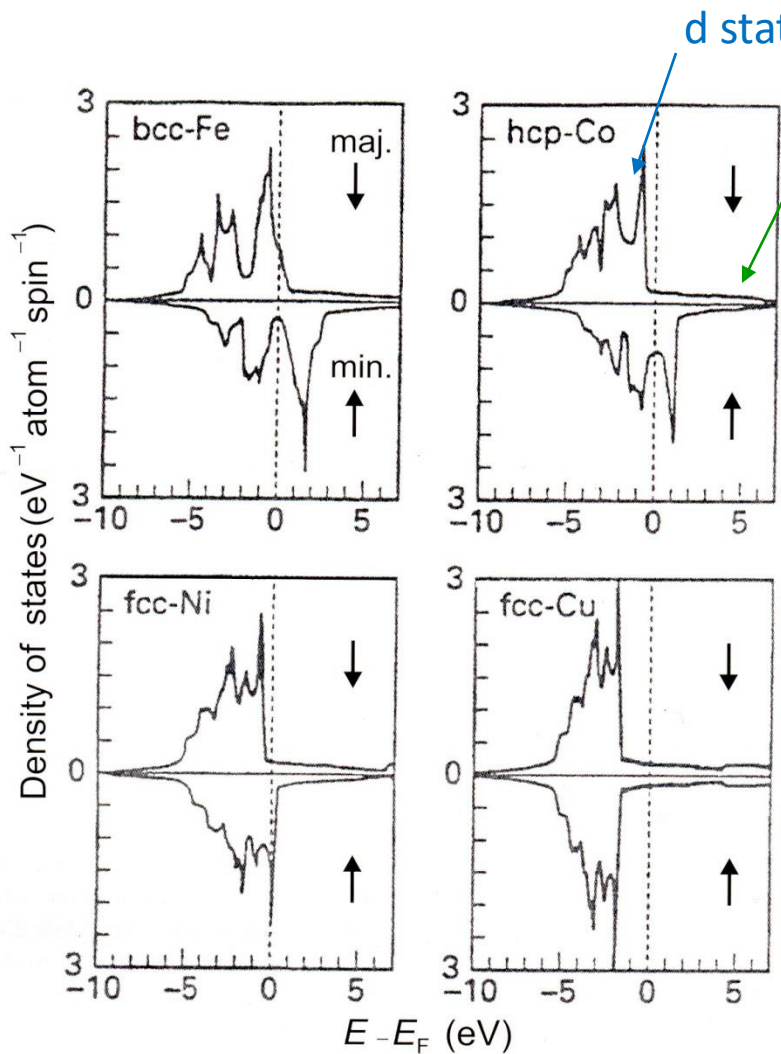
$$\text{Total electron energy variation } dE = \frac{\Delta^2}{2} n(E_F) - V_{ee} \frac{\Delta^2}{4} n^2(E_F) = \frac{\Delta^2}{2} n(E_F) \left(1 - \frac{V_{ee}}{2} n(E_F) \right)$$

If $dE < 0$ spontaneous magnetism appears: $1 - \frac{V_{ee}}{2} n(E_F) < 0$ **(Stoner criterion)**

It depends on V_{ee} and $n(E_F)$ (both are material dependent)



Exercise 10.2



s, p bands are extended (band width about 10 eV) → contribution of about 5% to the spin moment

d bands are narrow (band width about 3 eV) → their splitting strongly affects the magnetism

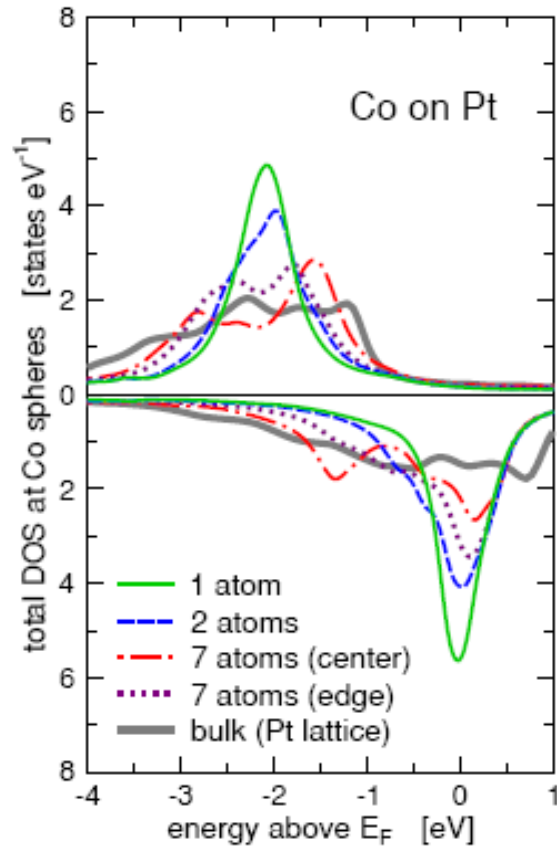
Redistribution of electrons in the different bands:
spin is smaller than in the gas phase case

	$n_{3d}+n_{4s}$	$n_{3d}(\downarrow)$	$n_{3d}(\uparrow)$	$n_{4s}(\downarrow)$	$n_{4s}(\uparrow)$	$h_{3d}(\downarrow)$	$h_{3d}(\uparrow)$	$\mu (\mu_B)$
Cr	6	2.7	2.7	0.3	0.3	2.3	2.3	0
Mn	7	3.2	3.2	0.3	0.3	1.8	1.8	0
Fe	8	4.8	2.6	0.3	0.3	0.2	2.4	2.2
Co	9	5.0	3.3	0.35	0.35	0.0	1.7	1.7
Ni	10	5.0	4.4	0.3	0.3	0.0	0.6	0.6
Cu	11	5.0	5.0	0.3	0.3	0.0	0.0	0

Filling of the \downarrow and \uparrow bands for 3d elements (n : electrons, h : holes)

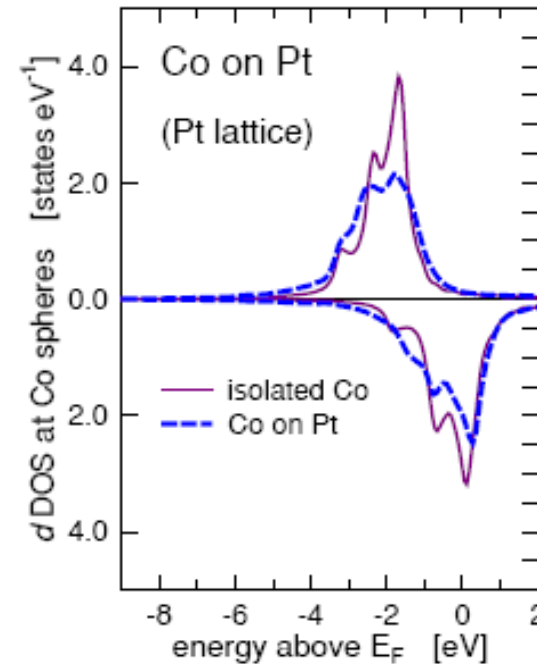


Size dependence



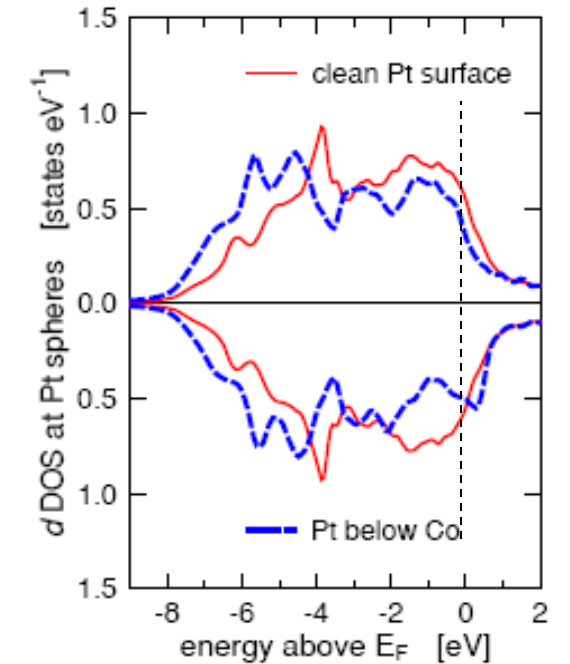
Increasing the cluster size (i.e. bond number), the DOS becomes broader

Hybridization effect



The Co DOS becomes broader due to hybridization (bond formation) with the Pt substrate

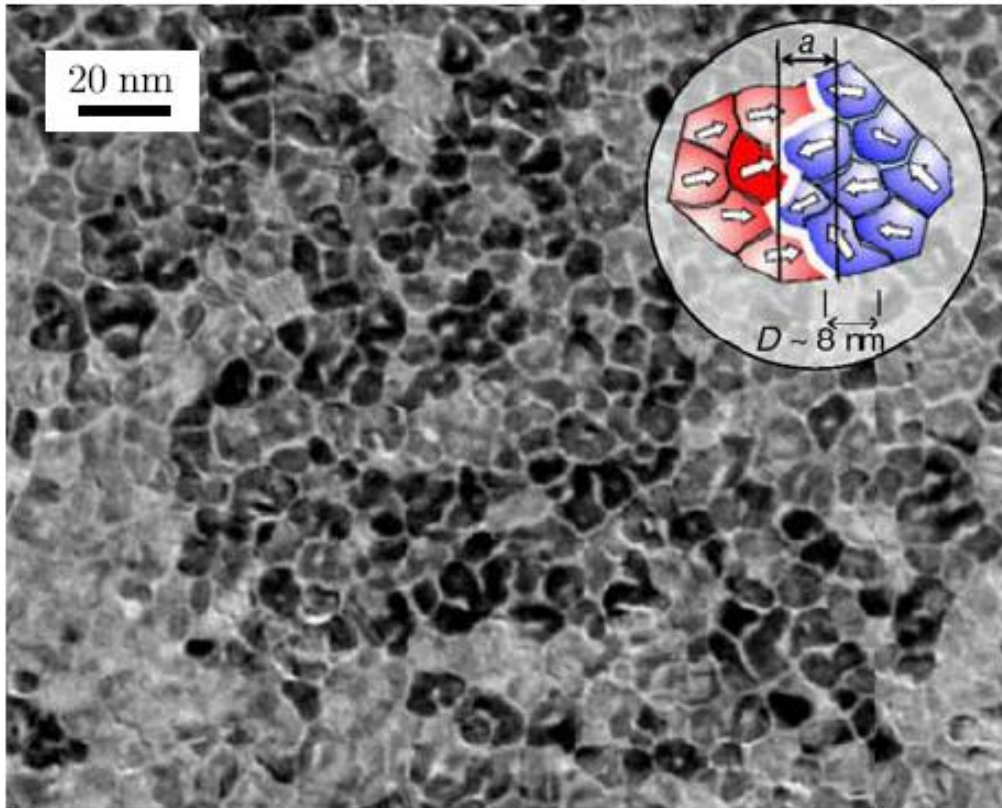
Co monolayer



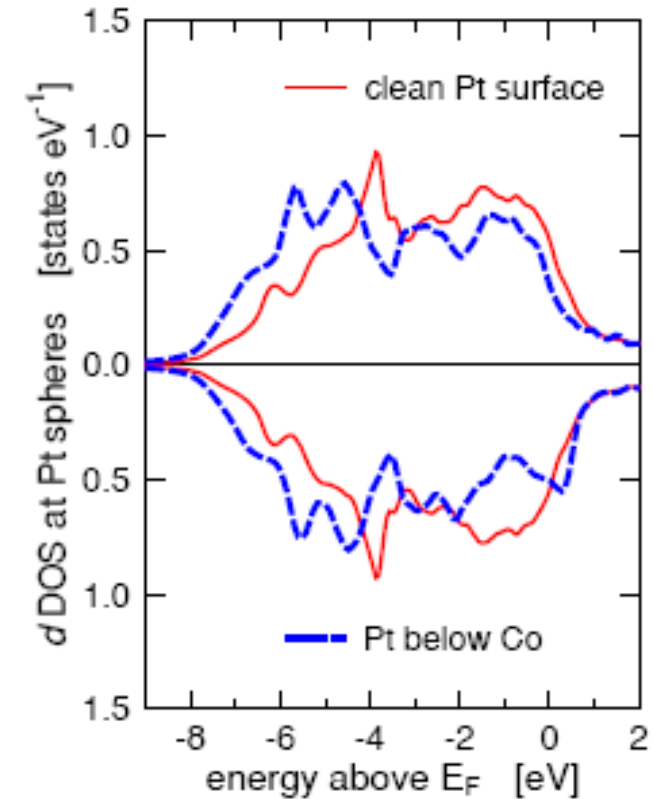
Asymmetry in the Pt DOS at E_F when covered by Co → induced magnetic moment



CoCrPt recording layer



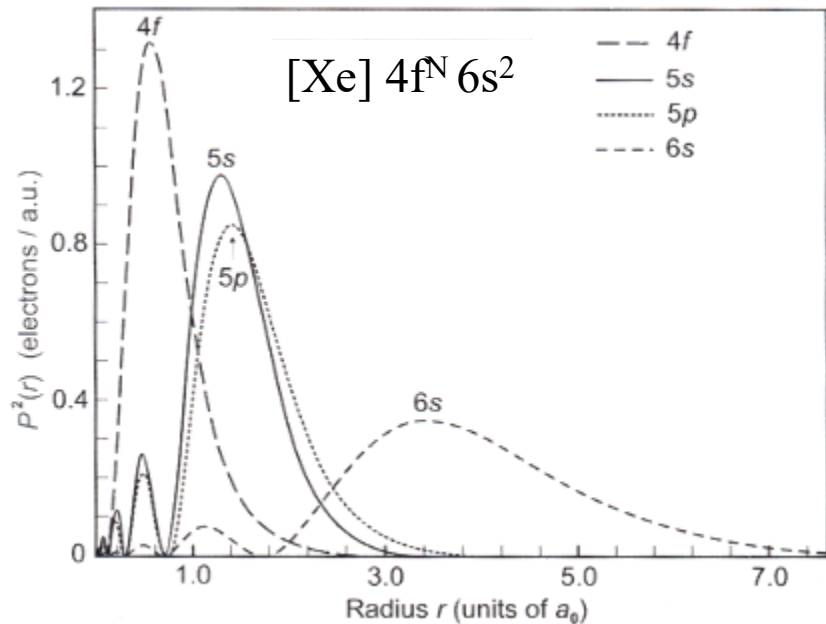
Pt covered by a Co monolayer



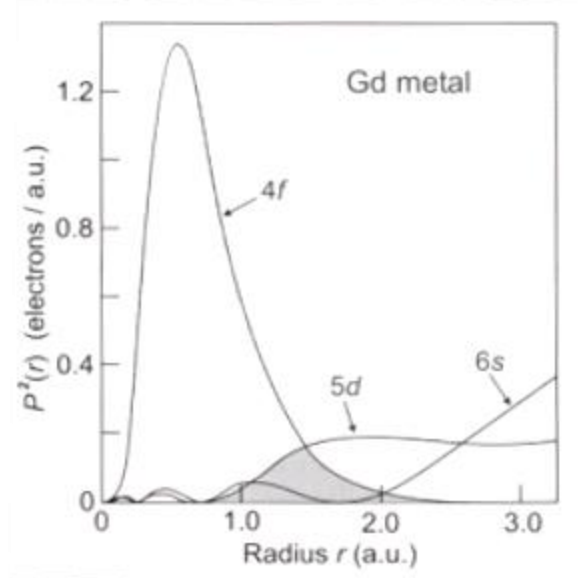
In a cluster the Pt DOS is narrower and due to hybridization with Co can develop a non negligible magnetic moment



Electronic configuration in atomic case: $[\text{Xe}] 4f^N 6s^2$
(exception for Gd: $[\text{Xe}] 4f^7 5d^1 6s^2$)



Radial distribution of the different orbitals



4f states are strongly localized at the core →
the 4f electrons do not participate to bonds

L unquenched

Magnetic moment defined by the 4f states
(Hund's rules hold)

The difference between atomic and bulk case is
the electronic configuration.

In bulk, rare earths have the configuration
 $[\text{Xe}] 4f^{N-1} 5d^1 6s^2$ with the 3 electrons in the
outer shells that can participate to bonds